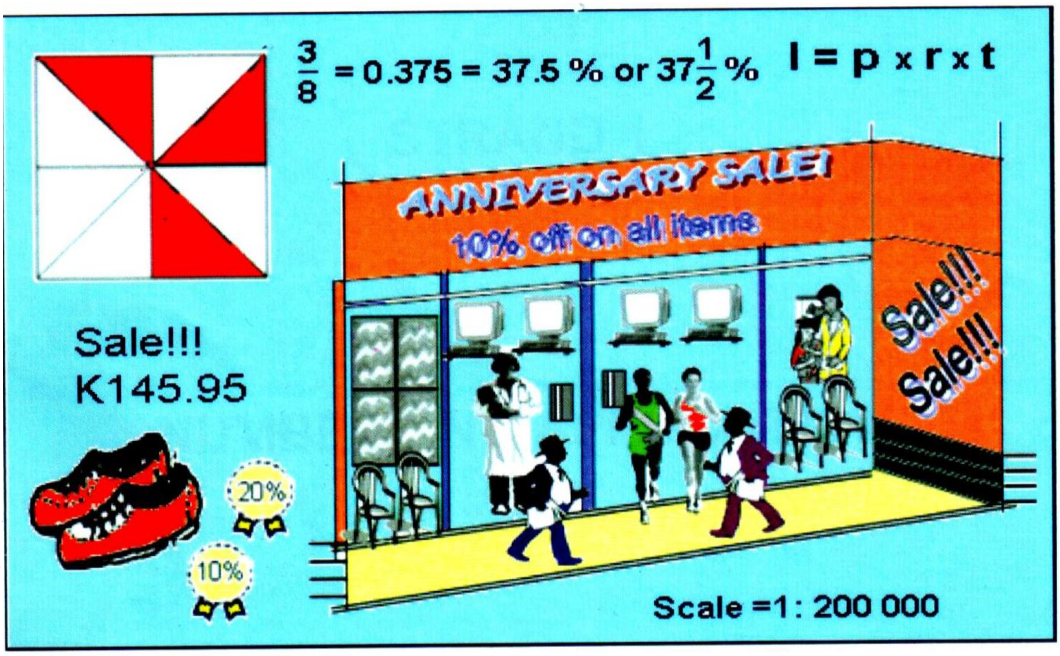
DEPARTMENT OF EDUCATION

GRADE 7

MATHEMATICS

STRAND 1



NUMBER AND APPLICATION

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PAPUA NEW GUINEA

GRADE 7

MATHEMATICS

STRAND 1

NUMBER AND APPLICATION

SUB-STRAND 1:	**FRACTIONS**
SUB-STRAND 2:	**DECIMALS**
SUB-STRAND 3:	**PERCENTAGES**
SUB-STRAND 4:	**RATIOS AND RATES**

Acknowledgements

We acknowledge the contributions of all Secondary and Upper Primary Teachers who in one way or another helped to develop this Course.

Special thanks to the Staff of the Mathematics Department of FODE who played active role in coordinating writing workshops, outsourcing lesson writing and editing processes involving selected teachers NCD.

We also acknowledge the professional guidance provided by the Curriculum Development and Assessment Division throughout the processes of writing and, the services given by the members of the Mathematics Review and Academic Committees.

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MR. DEMAS TONGOGO
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CONTENTS

SECRETARY'S MESSAGE

Achieving a better future by individual students and their families, communities or the nation as a whole, depends on the kind of curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum. The learning outcomes are student-centered with demonstrations and activities that can be assessed.

It maintains the rationale, goals, aims and principles of the national curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision by Flexible, Open and Distance Education as an alternative pathway of formal education.

The course promotes Papua New Guinea values and beliefs which are found in our Constitution and Government Policies. It is developed in line with the National Education Plans and addresses an increase in the number of school leavers as a result of lack of access to secondary and higher educational institutions.

Flexible, Open and Distance Education curriculum is guided by the Department of Education‟s Mission which is fivefold:

- to facilitate and promote the integral development of every individual
- to develop and encourage an education system that satisfies the requirements of Papua New Guinea and its people
- to establish, preserve and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make the education accessible to the poor and physically, mentally and socially handicapped as well as to those who are educationally disadvantaged.

The college is enhanced through this course to provide alternative and comparable pathways for students and adults to complete their education through a one system, two pathways and same outcomes.

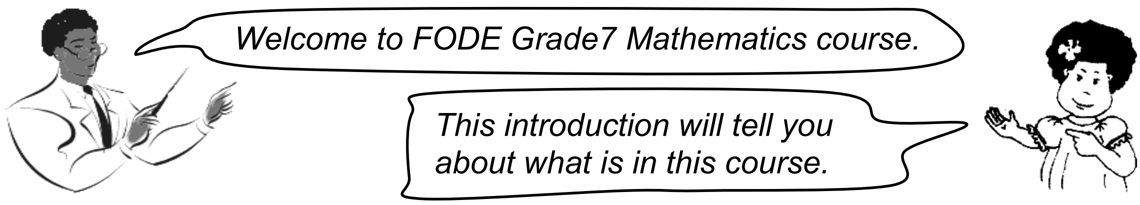
It is our vision that Papua New Guineans‟ harness all appropriate and affordable technologies to pursue this program.

I commend all the teachers, curriculum writers and instructional designers who have contributed towards the development of this course.

…………………………………………

DR. UKE KOMBRA, PhD
Acting Secretary for Education

COURSE INTRODUCTION



HOW TO STUDY YOUR GRADE 7 MATHEMATICS COURSE?

1. YOUR LESSONS

In Grade 7 Mathematics there are 6 books for you to study. Each book corresponds to one of the six strands of the course.

Strand 1: Number and Application
Strand 2: Space and Shapes
Strand 3: Measurements (1)
Strand 4: Measurements (2)
Strand 5: Chance and Data
Strand 6: Patterns and Algebra

Each strand is divided into 4 sub-strands and each sub-strand consists of 5 to 7 lessons.

Here is a list of the Strands in this Grade 7 course and the Sub-strands you will study:

STRAND NO.	STRAND	SUB-STRAND	TITLE
1	NUMBER AND APPLICATION	1	FRACTIONS
		2	DECIMALS
		3	PERCENTAGES
		4	RATIOS AND RATES
2	SPACE AND SHAPES	1	ANGLES
		2	SHAPES
		3	NETS
		4	TESSELLATION
3	MEASUREMENTS 1	1	LENGTH
		2	AREA
		3	VOLUME AND CAPACITY
		4	ESTIMATION
4	MEASUREMENTS 2	1	WEIGHTS
		2	TEMPERATURE
		3	TIME
		4	DIRECTIONS

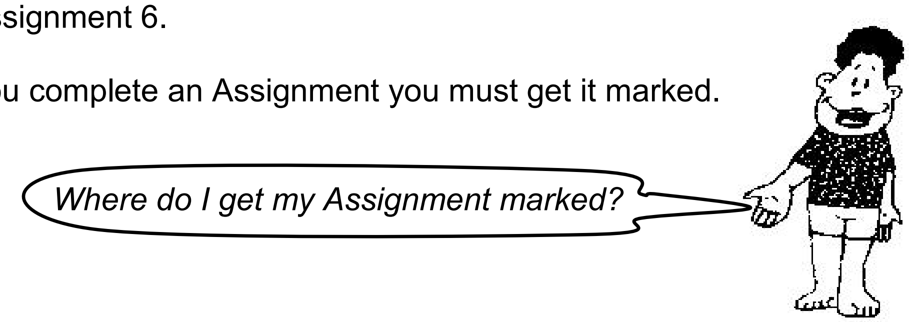
5	CHANCE AND DATA	1	STATISTICAL DATA
		2	SETS
		3	CHANCE AND PROBABILITY
		4	ERROR AND ACCURACY
6	PATTERNS AND ALGEBRA	1	NUMBER PATTERNS
		2	DIRECTED NUMBERS
		3	INDICES
		4	ALGEBRA

2. YOUR ASSIGNMENTS

In this course you will also do six ASSIGNMENTS.

You will study Strand 1 and do Assignment 1 at the same time. Then you will study Strand 2 and do Assignment 2 at the same time, and so on up to Strand 6 and Assignment 6.

When you complete an Assignment you must get it marked.

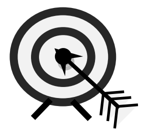


- **Students in a Registered Study Centre** must give their completed Assignments to their Supervisor for marking.
- **Students who study at Home by themselves but who live in the Provinces** must send their completed Assignments to their Provincial Centres for marking.

3. LESSON ICONS

Below are the icons used by FODE in its courses.


Lesson Introduction


Aims


Summary


Activities/Practice Exercises

STRAND 1: NUMBER AND APPLICATION

Dear Student,

This is the first Strand of the Grade 7 Mathematics Course.
It is based on the NDOE Upper Primary Mathematics Syllabus and Curriculum Framework for Grade 7.

This Strand consists of four Sub-strands:

Sub-strand 1: Fractions
Sub-strand 2: Decimals
Sub-strand 3: Percentages
Sub-strand 4: Ratios and Rates

Sub-strand 1 – **Fractions** – You will solve problems in fractions requiring any of the four operations including mixed numbers.

Sub-strand 2 – **Decimals** – You will learn to use decimals in solving problem sets in familiar contexts.

Sub-strand 3 – **Percentages** – You will learn to use percentages in a variety of real life situations.

Sub-strand 4 – **Ratios and Rates** – You will learn to convert ratios to fractions and decimals.

You will find that each lesson has reading materials to study, worked examples to help you, and a Practice Exercise. The answers to practice exercise are given at the end of each sub-strand.

All the lessons are written in simple language with comic characters to guide you and many worked examples to help you. The practice exercises are graded to help you to learn the process of working out problems.

We hope you enjoy learning the material in this Strand.

All the best!

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Strand.

Step 1: Start with SUB-STRAND 1 Lesson 1 and work through it.

Step 2: When you complete Lesson 1, do Practice Exercise 1.

Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of SUB-STRAND 1.

Step 4: Then, revise Lesson 1 and correct your mistakes, if any.

Step 5: When you have completed all these steps, tick the Lesson check-box on the Contents Page like this:

[√] Lesson 1: Comparing Fractions

Then go on to the next Lesson. Repeat the process until you complete all the lessons in Topic 1.

> As you complete each lesson, tick the check-box for that lesson, on the Contents Page, like this [√] .This helps you to check on your progress.

Step 6: Revise the Sub-strand using Sub-strand 1 Summary, then do Sub-strand Test 1 in Assignment 1.

Then go on to the next Sub-strand. Repeat the process until you complete all the four Sub-strands in Strand 1.

<u>Assignment:</u> (Four Sub-strand Tests and a Strand Test)
When you have revised each Sub-strand using the Sub-strand Summary, do the Sub-strand Test in your Assignment Book. The Course Book tells you when to do each Sub-strand Test.

When you have completed the four Sub-strand Tests, revise well and then attempt the Strand Test. The Assignment Book tells you when to do the Strand Test.

> The Sub-strand Tests and the Strand Test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment Book will count towards your final mark. If you score less than 50%, you will repeat that Assignment Book.

Remember, if you score less than 50% in three Assignments, your enrolment will be cancelled. So, work carefully and make sure that you pass all of the Assignments.

SUB-STRAND 1

FRACTIONS

SUB-STRAND 1: FRACTIONS

Introduction

You have already learned some things about fractions in Grade 6. In this Sub-strand, you will revise some of the things you did in Grade 6 and you will also learn many new things about fractions.

Whilst our exposure to the number system so far has dealt mostly with whole numbers, we also come across figures from time to time where the use of fractions cannot be avoided. Such terms as “halves”, “fourths” or “quarters” are used with reference to measurements like “quarter to twelve”, “half a kilometre”, “three-quarters full” and so on.

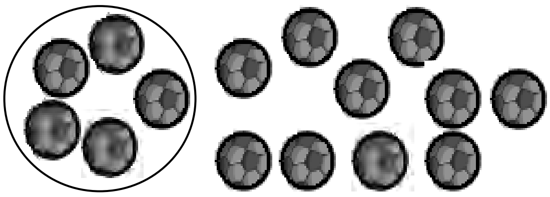
Consider the following questions:

What is three parts out of ten equal parts?

An apple cake was divided equally among three children. What part did each child get?

What part of a Kina is ten toea?

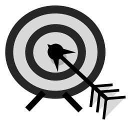
What part of the group has been circled?



The quantities above cannot be represented by whole numbers. We use fractions to represent them. Fractions supplement whole numbers which are sometimes inadequate for accurate measurement and computation. If a unit is divided into two or more parts, each equal part is the fractional part of the whole unit which is usually expressed in the form of a fraction.

Lesson 1: Comparing Fractions

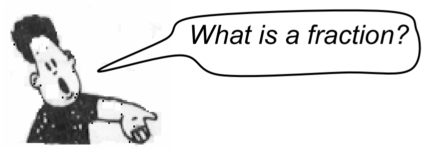
You learnt some things about fractions in your grade 6 Mathematics.

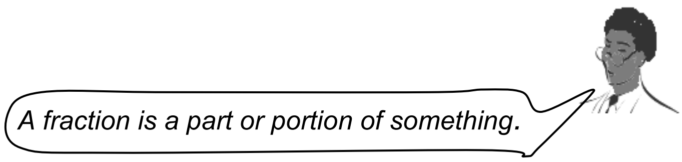


In this lesson you will:

- revise the meaning of fractions
- identify and discuss the kinds of fractions.
- differentiate between similar and dissimilar fractions
- compare similar and dissimilar fractions
- arrange fractions from lowest to highest or vice versa

First you will revise the meaning of fractions and the parts of fractions.

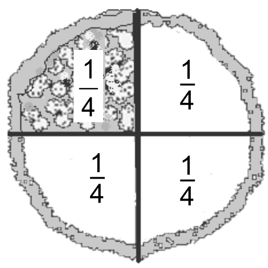




When we divide something into parts, each part is called a fraction of the whole part.

For example:

A mother bought a whole cake and shared it equally among her four children. The mother cut the cake into four equal parts and one part was given to each child, which was one-fourth of the whole cake as shown below.



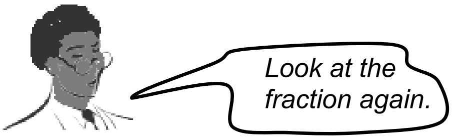
Shaded Part = $\mathbf{\frac{1}{4}}$ means **one** part of the cake which was divided into 4 equal parts.

Un-shaded Part = $\mathbf{\frac{3}{4}}$ means **three** parts of the cake which was divided into 4 equal parts.

Each of the 4 equal pieces is a <u>fraction</u> of the cake.

We say that each piece is one fourth of the cake.

We write one fourth like this: $\frac{1}{4}$



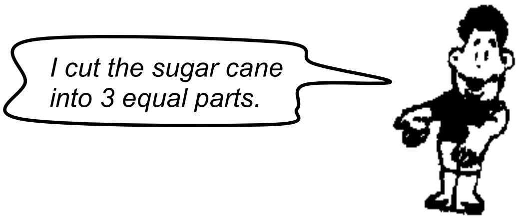
$\boxed{\dfrac{1}{4}}$

1 means one of the equal pieces

4 means the total number of equal pieces.

Now look at the other example.

Here is a sugar cane



Here are the equal parts

Each of the 3 equal parts is a <u>fraction</u> of the whole piece.

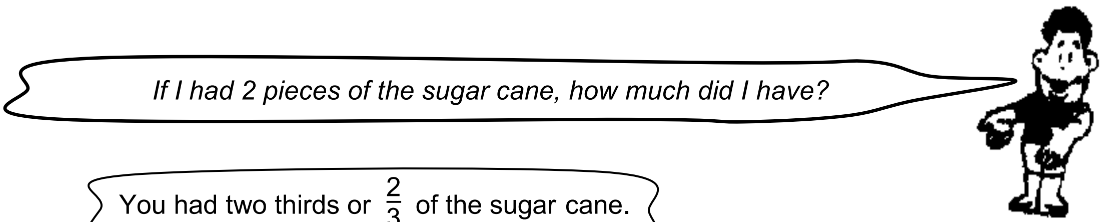
We say that each piece is one third of the whole piece.

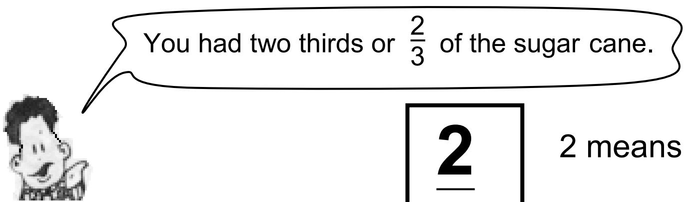
We write one third like this: $\dfrac{1}{3}$

$\boxed{\dfrac{1}{3}}$

1 means one of the equal pieces

3 means the total number of equal pieces.





$\boxed{\dfrac{2}{3}}$

2 means two of the equal pieces

3 means the total number of equal pieces.

The two parts of a fraction have special names. You learnt the names in Grade Six.

The top part of any fraction is referred to as the **numerator.** It tells us how many of the equal parts are to be taken.

The bottom part of any fraction is referred to as the **denominator**. It tells us how many equal parts the whole is divided into.

Look at the next page and see how common fractions are classified.

Common fractions are classified into the following classes:

1) **Unit fractions** are fractions where the numerator is always one (1).

Examples: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, etc.

2) **Proper fractions** are fractions whose *numerators are less than the denominators.* These fractions indicate values less than one (1).

Examples: $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$, etc.

3) **Improper fractions** are fractions whose *numerators are greater than or equal to the denominators.* These fractions indicate values equal to or greater than one (1).

Examples: $\frac{3}{2}, \frac{7}{3}, \frac{11}{4}, \frac{12}{5}$, etc.

4) **Fractions Written in Mixed Form or Mixed Numbers** are fractions written as a sum of a whole number and a fraction. An improper fraction can also be written as a mixed number or vice versa.

Examples: $1\frac{1}{2}, 2\frac{1}{3}, 2\frac{3}{4}, 2\frac{2}{5}$, etc

5) **Similar Fractions** are two or more fractions whose denominators are the same.

Examples: $\frac{3}{12}$, $\frac{5}{12}$ and $\frac{7}{12}$, $\frac{2}{5}$ and $\frac{3}{5}$ etc

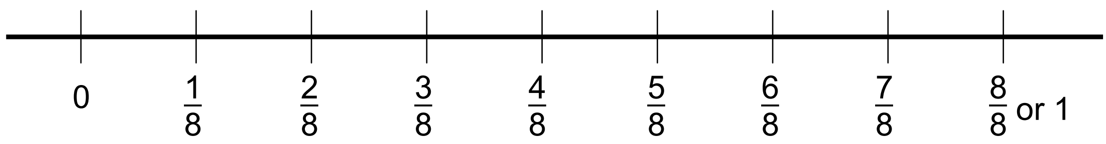
6) **Dissimilar fractions** are two or more fractions whose numerators and denominators are different.

Examples: $\frac{2}{8}$ and $\frac{3}{7}$; $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{8}$

In this lesson, you will also learn how to compare fractions.

Comparing Fractions

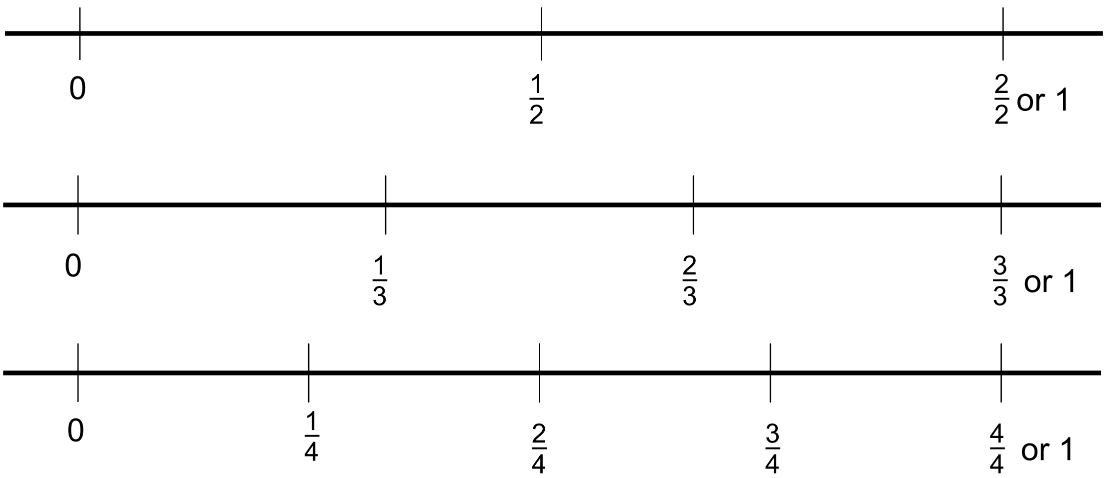
Below is a line segment showing points corresponding to 0 and 1. The distance is divided into eight equal parts.



From 0, compare the distances to $\frac{3}{8}$ and $\frac{5}{8}$; $\frac{1}{8}$ and $\frac{3}{8}$; $\frac{3}{8}$ and $\frac{7}{8}$.

How do you compare fractions with the same denominator?

Study the diagram below to answer the question.



In the diagram above you find three line segments which are divided into 2, 3 and 4 equal parts, respectively.

With 0 as the starting point, compare the distances to $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. Which of the points is nearest to 0? farthest from 0?

- **When two fractions have the same denominator, the fraction with the smaller numerator is less than the fraction with a greater numerator.**
- **When two fractions have the same numerator, the fraction with a greater denominator is less than the fraction with a smaller denominator.**

Here are some examples.

In each of the following pairs of fractions, which is greater?

a. $\frac{8}{13}$ or $\frac{7}{13}$ **answer:** $\mathbf{\frac{8}{13}}$

b. $\frac{5}{12}$ or $\frac{8}{12}$ **answer:** $\mathbf{\frac{8}{12}}$

c. $\frac{14}{18}$ or $\frac{14}{7}$ **answer:** $\mathbf{\frac{14}{7}}$

d. $\frac{75}{10}$ or $\frac{75}{20}$ **answer:** $\mathbf{\frac{75}{10}}$

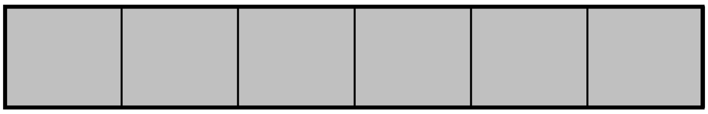
d. $\frac{7}{9}$ or $\frac{5}{9}$ **answer:** $\mathbf{\frac{7}{9}}$

NOW GO ON TO DO THE PRACTICE EXERCISE 1.

Practice Exercise 1

---

1. This rectangle is divided into 6 equal parts.



   (a) Each part of the rectangle is ________________ of the whole rectangle.
   (Write a fraction.)

   (b) Five parts of the rectangle is ________________ of the whole rectangle.
   (Write a fraction.)

---

2. Here is a fraction: $\frac{5}{8}$

   (a) The numerator of this fraction is ____________.

   (b) The denominator of this fraction is ___________.

3. What kind of fractions are the following:

   (a) $\frac{5}{8}$ ________________   (b) $\frac{5}{2}$ ________________

   (c) $\frac{1}{12}$ ________________   (d) $5\frac{3}{4}$ ________________

   (e) $\frac{2}{7}$ ________________

---

4. In each of the following pairs of fractions, which is the smaller one?

   (a) $\frac{7}{11}$ or $\frac{8}{11}$ (b) $\frac{5}{2}$ or $\frac{5}{5}$

   (c) $\frac{2}{7}$ or $\frac{5}{7}$ (d) $\frac{14}{18}$ or $\frac{7}{18}$

   (e) $\frac{10}{25}$ or $\frac{10}{3}$

5. Arrange the following fractions in increasing order. The first one is done for you.

(a) $\frac{1}{8}$, $\frac{1}{36}$, $\frac{1}{24}$, $\frac{1}{5}$, $\frac{1}{40}$ = $\mathbf{\frac{1}{40}, \frac{1}{36}, \frac{1}{24}, \frac{1}{8}, \frac{1}{5}}$

(b) $\frac{3}{4}$, $\frac{3}{21}$, $\frac{3}{7}$, $\frac{3}{8}$, $\frac{3}{5}$

(c) $\frac{7}{22}$, $\frac{3}{22}$, $\frac{19}{22}$, $\frac{4}{22}$, $\frac{11}{22}$

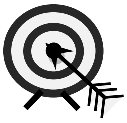
(d) $\frac{5}{8}$, $\frac{7}{8}$, $\frac{3}{8}$, $\frac{2}{8}$, $\frac{1}{8}$

(e) $\frac{6}{7}$, $\frac{6}{19}$, $\frac{6}{25}$, $\frac{6}{17}$, $\frac{6}{9}$

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 2: Renaming Fractions

You learnt the names of the parts of a fraction in Lesson 1.

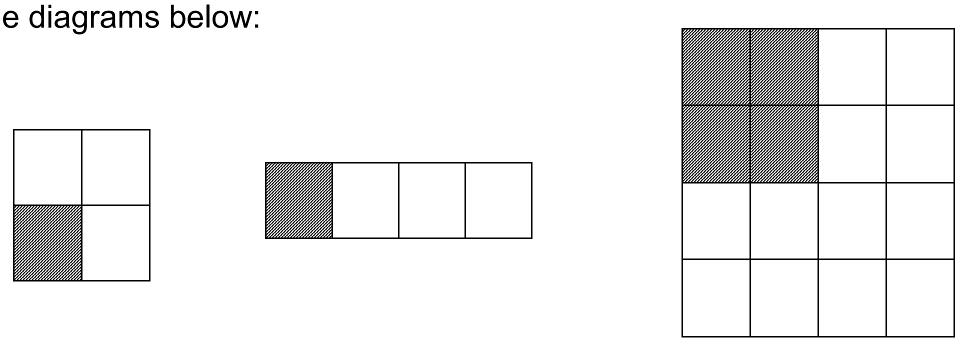


In this lesson you will:

- describe equivalent fractions
- find equivalent fractions
- change fractions to their highest and lowest terms.

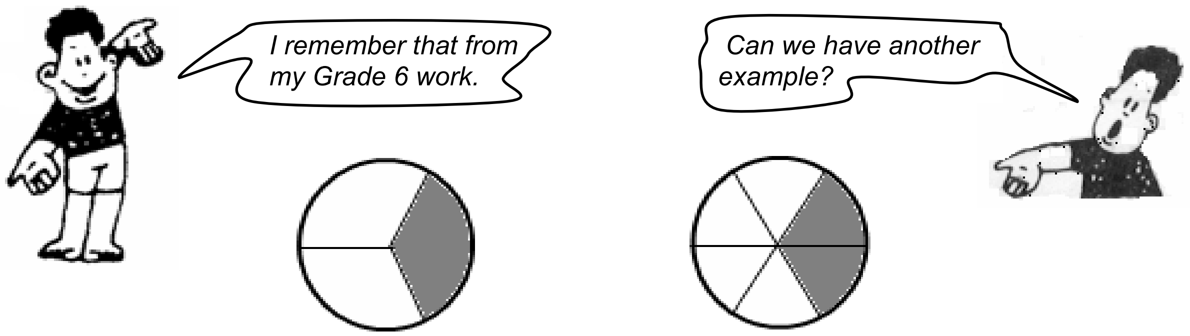
First, you will learn the meaning of equivalent fractions.

Look at the diagrams below:



Each diagram shows **one quarter** ($\frac{1}{4}$) shaded.

Remember that numbers like $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ are called **fractions**.



The shaded area is $\frac{1}{3}$ The shaded area is $\frac{2}{6}$

But both areas have the same size.

$$\mathbf{\frac{1}{3} = \frac{2}{6}}$$

One third and two sixth are **equivalent fractions**.

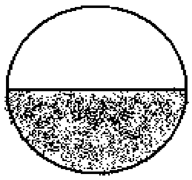
Look at the examples on the next page.

Example 1

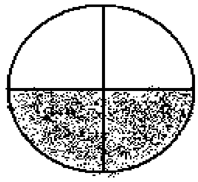
David and Eope have a cake each.

David and Eope ate the same amount.

Look at the diagrams of the cakes below:



David‟s cake has been cut into 2 equal parts.



Eope‟s cake has been cut into 4 equal parts.

So, one half is the same as two quarters.

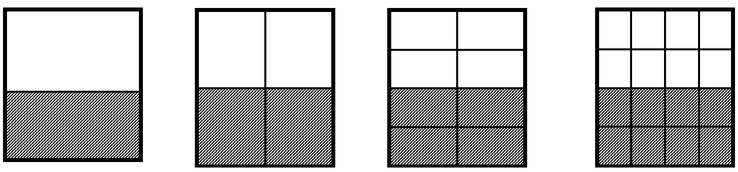
$$\frac{1}{2} = \frac{2}{4}$$

One half and two quarters are EQUIVALENT FRACTIONS.

Now look at the other example below.

Example 2

Look at these diagrams.

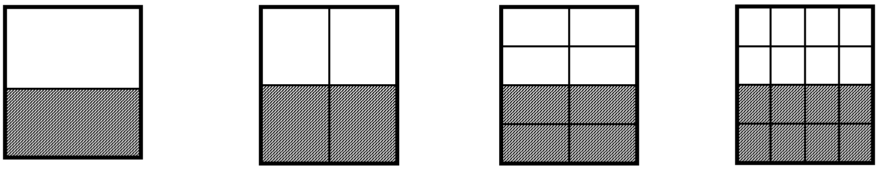


Each large shape is of the same size.

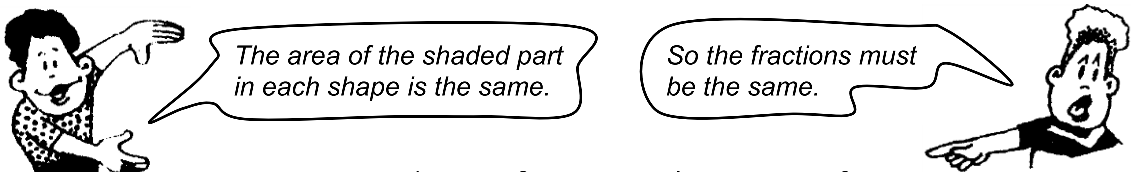
Each shape is divided into a different number of parts.

An equal amount of each shape has been shaded.

Look at the shapes again.



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{4}{8}$ $\frac{8}{16}$



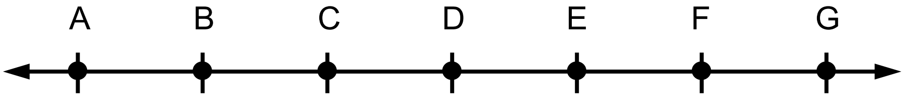
So, $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$

These are all Equivalent Fractions. They are all equal to each other. You will notice that in each of the equivalent fractions above the numerator is exactly half of the denominator.

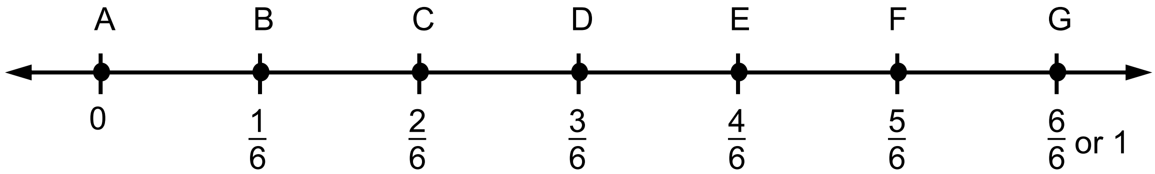
Equivalent fractions are fractions that have the same meaning or value.

Below you can see divided line segments showing the same information:

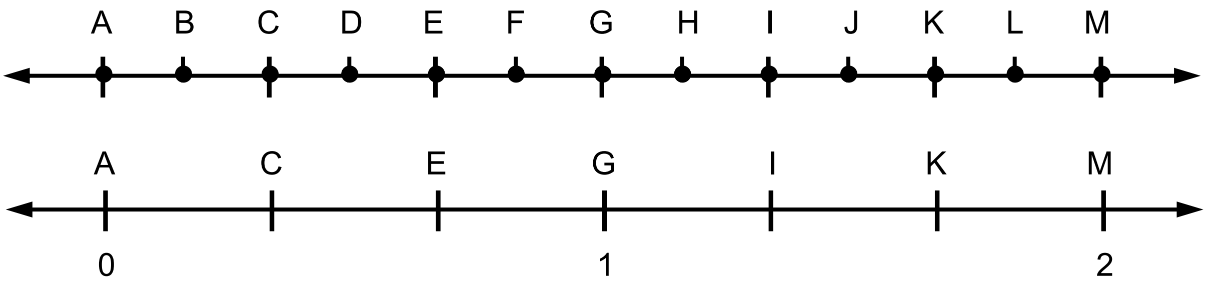
This line segment is divided into six equal parts. Therefore, each part is $\frac{1}{6}$ of the segment.



Match point A with 0 and point G with 1. Look at how the points can be matched with different fractions as shown.

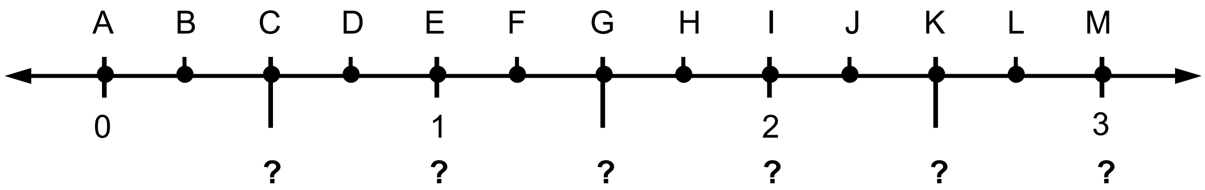


Line segment AG has been extended so that point M matches with number 2.



What fractions may be matched with the points H, I, J, K, L, and M?

If the same line segment AM is divided into three parts, what fractions match with points C, E, G, I, K, and M?



The examples show that

$$\frac{1}{3} = \frac{2}{6} \qquad 1 = \frac{6}{6} = \frac{3}{3} \qquad \frac{4}{3} = \frac{8}{6}$$

$$\frac{2}{3} = \frac{4}{6} \qquad 2 = \frac{12}{6} = \frac{6}{3} \qquad \frac{5}{3} = \frac{10}{6}$$

You can also divide the segment into more parts to obtain other fractions for the same points. Thus several fractions may have the same value, meaning they are equivalent.

You learned that $\frac{1}{3} = \frac{2}{6}$. Remember also that $\frac{2}{6} = \frac{1 \times 2}{3 \times 2}$ by factoring.

You also learned that $\frac{2}{3} = \frac{4}{6}$ and that $\frac{4}{6} = \frac{2 \times 2}{3 \times 2}$.

The examples show that you can obtain equivalent fractions by multiplying or dividing the numerator and denominator of a fraction by the same number (or common factor).

Example 1: $\frac{4}{3} = \frac{4 \times 2}{3 \times 2}$ or $\frac{8}{6}$

$\frac{4}{3} = \frac{4 \times 3}{3 \times 3}$ or $\frac{12}{9}$

$\frac{4}{3} = \frac{4 \times 4}{3 \times 4}$ or $\frac{16}{12}$

The fractions $\frac{8}{6}$, $\frac{12}{9}$, $\frac{16}{12}$ and $\frac{4}{3}$ are equivalent fractions; the simplest is $\frac{4}{3}$ since 4 and 3 have no common factor except 1.

So, we say that $\frac{4}{3}$ is in simplest form.

Example 2

$$\frac{10}{6} = \frac{5 \times 2}{3 \times 2} \text{ or } \frac{5}{3}$$

$$\frac{20}{12} = \frac{5 \times 4}{3 \times 4} \text{ or } \frac{5}{3}$$

$$\frac{25}{15} = \frac{5 \times 5}{3 \times 5} \text{ or } \frac{5}{3}$$

The simplest form, therefore is $\frac{5}{3}$.

Equivalent fractions may be obtained by multiplying or dividing the numerator and denominator of the given fraction by the same number.

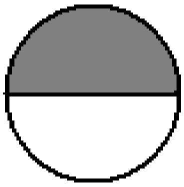
A fraction is in its simplest form when the numerator and denominator have no common factor except 1.

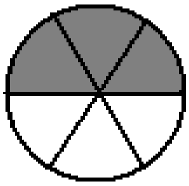
NOW DO PRACTICE EXERCISE 2

Practice Exercise 2

1) Write down the equivalent fractions represented by these diagrams.
The first one is done for you.

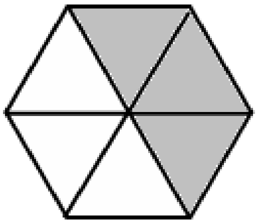
(a)

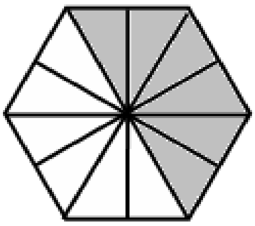




$\frac{1}{2}$ = $\frac{3}{6}$

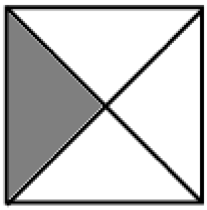
(b)

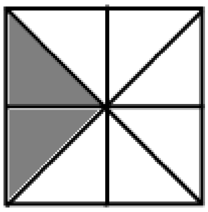




$\boxed{}$ = $\boxed{}$

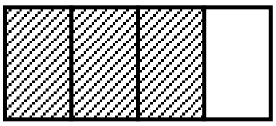
(c)

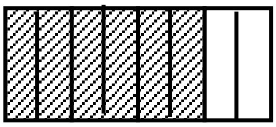




$\boxed{}$ = $\boxed{}$

(d)





$\boxed{}$ = $\boxed{}$

(2) Supply the missing number.

(a) $\frac{1}{5} = \frac{2}{\square}$

(b) $\frac{2}{5} = \frac{4}{\square}$

(c) $\frac{3}{5} = \frac{\square}{10}$

(d) $\frac{4}{5} = \frac{\square}{10}$

(e) $1 = \frac{\square}{10} = \frac{\square}{5}$

---

3) Change the following fractions to simplest form.

(a) $\frac{7}{42} =$

(b) $\frac{12}{40} =$

(c) $\frac{18}{27} =$

(d) $\frac{36}{48} =$

(e) $\frac{14}{35} =$

(f) $\frac{21}{30} =$

(g) $\frac{45}{54} =$

(h) $\frac{24}{72} =$

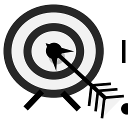
(i) $\frac{33}{44} =$

(j) $\frac{26}{65} =$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 3: Renaming Mixed Numbers

In the previous lesson, you learnt how to change a fraction to its simplest forms.



In this lesson you will:

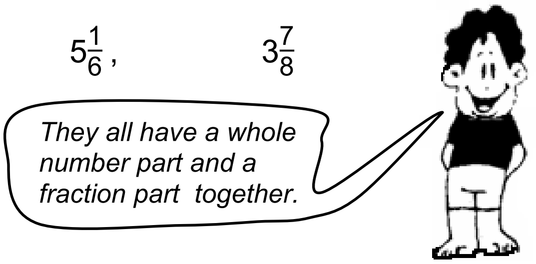
- differentiate between mixed numbers and improper fractions
- change mixed numbers to improper fractions and vice versa

First you will learn about mixed numbers.

Here are some examples of mixed numbers:

$1\frac{1}{2}$, $2\frac{3}{4}$, $5\frac{1}{6}$, $3\frac{7}{8}$

What do you notice about all of them?

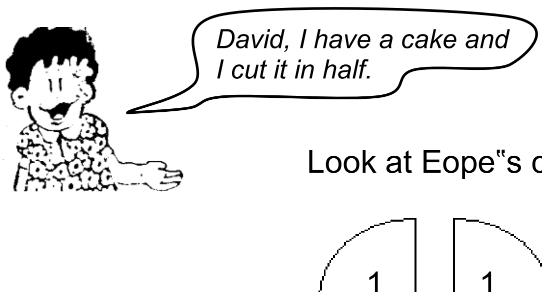


Mixed numbers are fractions written as the sum of a whole number and a fraction.

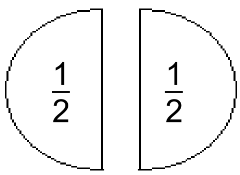
$1\frac{1}{2}$ is the same as $1 + \frac{1}{2}$

$2\frac{3}{4}$ is the same as $2 + \frac{3}{4}$

$5\frac{1}{6}$ is the same as $5 + \frac{1}{6}$



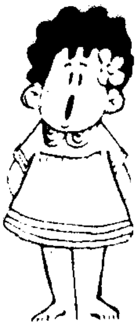
Look at Eope‟s cake:



Eope has 2 halves in one whole

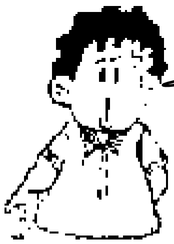
So, $1 = \frac{2}{2}$

1 whole = 2 halves



$$1 = \frac{7}{7}$$

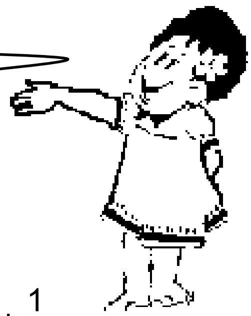
1 whole = 7 sevenths



$$1 = \frac{5}{5}$$

1 whole = 5 fifths

David and Eope like to test each other with their Mathematics work. Maybe you can find a friend who could test you.



$1\frac{1}{3}$ is the same as: $1 + \frac{1}{3}$

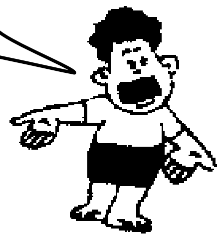
$$\text{So, } 1\frac{1}{3} = 1 + \frac{1}{3} = \frac{3}{3} + \frac{1}{3}$$

$$1\frac{1}{3} = \frac{4}{3}$$

Here are some more examples showing you how to change mixed numbers into fractions.

Example 1

Change $2\frac{7}{8}$ into a fraction.



$$2\frac{7}{8} = 2 + \frac{7}{8}$$

$$\text{So, } 2\frac{7}{8} = \frac{16}{8} + \frac{7}{8}$$

change into eighths

Both numbers have the same denominator.

$$= \frac{16 + 7}{8}$$

$$= \mathbf{\frac{23}{8}} \quad \textbf{ANSWER}$$

Example 2

Change $3\frac{1}{4}$ into fractions.

$$3\frac{1}{4} = 3 + \frac{1}{4}$$

$$= \frac{12}{4} + \frac{1}{4}$$

$$= \frac{12 + 1}{4}$$

$$= \mathbf{\frac{13}{4}} \quad \textbf{ANSWER}$$

$1 = \frac{4}{4}$, so

$3 = \frac{3 \times 4}{4} = \frac{12}{4}$

Example 3

Change $3\frac{1}{2}$ into a fraction.

$$3\frac{1}{2} = 3 + \frac{1}{2}$$

$$= \frac{6}{2} + \frac{1}{2}$$

$$= \mathbf{\frac{7}{2}} \quad \textbf{ANSWER}$$

$1 = \frac{2}{2}$, so

$3 = \frac{3 \times 2}{2} = \frac{6}{2}$

Example 4

Change $5\frac{2}{3}$ into a fraction.

$$5\frac{2}{3} = 5 + \frac{2}{3}$$

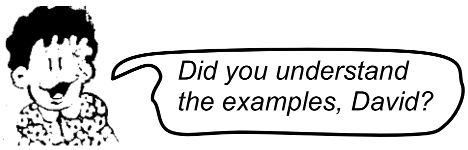
$$= \frac{15}{3} + \frac{2}{3}$$

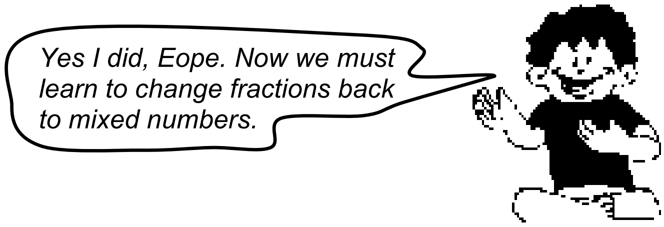
$$= \mathbf{\frac{17}{3}} \quad \textbf{ANSWER}$$

$$\left(\begin{array}{l} 1 = \frac{3}{3}, \text{ so} \\ 5 = \frac{5 \times 3}{3} = \frac{15}{3} \end{array} \right)$$

Note that the fractions $\frac{23}{8}$, $\frac{13}{4}$, $\frac{7}{2}$, $\frac{17}{3}$ have numerators that are greater than the denominators. Such fractions are called **improper fractions or fractions greater than one**.

Improper fractions are fractions whose numerators are greater than or equal to the denominators.





Example 5

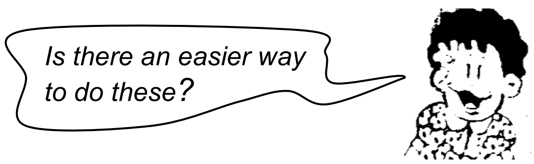
Change the fraction $\frac{5}{2}$ into mixed number.

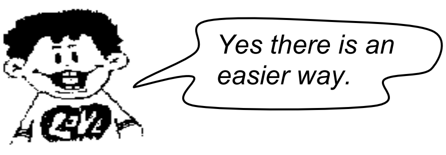
$\frac{5}{2}$ means <u>5 halves</u>

$$\left(\begin{array}{l} 2 \text{ halves} = 1 \text{ whole} \\ 4 \text{ halves} = 2 \text{ whole} \end{array} \right)$$

So, $\frac{5}{2}$ is the same as 2 whole ones plus one half.

$$\frac{5}{2} = 2\frac{1}{2}$$



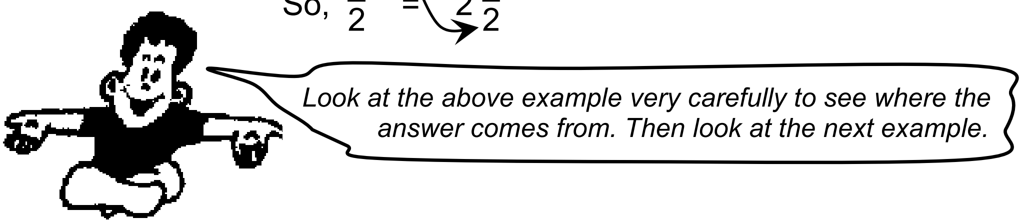


$\frac{5}{2}$ also means $5 \div 2$.

Write it like this:

$$\begin{array}{r|l} & 2 \text{ remainder } 1 \\ \hline 2 & 5 \end{array}$$

So, $\frac{5}{2} = 2\frac{1}{2}$



Example 6

Change $\frac{7}{3}$ into a mixed number.

$\frac{7}{3}$ also means: $7 \div 3$

$$\begin{array}{r|l} & 2 \text{ remainder } 1 \\ \hline 3 & 7 \end{array}$$

So, $\frac{7}{3} = 2\frac{1}{3}$

Example 7

Change $\frac{15}{4}$ into a mixed number.

$\frac{15}{4}$ also means: $15 \div 4$

$$\begin{array}{r|l} & 3 \text{ remainder } 3 \\ \hline 4 & 15 \end{array}$$

So, $\frac{15}{4} = 3\frac{3}{4}$

NOW DO PRACTICE EXERCISE 3

Practice Exercise 3

A. Change these mixed numbers into improper fractions. The first one has been started for you.

1. $1\frac{3}{4} = 1 + \frac{3}{4}$

   $= \frac{\quad}{4} + \frac{3}{4}$

   $= \frac{\quad + \quad}{4}$

   $=$

2. $2\frac{1}{5} =$

3. $8\frac{1}{2} =$

4. $3\frac{2}{3} =$

5. $4\frac{2}{7} =$

6. $5\frac{5}{6} =$

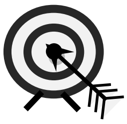
B. Change the following improper fractions to mixed numbers:

1. $\frac{7}{2}$ =	2. $\frac{4}{3}$ =
3. $\frac{9}{8}$ =	4. $\frac{7}{6}$ =
5. $\frac{11}{2}$ =	6. $\frac{5}{3}$ =

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 4: Addition of Fractions and Mixed Numbers

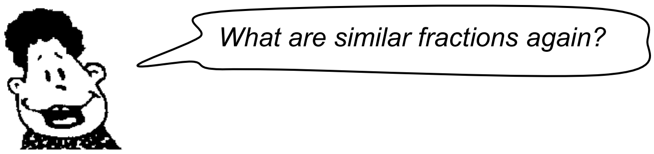
You learned to rename fractions and mixed numbers in the previous lessons.



In this lesson you will:

- add fractions and mixed numbers with the same denominator
- add fractions and mixed numbers with different denominators

First we will learn to add similar fractions.

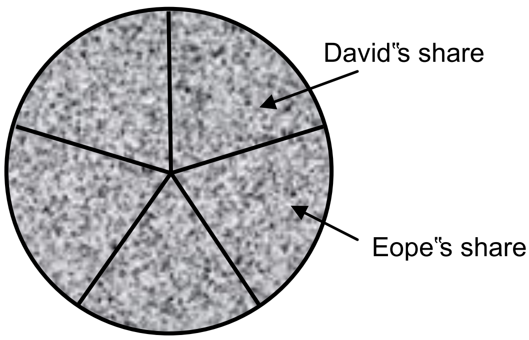


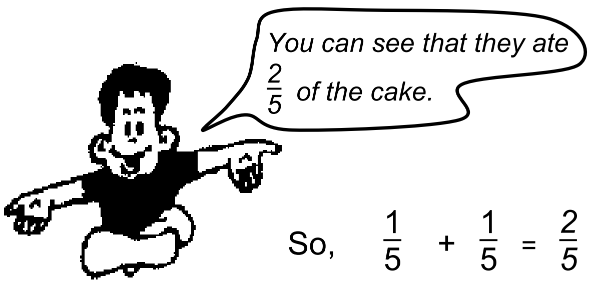
Similar fractions are fractions with the same denominator.

Example 1

David and Eope have a cake. David and Eope ate $\frac{1}{5}$ each . How much of the cake did they eat?

Here is the cake.





So, $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

Look at the other examples on the next page.

Example 2

Add: $\frac{3}{8} + \frac{2}{8}$ ← *These fractions have the same denominators.*

So, we write: $\frac{3}{8} + \frac{2}{8} = \frac{3 + 2}{8}$ ← Add the numerators

$= \mathbf{\frac{5}{8}}$ **ANSWER**

To add similar fractions, add the numerators over the same denominator.

We can also add two or more fractions easily if their denominators are the same.

Look at the examples below.

Example 3

Add: $\frac{2}{9} + \frac{1}{9} + \frac{4}{9} = \boxed{}$

All the denominators are the same.

So, we write: $\frac{2}{9} + \frac{1}{9} + \frac{4}{9} = \frac{2 + 1 + 4}{9}$

$= \mathbf{\frac{7}{9}}$ **ANSWER**

Example 4

Add: $\frac{1}{8} + \frac{2}{8} + \frac{3}{8} = \boxed{}$

All the denominators are the same.

So, we write: $\frac{1}{8} + \frac{2}{8} + \frac{3}{8} = \frac{1 + 2 + 3}{8}$

$= \frac{6}{8}$ $\quad \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$

$= \mathbf{\frac{3}{4}}$ **ANSWER**

Example 5

Add: $\frac{4}{5} + \frac{2}{5} + \frac{1}{5} = \square$

All the denominators are the same.

So, we write: $\frac{4}{5} + \frac{2}{5} + \frac{1}{5} = \frac{4 + 2 + 1}{5}$

$= \frac{7}{5}$ (change to mixed number)

$= \mathbf{1\frac{2}{5}}$ **ANSWER**

When adding fractions, you must always look at your answer and ask two questions:

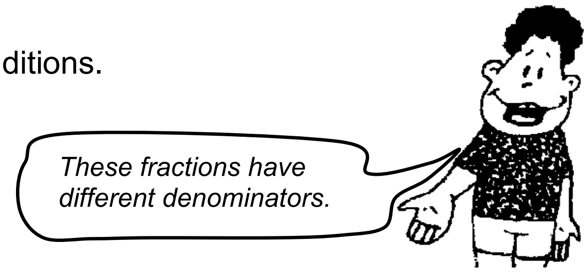
1. Can I simplify the answer any more?
2. Can I change the answer to a mixed number?

If the answer is NO for these two questions, then you have your final answer.

If the answer is YES for these two questions, then you have to do the following:

1. If the denominator on the answer is smaller than the numerator, you must change the fraction to a mixed number.
2. If the denominator on the answer is bigger than the numerator, you must write the fraction in its lowest term.

Now we will have a look at some harder additions.



Example 6

Add these fractions: $\frac{1}{3} + \frac{1}{4} = \square$

The denominators are different. We have to make them the same.

Follow the steps:

$\frac{1}{3} + \frac{1}{4} = \square$

STEP 1: The LCM of 3 and 4 is 12. (Lowest Common Multiple is the lowest multiple of the given denominators.)

STEP 2: Change both the fractions into equivalent fractions with denominators of 12.

$$\frac{1}{3} \xrightarrow{\times 4} = \frac{4}{12} \quad \text{and} \quad \frac{1}{4} \xrightarrow{\times 3} = \frac{3}{12}$$

STEP 3: The denominators are now the same. Add the numerators of the fractions.

$$\frac{4}{12} + \frac{3}{12} = \frac{4 + 3}{12}$$

$$= \mathbf{\frac{7}{12}} \quad \textbf{ANSWER}$$

Dissimilar fractions are fractions having different denominators.

Example 7

Add these fractions: $\frac{1}{2} + \frac{3}{4} = \square$

STEP 1: LCM of 2 and 4 is 4.

STEP 2: $\frac{1}{2} = \frac{2}{4}$

$\frac{3}{4} = \frac{3}{4}$

Equivalent fractions with the same denominators

STEP 3: $\frac{2}{4} + \frac{3}{4} = \frac{2 + 3}{4}$ (Add the numerators)

$= \frac{5}{4}$ (Change to mixed number)

$= \mathbf{1\frac{1}{4}}$ **ANSWER**

The next example will show you how to simplify your answer.

Example 8

Add: $\frac{1}{8} + \frac{3}{6} = \boxed{}$ LCM of 8 and 6 is 24.

So, we write: $\frac{1}{8} + \frac{3}{6} = \frac{3}{24} + \frac{12}{24}$ $\left[\frac{1}{8} = \frac{3}{24} \;,\; \frac{3}{6} = \frac{12}{24}\right]$

$= \frac{3 + 12}{24}$

$= \frac{15}{24}$ (Simplify by dividing 15 and 24 by 3)

$= \mathbf{\frac{5}{8}}$ **ANSWER**

Example 9

Add: $\frac{2}{3} + \frac{5}{6} = \boxed{}$ LCM of 3 and 6 is 6

So, we write: $\frac{2}{3} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6}$ $\left[\frac{2}{3} = \frac{4}{6} \;,\; \frac{5}{6} = \frac{5}{6}\right]$

$= \frac{4 + 5}{6}$

$= \frac{9}{6}$ (Simplify by dividing 9 and 6 by 3)

$= \frac{3}{2}$ (Change to Mixed number)

$= \mathbf{1\frac{1}{2}}$ **ANSWER**

A mixed number is the sum of a whole number and a fraction.

Example 10

Add: $2\frac{2}{7} + 5\frac{3}{7} = \square$

$$2\frac{2}{7} + 5\frac{3}{7} = \frac{16}{7} + \frac{38}{7}$$ (Change to fractions)

$$= \frac{16 + 38}{7}$$ (Add the numerators)

$$= \frac{54}{7}$$ (Change to mixed number)

$$= \mathbf{7\frac{5}{7}}$$ **ANSWER**

Example 11

Add: $\frac{1}{3} + 1\frac{1}{2} = \square$

We change mixed numbers to fractions:

$$\frac{1}{3} + 1\frac{1}{2} = \frac{1}{3} + \frac{3}{2} \longleftarrow \left[1\frac{1}{2} = \frac{3}{2}\right]$$

LCM of 3 and 2 is 6.

$$= \frac{2}{6} + \frac{9}{6} \qquad \left[\frac{1}{3} = \frac{2}{6},\ \frac{3}{2} = \frac{9}{6}\right]$$

$$= \frac{2 + 9}{6}$$ (Add numerators)

$$= \frac{11}{6}$$ (Change to mixed number)

$$= \mathbf{1\frac{5}{6}}$$ **ANSWER**

Example 12

Add: $2\frac{1}{4} + 1\frac{3}{8} = \boxed{}$

$2\frac{1}{4} + 1\frac{3}{8} = \frac{9}{4} + \frac{11}{8}$ (Change mixed numbers to fractions)

$= \frac{18}{8} + \frac{11}{8}$ (LCM of 4 and 8 is 8)

$= \frac{18 + 11}{8}$ (Add numerators)

$= \frac{29}{8}$ (Change to a mixed number)

$= \mathbf{3\frac{5}{8}}$ **ANSWER**

REMEMBER:

In adding fractions, only similar fractions can be added. If fractions are dissimilar, change the fractions to similar fractions first, then, add the numerators over the same denominator and express the answer in its simplest form. Addition of mixed numbers follows the rules of fractions.

A fraction is in its simplest form, when the numerator and the denominator have no common factor except 1.

NOW DO PRACTICE EXERCISE 4.

Practice Exercise 4

Add the following fractions and reduce your answer to its simplest form.

(a) $\frac{5}{12} + \frac{4}{12} = \boxed{}$

ANSWER =

(b) $\frac{2}{5} + \frac{3}{5} + \frac{4}{5} = \boxed{}$

ANSWER =

(c) $\frac{1}{8} + \frac{3}{8} + \frac{2}{8} = \boxed{}$

ANSWER =

(d) $\frac{2}{6} + \frac{1}{6} + \frac{5}{6} = \boxed{}$

ANSWER =

(e) $\frac{2}{3} + \frac{3}{5} + \frac{1}{2} = \boxed{}$

ANSWER =

(f) $\frac{49}{100} + \frac{9}{10} = \boxed{}$

ANSWER =

(g) $\frac{2}{3} + \frac{1}{4} + \frac{1}{5} = \boxed{}$

ANSWER =

(h) $\frac{2}{3} + \frac{7}{10} = \boxed{}$

ANSWER =

(i) $\frac{4}{5} + 5\frac{3}{5} = \square$

ANSWER =

(j) $4\frac{5}{8} + 6\frac{7}{8} = \square$

ANSWER =

(k) $2\frac{4}{5} + \frac{3}{10} = \square$

ANSWER =

(l) $\frac{5}{6} + 7\frac{1}{8} = \square$

ANSWER =

(m) $2\frac{2}{3} + 4\frac{1}{2} + \frac{3}{4} = \square$

ANSWER =

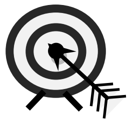
(n) $2\frac{1}{4} + 6\frac{3}{8} + \frac{2}{3} = \square$

ANSWER =

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 5: Subtraction of Fractions and Mixed Numbers

You learnt to add fractions and mixed numbers in Lesson 4.



In this lesson you will:

- subtract fractions and mixed numbers with the same denominators
- subtract fractions and mixed numbers with different denominators

In subtracting fractions, the method is exactly the same as in adding fractions, except that we subtract the numerators.

Look at the first example.

Example 1

Subtract: $\frac{5}{6} - \frac{3}{6} = \boxed{}$

The denominators are the same.

So, we write: $\frac{5}{6} - \frac{3}{6} = \frac{5 - 3}{6}$ (Subtract numerators)

$= \frac{2}{6}$ (Simplify by dividing 2 and 6 by 2)

$= \mathbf{\frac{1}{3}}$ **ANSWER**

Example 2

Subtract: $\frac{9}{10} - \frac{7}{10} = \boxed{}$

The denominators are the same.

So, we write: $\frac{9}{10} - \frac{7}{10} = \frac{9 - 7}{10}$ (Subtract numerators)

$= \frac{2}{10}$ (Simplify by dividing 2 and 10 by 2)

$= \mathbf{\frac{1}{5}}$ **ANSWER**

Example 3

Subtract: $\frac{7}{8} - \frac{1}{3} = \boxed{}$

LCM of 8 and 3 is 24.

So, we write: $\frac{7}{8} - \frac{1}{3} = \frac{21}{24} - \frac{8}{24}$ $\left[\frac{7}{8} = \frac{21}{24}, \frac{1}{3} = \frac{8}{24} \right]$

$= \frac{21 - 8}{24}$ (Subtract numerators)

$= \mathbf{\frac{13}{24}}$ **ANSWER**

Example 4

Subtract: $\frac{11}{12} - \frac{3}{4} = \boxed{}$

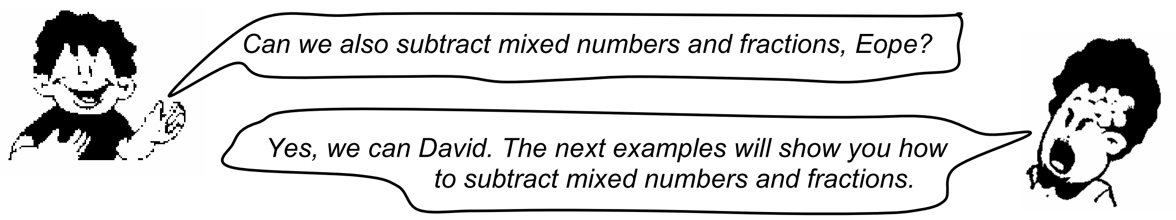
LCM of 12 and 4 is 12.

So, we write: $\frac{11}{12} - \frac{3}{4} = \frac{11}{12} - \frac{9}{12}$ $\left[\frac{11}{12} = \frac{11}{12}, \frac{3}{4} = \frac{9}{12} \right]$

$= \frac{11 - 9}{12}$ (Subtract numerators)

$= \frac{2}{12}$ (Simplify)

$= \mathbf{\frac{1}{6}}$ **ANSWER**



Example 5

Subtract: $2\frac{3}{8} - 1\frac{1}{5} = \square$

We write: $2\frac{3}{8} - 1\frac{1}{5} = \frac{19}{8} - \frac{6}{5}$ (Change to improper fractions)

LCM of 8 and 5 is 40.

$= \frac{95}{40} - \frac{48}{40}$ $\left[\frac{19}{8} = \frac{95}{40}, \frac{6}{5} = \frac{48}{40}\right]$

$= \frac{95 - 48}{40}$ (Subtract numerators)

$= \frac{47}{40}$ (Change to mixed number)

$= \mathbf{1\frac{7}{40}}$ **ANSWER**

Example 6

Subtract: $4\frac{1}{5} - 2\frac{7}{10} = \square$

We write: $4\frac{1}{5} - 2\frac{7}{10} = \frac{21}{5} - \frac{27}{10}$ (Change to improper fractions)

LCM of 10 and 5 is 10.

$= \frac{42}{10} - \frac{27}{10}$ $\left[\frac{21}{5} = \frac{42}{10}, \frac{27}{10} = \frac{27}{10}\right]$

$= \frac{42 - 27}{10}$ (Subtract numerators)

$= \frac{15}{10}$ (Simplify)

$= \frac{3}{2}$ (Change to mixed number)

$= \mathbf{1\frac{1}{2}}$ **ANSWER**

Subtraction of fractions and mixed numbers follow the rules of adding fractions and mixed numbers except that you always subtract the numerators.

NOW DO PRACTICE EXERCISE 5

Practice Exercise 5

Subtract the following fractions and simplify your answers where necessary.

1. $\frac{9}{10} - \frac{3}{10} = \boxed{}$

2. $\frac{8}{15} - \frac{3}{15} = \boxed{}$

3. $\frac{11}{16} - \frac{5}{16} = \boxed{}$

4. $\frac{13}{24} - \frac{1}{4} = \boxed{}$

5. $\frac{9}{10} - \frac{3}{5} = \boxed{}$

6. $2\frac{5}{8} - \frac{3}{8} = \boxed{}$

7. $5\frac{1}{6} - \frac{2}{3} = \boxed{}$

8. $3\frac{1}{2} - 2\frac{5}{8} = \boxed{}$

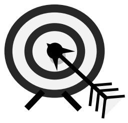
9. $6\frac{2}{3} - 2\frac{3}{4} = \boxed{}$

10. $8\frac{4}{5} - 7\frac{2}{3} = \boxed{}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 6: Multiplication of Fractions and Whole Numbers

In the last two lessons, you learnt how to add and subtract fractions and mixed numbers.



In this lesson you will:

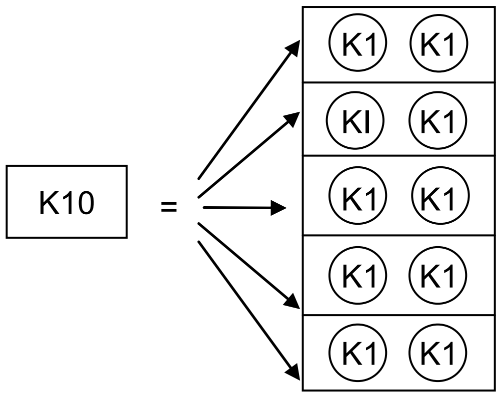
- multiply fractions and whole numbers

First, you will look at multiplying a whole number and a fraction.

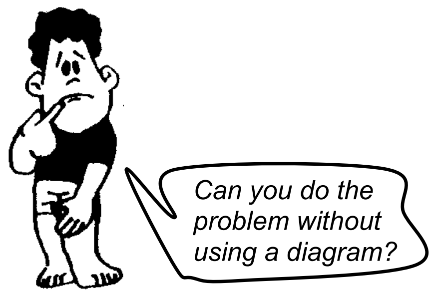
Look at David‟s problem below:

David has K10 and he wants to divide it equally between 5 people.
How much does each person get?

Here is a diagram showing you the problem.



So each person gets K2.00



There are 5 people to share the money, so each person gets,

$\frac{1}{5}$ (one fifth) of the money

Each person gets $\frac{1}{5}$ of K10.

In Mathematics, the word “of” is a short way of writing “multiplied by” (x).

So, David wrote:

Each person gets $\frac{1}{5}$ x K10

Note: Multiplying by $\frac{1}{5}$ is the same as dividing by 5.

So now, David wrote:

Each person gets $\frac{K10}{5}$ or (K10 ÷ 5)

Each person gets K2. $\left[\frac{K10}{5} = K2\right]$

Example 2

What is $\frac{1}{3}$ of 21?

We write: $\frac{1}{3}$ of 21 = $\frac{1}{3}$ x 21 (Remember: "of" means, multiply)

= $\frac{21}{3}$ or (21 ÷ 3)

= **7** **ANSWER**

Example 3

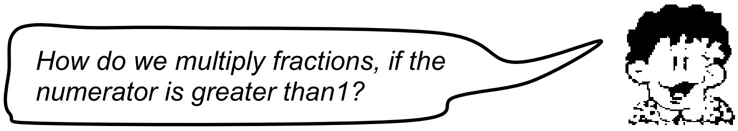
What is $\frac{1}{10}$ of 200?

We write: $\frac{1}{10}$ of 200 = $\frac{1}{10}$ x 200 (Remember: "of" means, multiply)

= $\frac{200}{10}$ or (200 ÷ 10)

= **20** **ANSWER**

Notice that all the fractions we have done had a numerator of 1. Sometimes the denominators were different but all the numerators were greater than 1.



The next lot of examples will show you how to do that.

Example 4

What is $\frac{2}{3}$ of 12?

We write: $\frac{2}{3}$ of 12 = $\frac{2}{3}$ x 12

= $\frac{2 \times 12}{3}$ (Multiply the numerators)

= $\frac{24}{3}$ (Simplify by dividing 24 by 3)

= **8** **ANSWER**

Example 5

What is $\frac{3}{4}$ of 18?

We write: $\frac{3}{4}$ of 18 $= \frac{3}{4} \times 18$

$= \frac{3 \times 18}{4}$ (Multiply the numerators)

$= \frac{54}{4}$ (simplify by dividing 54 and 4 by 2)

$= \frac{27}{2}$ (change to mixed fraction)

$\mathbf{= 13\frac{1}{2}}$ **ANSWER**

A whole number is understood to have a denominator of 1.

To multiply a fraction and a whole number, simply multiply the numerators and express the answer to its simplest form.

NOW DO PRACTICE EXERCISE 6

Practice Exercise 6

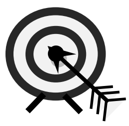
Find the numbers that go in the boxes. Show your working out.

(1) $\frac{1}{3}$ of 105 = $\square$	(2) $\frac{1}{9}$ of 108 = $\square$
(3) $\frac{1}{15}$ of 45 = $\square$	(4) $\frac{1}{4}$ of 72 = $\square$
(5) $\frac{1}{7}$ of 84 = $\square$	(6) $\frac{2}{3}$ of 21 = $\square$
(7) $\frac{2}{5}$ of 40 = $\square$	(8) $\frac{3}{4}$ of 90 = $\square$
(9) $\frac{5}{6}$ of 14 = $\square$	(10) $\frac{2}{9}$ of 20 = $\square$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 7: Multiplication of Fractions and Mixed Numbers

You learnt to multiply fractions and whole numbers in Lesson 6.



In this lesson you will:

- multiply a fraction by another fraction
- multiply a fraction by a mixed number or vice versa
- multiply a mixed number by another mixed number

Here we will first learn the basic principles of multiplying two fractions.

Let us look at the following examples.

Example 1

Multiply: $\frac{1}{2} \times \frac{2}{3} = \boxed{}$

$$\frac{1}{2} \times \frac{2}{3} = \frac{1 \times 2}{2 \times 3} \qquad \left[\frac{\text{multiply numerators}}{\text{multiply denominators}}\right]$$

$$= \frac{2}{6} \qquad \text{(Simplify)}$$

$$\mathbf{= \frac{1}{3}} \qquad \textbf{ANSWER}$$

Example 2

Multiply: $\frac{7}{8} \times \frac{2}{5} = \boxed{}$

$$\frac{7}{8} \times \frac{2}{5} = \frac{7 \times 2}{8 \times 5} \qquad \left[\frac{\text{multiply numerators}}{\text{multiply denominators}}\right]$$

$$= \frac{14}{40} \qquad \text{(simplify)}$$

$$\mathbf{= \frac{7}{20}} \qquad \textbf{ANSWER}$$

I see. The multiplication of fractions is multiplying the numerators and multiplying the denominators, and then simplifying the answer to its simplest form.

That"s right. There is no need to find the common denominator of the fractions in *multiplication*.

Here are other examples. This time we have a whole number and a mixed number.

Example 3

Multiply: 5 by $1\frac{1}{4} = \boxed{}$

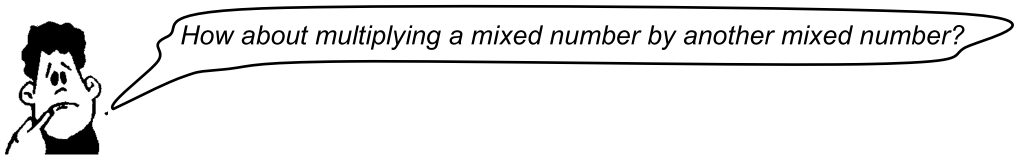
$5 \times 1\frac{1}{4} = 5 \times \frac{5}{4}$ (Change mixed number to fraction)

$= \frac{5 \times 5}{4}$ (Multiply numerators)

$= \frac{25}{4}$ (Change to mixed number)

$= \mathbf{6\frac{1}{4}}$ **ANSWER**

> **REMEMBER:**
>
> **To multiply a fraction and a whole number, multiply their numerators and express the answer to its simplest form.**



The following examples will show you how to multiply two mixed numbers.

Example 4

Multiply: $1\frac{4}{5}$ by $1\frac{1}{2} = \boxed{}$

Change mixed numbers to fractions

$1\frac{4}{5} \times 1\frac{1}{2} = \frac{9}{5} \times \frac{3}{2}$ $\left[1\frac{4}{5} = \frac{9}{5}, 1\frac{1}{2} = \frac{3}{2} \right]$

$= \frac{9 \times 3}{5 \times 2}$ $\left[\frac{\text{Multiply numerators}}{\text{Multiply denominators}} \right]$

$= \frac{27}{10}$ (Change to mixed number)

$= \mathbf{2\frac{7}{10}}$ **ANSWER**

Example 5

Multiply: $2\frac{1}{4}$ by $1\frac{1}{3}$ = $\boxed{}$

Change mixed numbers to fractions.

$2\frac{1}{4} \times 1\frac{1}{3} = \frac{9}{4} \times \frac{4}{3}$ $\left[2\frac{1}{4} = \frac{9}{4}, 1\frac{1}{3} = \frac{4}{3}\right]$

$= \frac{9 \times 4}{4 \times 3}$ $\left[\frac{\text{(Multiply numerators)}}{\text{(Multiply denominators)}}\right]$

$= \frac{36}{12}$ (Simplify)

$= \mathbf{\frac{3}{1}}$ **or 3 ANSWER**

REMEMBER:

In multiplication of fractions:

- **There is no need to find the common denominators of the fractions. The basic principle (rule) is to multiply both the numerators and the denominators and the products become the numerator and denominator of the answer expressed in simplest form.**
- **In multiplying a whole number by a mixed number, change the mixed number to an improper fraction and find the product.**
- **If both factors are mixed numbers, change each mixed number to an improper fraction before multiplying.**

NOW DO PRACTICE EXERCISE 7

Practice Exercise 7

Multiply the following fractions and write your answers in simplest form.

1) $\frac{3}{4} \times \frac{2}{3} = \square$

(2) $\frac{3}{8} \times \frac{4}{5} = \square$

(3) $6 \times 2\frac{3}{4} = \square$

(4) $5\frac{1}{2} \times 4 = \square$

(5) $3\frac{1}{3} \times 6\frac{1}{2} = \square$

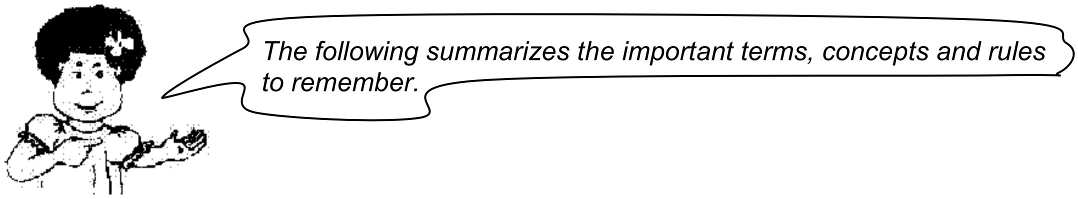
(6) $4\frac{4}{5} \times 8\frac{1}{3} = \square$

(7) $4\frac{1}{5} \times 2\frac{6}{7} = \square$

(8) $7\frac{1}{4}$ X $3\frac{3}{5} = \square$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

SUB-STRAND 1: SUMMARY



- The numerator is the number written on the top part of any fraction. It indicates the number of parts.
- The denominator is the number written at the bottom part of any fraction. It indicates the total number of parts the whole is divided into
- Equivalent fractions are fractions that have the same meaning or value. They are fractions equal to each other.
- The size of a fraction is unchanged if both the numerator and the denominator are multiplied by the same number.
- Example: $\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$ …. $\frac{3}{8}$ and $\frac{15}{40}$ are equivalent fractions.
- A mixed number is the sum of a whole number and a fraction.
- Improper fractions are fractions that have numerators that are greater than the denominators.
- Mixed numbers may be expressed as improper fractions or vice versa.
- In adding or subtracting fractions, **only similar fractions can be added or subtracted.**
- Fractions are similar if they have the same denominators. Addition and subtraction of mixed numbers follow the rules for whole numbers and for fractions.
- Before adding or subtracting fractions, express all fractions with the same denominator. This is called a **common denominator.** Always try to use the lowest common denominator or (LCM).
- In multiplying fractions, the product of two fractions is equal to the product of their numerators divided by the product of their denominators.

REVISE LESSONS 1-7 THEN DO SUB-STRAND TEST 1 IN ASSIGNMENT 1.

ANSWERS TO PRACTICE EXERCISES 1-7

Practice Exercise 1

1. a) $\frac{1}{6}$ b) $\frac{5}{6}$

2. a) 5 b) 8

3. a) proper fraction
   b) improper fraction
   c) unit fraction
   d) mixed number
   e) proper fraction

4. a) $\frac{7}{11}$ b) $\frac{5}{5}$ c) $\frac{2}{7}$ d) $\frac{7}{18}$ e) $\frac{10}{25}$

5. a) $\frac{1}{40}, \frac{1}{36}, \frac{1}{24}, \frac{1}{8}, \frac{1}{5}$ d) $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$

   b) $\frac{3}{21}, \frac{3}{8}, \frac{3}{7}, \frac{3}{5}, \frac{3}{4}$ e) $\frac{6}{25}, \frac{6}{19}, \frac{6}{17}, \frac{6}{9}, \frac{6}{7}$

   c) $\frac{3}{22}, \frac{4}{22}, \frac{7}{22}, \frac{11}{22}, \frac{19}{22}$

Practice Exercise 2

1) (a) $\frac{1}{2} = \frac{3}{6}$ (b) $\frac{3}{6} = \frac{6}{12}$

   (c) $\frac{1}{4} = \frac{2}{8}$ (d) $\frac{3}{4} = \frac{6}{8}$

2. (a) $\frac{1}{5} = \frac{2}{10}$ (b) $\frac{2}{5} = \frac{4}{10}$ (c) $\frac{3}{5} = \frac{6}{10}$

   (d) $\frac{4}{5} = \frac{8}{10}$ (e) $1 = \frac{10}{10} = \frac{5}{5}$

3. (a) $\frac{7}{42} = \frac{1}{6}$ (f) $\frac{21}{30} = \frac{7}{10}$

(b) $\frac{12}{40} = \frac{3}{10}$ (g) $\frac{45}{54} = \frac{5}{6}$

(c) $\frac{18}{27} = \frac{2}{3}$ (h) $\frac{24}{72} = \frac{1}{3}$

(d) $\frac{36}{48} = \frac{3}{4}$ (i) $\frac{33}{44} = \frac{3}{4}$

(e) $\frac{14}{35} = \frac{2}{5}$ (j) $\frac{26}{65} = \frac{2}{5}$

Practice Exercise 3

A. (1) $1\frac{3}{4} = \frac{7}{4}$ (2) $2\frac{1}{5} = \frac{11}{5}$ (3) $8\frac{1}{2} = \frac{17}{2}$

(4) $3\frac{2}{3} = \frac{11}{3}$ (5) $4\frac{2}{7} = \frac{30}{7}$ (6) $5\frac{5}{6} = \frac{35}{6}$

B. (1) $\frac{7}{2} = 3\frac{1}{2}$ (2) $\frac{4}{3} = 1\frac{1}{3}$ (3) $\frac{9}{8} = 1\frac{1}{8}$

(4) $\frac{7}{6} = 1\frac{1}{6}$ (5) $\frac{11}{2} = 5\frac{1}{2}$ (6) $\frac{5}{3} = 1\frac{2}{3}$

Practice Exercise 4

(a) $\frac{3}{4}$ (b) $1\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $1\frac{1}{3}$

(e) $1\frac{23}{30}$ (f) $1\frac{39}{100}$ (g) $1\frac{7}{60}$ (h) $1\frac{11}{30}$

(i) $6\frac{2}{5}$ (j) $11\frac{1}{2}$ (k) $3\frac{1}{10}$ (l) $7\frac{23}{24}$

(m) $7\frac{11}{12}$ (n) $9\frac{7}{24}$

Practice Exercise 5

(1)	$\frac{3}{5}$	(2)	$\frac{1}{3}$	(3)	$\frac{3}{8}$	(4)	$\frac{7}{24}$
(5)	$\frac{3}{10}$	(6)	$2\frac{1}{4}$	(7)	$4\frac{1}{2}$	(8)	$\frac{7}{8}$
(9)	$3\frac{11}{12}$	(10)	$1\frac{2}{15}$				

Practice Exercise 6

(1)	35	(2)	12	(3)	3	(4)	18
(5)	12	(6)	14	(7)	16	(8)	$67\frac{1}{2}$
(9)	$11\frac{2}{3}$	(10)	$4\frac{4}{9}$				

Practice Exercise 7

(1)	$\frac{1}{2}$	(2)	$\frac{3}{10}$	(3)	$16\frac{1}{2}$	(4)	22
(5)	$21\frac{2}{3}$	(6)	40	(7)	12	(8)	$26\frac{1}{10}$

END OF SUB-STRAND 1

SUB-STRAND 2

DECIMALS

Lesson 8:	**Decimals and Place Value**
Lesson 9:	**Addition and Subtraction of Decimals**
Lesson 10:	**Multiplication of Decimals**
Lesson 11:	**Division of Decimals**
Lesson 12:	**Changing Fractions to Decimals**
Lesson 13:	**Application of Decimals**

SUB-STRAND 2: DECIMALS

Introduction

In the Lower Primary Mathematics course, you learnt about the decimal system and how to read decimal numbers like hundreds, tens, ones, tenths and hundredths from a place value table.

This sub-strand will help you revise and then extend your knowledge on the decimal system and all the operations involved.

In the decimal system, the **place value** means that the value of any figure is dependent on its placement.

For example, a figure is ten (10) times greater than the value of a similar figure placed in a position immediately to its right. Hence, the value of the figure 3 in the three-digit number 300 is ten times the value of the figure 3 in the two-digit number 30.

When figures are expressed in decimals, the whole number part is separated from the fractional number part by a decimal point. For example in the number 134.56, the number 134 is separated by the decimal point from the fractional part 56.

Here is the Place Value table to illustrate the place value of the figure 134.56 in the decimal system.

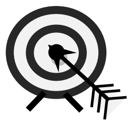
Thousands	Hundreds	Tens	Ones	Tenths	hundredths
	1	3	4 •	5	6

In this Sub-strand you will:

- revise and then extend your knowledge about decimals
- add, subtract, multiply and divide decimals
- change common fractions to decimal fractions
- apply decimals in working out problems involving money and measurements

Lesson 8: Decimals and Place Value

In Lower Primary you learnt about hundreds, tens, ones, tenths, hundredths and thousandths.



In this lesson you will:

- read and write the place value of digits in decimals
- write decimals in different ways
- represent decimals by using the mm^2 grid paper and
- make measurements using cm and m, kg and g or using the abacus

Here is the place value table

Hundreds 100	Tens 10	Ones 1	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$

DECIMAL POINT

We can write fractions in our place value table.

Look at the examples.

Example 1

This is how we write $\frac{2}{1000}$ (two thousandths) in a place value table.

Ones •	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0 •	0	0	2

Put 2 in the place for thousandths and zeros in the other places.

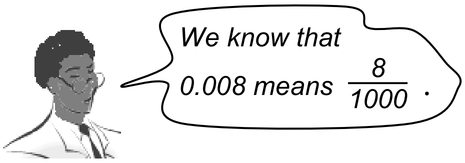
So, $\frac{2}{1000} = 0.002$.

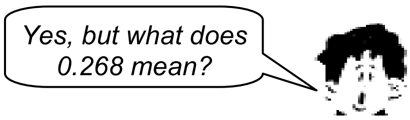
We call 0.002 a DECIMAL FRACTION.

Example 2

The table below shows you how to write and say some decimal fractions.

Number	Decimal Fraction	How we say it.
$\frac{1}{1000}$	0.001	One thousandths or Zero point zero, zero one.
$\frac{8}{1000}$	0.008	Eight thousandths or Zero point zero, zero eight
$\frac{5}{1000}$	0.005	Five thousandths or Zero point zero, zero five.





Let us write 0.268 on a place value table.

Ones	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0 •	2	6	8

In words: Zero point two six eight

In expanded form 0.268 means 2 lots of tenths plus 6 lots of hundredths plus 8 lots of thousandths.

$$= 2 \times \frac{1}{10} + 6 \times \frac{1}{100} + 8 \times \frac{1}{1000}$$

$$= \frac{2}{10} + \frac{6}{100} + \frac{8}{1000}$$

Now change these fractions to equivalent fractions before you add them.

L.C.D. is 1000

$$\frac{2}{10} \underset{\times 100}{\overset{\times 100}{=}} \frac{200}{1000} \qquad \frac{6}{100} \underset{\times 10}{\overset{\times 10}{=}} \frac{60}{1000} \qquad \frac{8}{1000} \underset{\times 1}{\overset{\times 1}{=}} \frac{8}{1000}$$

So, 0.268 $= \frac{200}{1000} + \frac{60}{1000} + \frac{8}{1000}$

$= \frac{200 + 60 + 8}{1000}$ (Add numerators)

$= \frac{268}{1000}$

Here are some more examples showing you how to write and say decimal fractions.

Fraction	Decimal Fraction	How we say it
$\frac{49}{1000}$	0.049	Forty nine thousandths or Zero point zero four nine
$\frac{123}{1000}$	0.123	One hundred twenty three thousandths or Zero point one, two, three
$1\frac{6}{1000}$	1.006	One and six thousandths or One point zero, zero, six
$2\frac{23}{1000}$	2.023	Two and twenty three thousandths or Two point zero two three

REMEMBER:

In the Decimal system,

- **Place value means that the value of the figure is dependent on its placement.**
- **The value of the number is ten times greater than the figure placed immediately to its right. Whole numbers are separated from the fractional part by a decimal point.**
- **In reading decimals, the decimal point is read as "and" or "point" which indicates fractions less than one or by reading the numbers just like reading your telephone number.**
- Example: 2.3456 is read as **"two point three four five six."** or **"two and three thousand four hundred fifty six ten-thousandths."**

NOW DO PRACTICE EXERCISE 8

Practice Exercise 8

1. Write the following numbers in expanded decimal form.

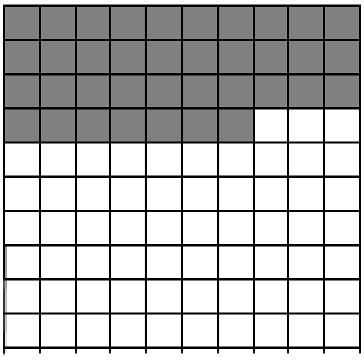
   (a) 5.85
   (b) 1.008
   (c) 21.8
   (d) 32.0006
   (e) 3.038

2. What is the place value of **6** in the following numbers?

   (a) 648.3
   (b) 95.61
   (c) 6.482
   (d) 1.346118
   (e) 0.00069

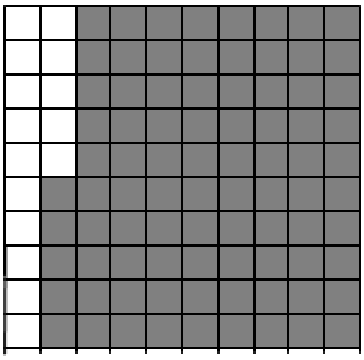
3. Each of the following one hundred square figures represents one whole. What decimal is represented by the shaded region in each figure? The first one is done for you.

(a)



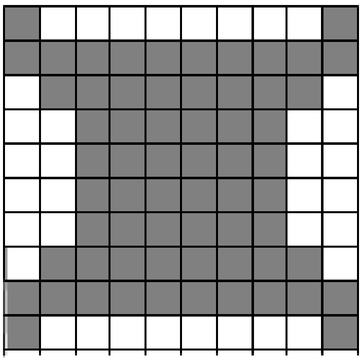
Answer: $\frac{37}{100}$ 0.37

(b)



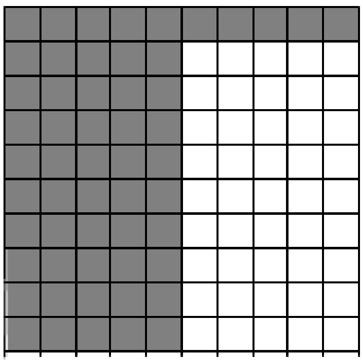
Answer: ________

(c)



Answer: ________

(d)



Answer: ________

4. Express each sum in correct decimal fraction. The first one is done for you.

(a) $\frac{6}{10} + \frac{4}{100} =$ **0. 64**

(b) $\frac{2}{10} + \frac{3}{100} + \frac{8}{1000} =$ ________

(c) $8 + \frac{3}{10} + \frac{1}{100} =$ ________

(d) $12 + \frac{8}{10} + \frac{4}{100} + \frac{6}{1000} =$ ________

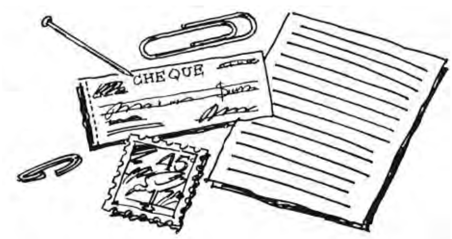
(e) $2 + \frac{5}{10} + \frac{9}{1000} =$ ________

---

5. Problem solving:

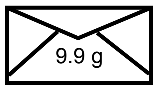
The following are typical of what we put in envelopes. The total weight (in grams) is printed on each envelope, and the envelope may contain more than one of each object.

pin 0.06 g
stamp 0.13 g
lined notepaper 3.69 g
staple 0.07 g
cheque 0.97 g
paper clip 0.22 g



If a standard envelope weighs 2.23 g, identify the contents of these envelopes from their total weight.

(a) Contents: 2 ______, 1 _______ and 1_______



Answer: 2 lined papers,1 paper clip and 1staple
(2 x3.69 g + 0.22 g + 0.07g + 2.23 g = 9.9 g)

(b) Contents: 1 ______ and 1 _______

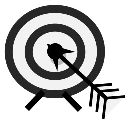
(c) Contents: 2 ______ , 1 ______ , and 5 ______

(d) Contents: 3 ______, 2 _______, 1 _______, 1 _______ and 1 _______

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 9: Addition and Subtraction of Decimals

You learnt about addition and subtraction in Lower Primary.



In this lesson you will:

- add and subtract decimal numbers with different decimal places.

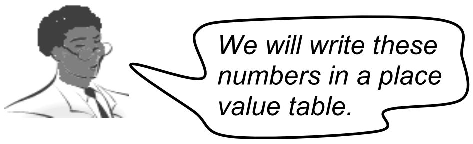
You will be adding and subtracting decimal fractions like you did in Lower Primary. These decimals will be more challenging than the ones you did in Lower Primary. First you will do addition of decimals.

Adding Decimals

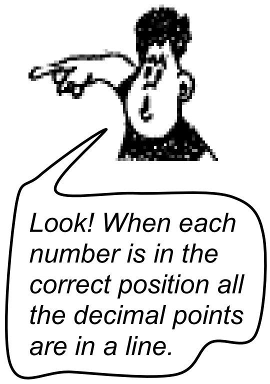
Look at the examples below.

Example 1

Add: 2.06 + 41.029 + 9.008 = ☐



TENS	ONES	TENTHS	HNUDREDTHS	THOUSNDTHS
	2	0	6	
4	1	0	2	9
	9	0	0	8

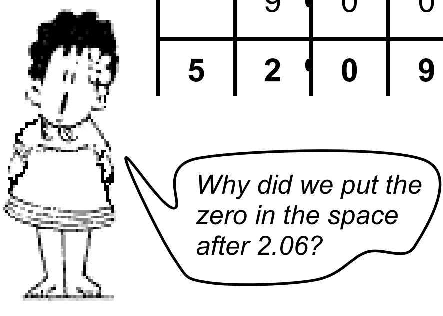


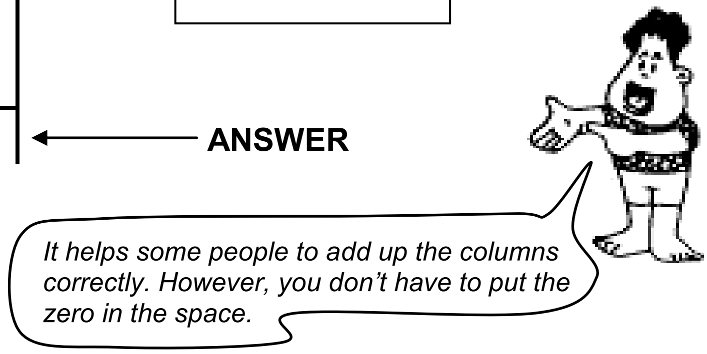
Here is the table again:

TENS	ONES	TENTHS	HNUDREDTHS	THOUSNDTHS
	2	0	6	0
4	1	0	2	9
	9	0	0	8
5	**2**	**0**	**9**	**7**

PUT A ZERO IN THE SPACE.

ANSWER





Example 2

Add:
```
   45.678
+ 130.020
   69.706
  -------
  245.404   ANSWER
```

Example 3

Add: 126. 029 + 384.17 + 49.7 + 1.002 = ☐

Write the numbers in columns so that all the decimal points are in a line.

```
  126.029
  384.170
+  49.700
    1.002
  -------
  560.901   ANSWER
```

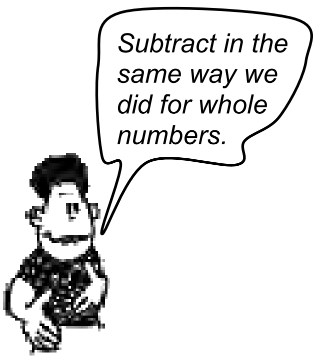
We put zeros in the spaces.

Subtracting Decimals

The method for subtracting decimals is like the method for adding decimals. We must always write the numbers in place value positions with the decimal points in a line.

Example 1

Subtract: 4 9 . 1 6 8 - 2 2 . 7 3 4 = ☐



tens	Ones	Tenths	Hundredths	thousandths
4	9 •	1	6	8
2	2 •	7	3	4
2	**6 •**	**4**	**3**	**4**

ANSWER

Example 2

Subtract: 7 . 8 4 - 2 . 2 6 5 = $\boxed{}$

Write the numbers in place value position.

$$\begin{array}{r} {}^{7\ 13\ 10} \\ 7\,.\,\cancel{8}\,\cancel{4}\,\cancel{0} \\ -\ \underline{2\,.\,2\,6\,5} \\ \mathbf{5\,.\,5\,7\,5} \end{array}$$

PUT ZERO IN THE SPACE

5 . 5 7 5 ANSWER

REMEMBER:

To add or subtract decimals, use the PUP rule: place Points Under Points. An empty space may be filled with zero.

NOW DO PRACTICE EXERCISE 9

Practice Exercise 9

(1) Do the following additions. Put zeros in the spaces after the decimal points.

(a)
```
   26.14
    2.009
+ 17.4
________
```

(b)
```
  126.029
  384.17
+  49.7
________
```

(c)
```
   84.92
  189.555
    6.23
+ 700.00
________
```

(d)
```
    59.839
    17.05
   121.73
+   10.118
__________
```

(e)
```
   55.678
    2.803
   39.482
+   7.599
_________
```

(2) Do the following additions. Write the numbers in their place value positions with decimal points in a line. The first one is done for you.

(a) 2. 8 + 4 +9.005 = $\boxed{15.805}$

```
  2. 800
  4. 000
+ 9. 005
  15.805
```

PUT ZEROS IN ALL THE SPACES

(b) 8.6 + 2 + 3.87 + 0.095 = $\boxed{}$

(c) 20.02 + 151.093 + 66.6 + 7.931 = $\boxed{}$

(d) 85 + 17.8 + 51.93 + 68.782 = $\boxed{}$

(e) 181. 26 + 394.05 + 799.015 + 206. 895 + 17 = $\boxed{}$

(3) Do the following subtractions. (Put zeros in the spaces after the decimal points.)

(a) $\begin{array}{r} 49.16 \\ -\ \ 2.8 \\ \hline \\ \hline \end{array}$

(b) $\begin{array}{r} 141.384 \\ -\ 29.76 \\ \hline \\ \hline \end{array}$

(c) $\begin{array}{r} 46.19 \\ -\ \ 2.93 \\ \hline \\ \hline \end{array}$

(d) $\begin{array}{r} 105.46 \\ -\ 22.9 \\ \hline \\ \hline \end{array}$

(e) $\begin{array}{r} 58.32 \\ -\ \ 9.43 \\ \hline \\ \hline \end{array}$

---

(4) Do the following subtractions. Write the numbers in place value position first. (Put zeros in the spaces after the decimal points.)

(a) 7.93 – 2.83 = ☐

(b) 46.098 – 23.73 = ☐

(c) 86.8 – 7.78 = ☐

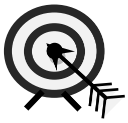
(d) 239.108 – 15.209 = ☐

(e) 92.018 – 36.004 = ☐

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 10: Multiplication of Decimals

You learnt about addition and subtraction of decimal numbers in Lesson 9.



In this lesson you will:

- multiply decimal numbers by whole numbers
- multiply decimal numbers by other decimal numbers

First we will learn to multiply decimals by 10, 100 and 1000.

What do you notice about the answer?

Multiplying by 10

Here is a decimal number: 32.86

Multiply 32.86 by 10.

32.86 x 10 = 328.6

The digits are the same but the decimal point is in a different place.

One zero

The decimal point is moved one place to the right.

32 . 86 x 10 = 328 . 6

Example 2

Multiply 205.1 by 10

205.1 x 10 = 2051.

= 2051 ANSWER

10 has 1 zero so we move the decimal point 1 place to the right.

What about multiplying a decimal by 100 or by 1000?

Multiplying by 100

100 has 2 zeros so we move the decimal point 2 places to the right.

Example 1

Multiply 32 . 86 by 100 ← Two zeros

32 . 86 x 100 = 3286. ← The decimal point is moved two places to the right

ANSWER: 3286

Example 2

Multiply 205 .1 by 100

205 .1 x 100 = 20510 .

ANSWER: 20510

Multiplying by 1000

1000 has 3 zeros so we move the decimal point 3 places to the right.

Example 1

Multiply 32 . 86 by 1000 ← Three zeros

32 . 86 x 1000 = 3286 . ← The decimal point is moved three places to the right

ANSWER: 32860

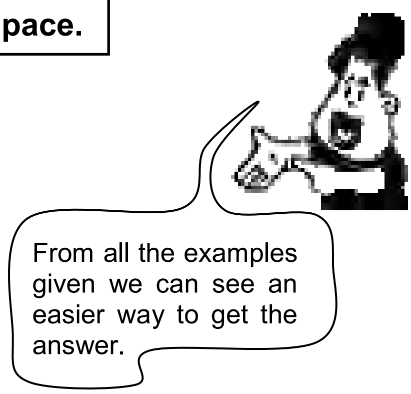
NOTE: We must add zero to fill up the space.

Example 2

Multiply 205 .1 by 1000

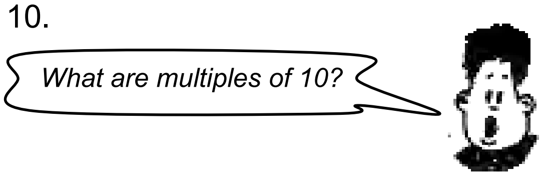
205 .1 x 1000 = 2051 .

ANSWER: 205100



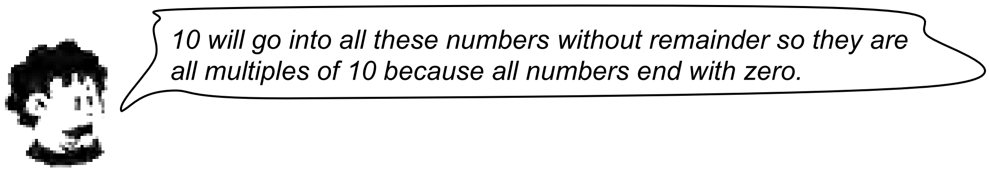
When we multiply by 10, 100 and 1000, we move the decimal point one, two or three places to the right respectively (in that order).

If you know how to multiply decimals by 10,100 or 1000, you can now learn to multiply decimals by multiples of 10.



Look at the numbers below.

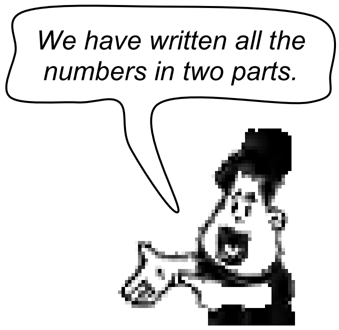
20 600 90 3000 400 160 500



We can write these multiples of 10 in another way.

Look at the numbers again.

20	=	10	x	2
90	=	10	x	9
160	=	10	x	16
400	=	100	x	4
500	=	100	x	5
600	=	100	x	6
3000	=	1000	x	3



All numbers ending in zero are multiples of 10 and all numbers divisible by 10 are multiples of ten.

Now we will learn how to multiply decimals by multiples of 10.

Look at the following examples.

Example 1

Multiply 4.9 x 20

4.9 x 20

4.9 x 10 x 2 — **Write 10 x 2 or 20**

49 x 2 — **Multiply 4.9 x 10**

98 — **Multiply 49 x 2**

ANSWER: 4.9 x 20 = 98

Example 2

Multiply 0.09 by 400

0.09 x 400

$\underbrace{0.09 \times 100} \times 4$ **Write 100 x 4 or 400**

$\underbrace{9 \times 4}$ **Multiply 0.09 x 100**

36 **Multiply 9 x 4**

Therefore, 0.09 x 400 = 36 **ANSWER**

Example 3

Multiply 12. 9 by 7000

12. 9 x 7000

$\underbrace{12.9 \times 1000} \times 7$

$\underbrace{12900 \times 7}$

90300 **ANSWER**

Example 4

Multiply 7.81 by 200

7.81 x 200

7.81 x 100 x 2

781 x 2

1562 **ANSWER**

Example 5

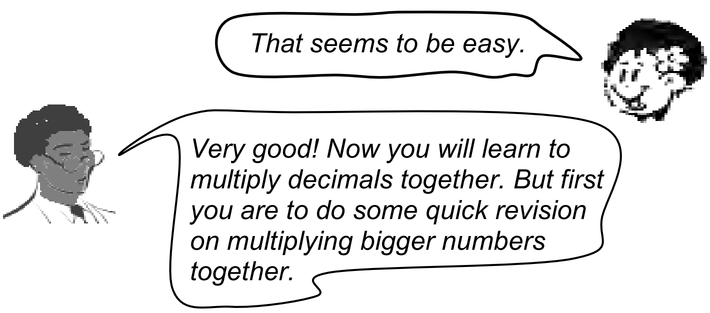
Multiply 0.24 by 300

0.24 x 300

0.24 x 100 x 3

24 x 3

72 **ANSWER**



Here are some examples.

	WORKING OUT
(a) 58 x 26 = 1508	58 x 26 348 (58 x 6) 1160 (58 x 20) **1508**
(b) 793 x 12 = 9516	793 x 12 1586 (793 x 2) 7930 (793 x 10) **9516**
(c) 685 x 731 = 500 735	685 x 731 685 (685 x 1) 20 550 (685 x 30) 479 500 (685 x 700) **500 735**

Before we do some decimal multiplication there is one more thing we must look at.

We know that

$$\frac{12}{10} = 12 \div 10 = 1.2$$

So, $\frac{12}{10} = 1.2$ or $1.2 = \frac{12}{10}$

We can write any decimal number as a whole number divided by 10, 100 or 1000, etc.

Here are some examples.

a. $9.6 = \frac{96}{10}$

b. $33.8 = \frac{338}{10}$

c. $0.67 = \frac{67}{100}$

d. $0.546 = \frac{546}{1000}$

Multiplying decimals by whole numbers

Example 1

Multiply 14 x 1.2

$$14 \times 1.2 = 14 \times \frac{12}{10}$$

$$= \frac{14 \times 12}{10} \qquad \boxed{1.2 = \frac{12}{10}}$$

$$= \frac{168}{10} \qquad \boxed{14 \times 12 = 168}$$

= 16.8 ANSWER

Example 2

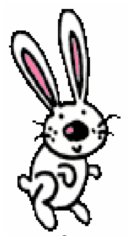
Multiply 68 x 4.3

$$68 \times 4.3 = 68 \times \frac{43}{10}$$

$$= \frac{68 \times 43}{10}$$

$$= \frac{2924}{10}$$

= 292.4 ANSWER



WORKING OUT

```
     68
   x 43
    204
 + 2720
   2924

    235
  x 143
    705   (235 x 3)
   9400   (235 x 40)
+ 23500   (235 x 100)
  33605
```

Example 3

Multiply 235 x 1.43

$$235 \times 1.43 = 235 \times \frac{143}{100}$$

$$= \frac{235 \times 143}{100}$$

$$= \frac{33\,605}{100}$$

= **336.05 ANSWER**

The next examples will show you how to multiply decimals together.

Example 4

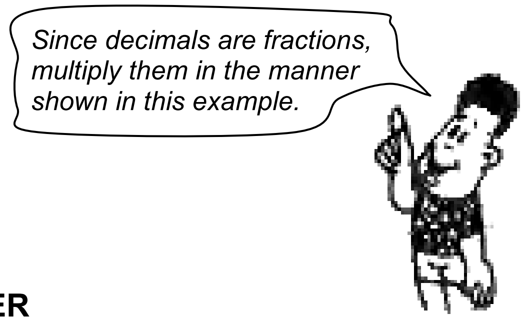
Multiply 8.6 x 9.2

$$8.6 \times 9.2 = \frac{86}{10} \times \frac{92}{10}$$

$$= \frac{86 \times 92}{10 \times 10}$$

$$= \frac{7912}{100}$$

= 79.12 ANSWER



These final problems are the same as the first but a little bit harder.

Example 5

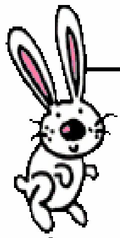
Multiply 2.68 x 1.3

$$2.68 \times 1.3 = \frac{268}{100} \times \frac{13}{10}$$

$$= \frac{268 \times 13}{100 \times 10}$$

$$= \frac{3484}{1000}$$

= 3.484 ANSWER



WORKING

268	100
x 13	x 10
804	1000
+ 2680	
3484	
4.38	100
x 1.9	x 10
3842	1000
4380	
8322	

Example 6

Multiply 4.38 x 1.9

$$4.38 \times 1.9 = \frac{438}{100} \times \frac{19}{10}$$

$$= \frac{438 \times 19}{100 \times 10}$$

$$= \frac{8322}{1000}$$

= 8.322 ANSWER

From all the examples that have been given we can see the relationship between the number of decimal places to the right of the decimal point in the factors (numbers used in multiplication) and in the product (the answer in multiplication).

When multiplying decimals, the number of decimal places in the product equals the sum of the number of decimal places in the factors.

NOW DO PRACTICE EXERCISE 10

Practice Exercise 10

(1) Write down 10 more numbers that 10 can divide into evenly.

(a)	_________20_________	(f)	____________________
(b)	____________________	(g)	____________________
(c)	____________________	(h)	____________________
(d)	____________________	(i)	____________________
(e)	____________________	(j)	____________________

(2) Fill in the spaces.

(a)	80 = 10 x ____________	(f)	900 = 100 x ____________
(b)	30 = 10 x ____________	(g)	____________ = 10 x 7
(c)	200 = ____________ x 2	(h)	150 = ____________ x 15
(d)	700 = ____________ x 7	(i)	____________ = 1000 x 5
(e)	110 = ____________ x 11	(j)	120 = 10 x ____________

(3) Write the missing numbers in the boxes.

(a) $2.2 = \frac{22}{\square}$ (b) $8.7 = \frac{87}{\square}$

(c) $32.1 = \frac{32.1}{\square}$ (d) $1.21 = \frac{121}{\square}$

(e) $7.4 = \frac{74}{\square}$ (f) $0.36 = \frac{\square}{100}$

(g) $0.103 = \frac{\square}{1000}$ (h) $12.92 = \frac{\square}{100}$

(4) Do the following multiplications. The first one is started for you.

	WORKING OUT
(a) $35 \times 2.6 = 35 \times \frac{\quad}{10}$ $= \frac{\quad \times \quad}{10}$ $= \frac{\quad}{10}$ $=$ ANSWER	
(b) $59 \times 3.3 =$	
(c) $86 \times 4.9 =$	
(d) $52 \times 1.6 =$	
(e) $8.9 \times 22 =$	

(5) Do the following multiplications. The first one has been started for you.

	WORKING OUT
(a) $1.4 \times 2.6 = \frac{14}{10} \times \frac{26}{10}$ $= \frac{\quad \times \quad}{\quad \times \quad}$ $= \underline{\qquad\qquad}$ $=$ ANSWER	
(b) $8.3 \times 3.9 =$	
(c) $8.23 \times 0.4 =$	
(d) $8.6 \times 4.53 =$	
(e) $7.33 \times 3.33 =$	

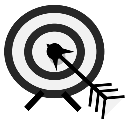
6. Find the product of the following:

(a) 0.13 x 4 = ________	(b) 0.14 x 5 = ________	(c) 0.21 x 6 = _________
(d) 3 x 0.42 = ________	(e) 8 x 0.36 = ________	(f) 0.34 x 7 = ________
(g) 0.81 x 12 = _______	(h) 0.4 x 0.83 = _______	(i) 6.2 x 4 = __________
(j) 0.6 x 4.72 = _______	(k) 0.5 x 0.75 =_______	(l) 5.4 x 9 = __________
(m) 0.08 x 6.24 = ______	(n) 0.02 x 26 = _______	(o) 0.18 x 83.9 = ______
(p) 0.13 x 2.06 = ______	(q) 0.07 x 37.4 = ______	(r) 467 x 0.94 = ______
(s) 0.14 x 27.3 = ______	(t) 0.05 x 263 = _______	(u) 0.6 x 0.27 = _______
(v) 0.21 x 183 = ______	(w) 0.12 x 27.3 = _____	(x) 0.14 x 47.9 = ______

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 11: Division of Decimals

In the previous lesson, you learnt that when you multiply decimals by 10, 100 or 1000, you move the decimal point to the correct number of places to the right.



In this lesson you will:

- divide decimals by whole numbers
- divide decimals by decimals.

Now we will learn to divide decimals by 10, 100 or 1000.

Dividing by 10

Example 1 Divide 27.81 by 10

27.81 ÷ 10 = 2.781

The decimal point is moved one place to the left.

One zero

27.81 ÷ 10 = 2.781

10 has 1 zero so we move the decimal point 1 place to the left.

Once again, you notice that the numbers are the same but the decimal point has moved.

What if we divide a decimal number by 100 or 1000?

It's like multiplying except we move the decimal point to the left each time.

Dividing by 100

100 has 2 zeros so we move the decimal point 2 places to the left.

Example 1

Divide 27.81 by 100 ← Two zeros

27.81 ÷ 100 = .2781 ← The decimal point is moved two places to the left.

ANSWER: 0.2781

When there is no number in front of a decimal point we always write zero.

Dividing by 1000

1000 has 3 zeros, so we move the decimal point 3 places to the left.

Example 1

Divide 27.81 by 1000 ← Three zeros

27.81 ÷ 1000 = .2781 ← The decimal point is moved three places to the left

ANSWER: 0.02781

We must write zero to fill up the space and a zero before the decimal point.

1) Divide 46.3 by 10

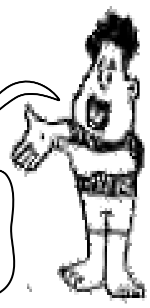
46.3 ÷ 10 = 4.63

2) Divide 46.3 by 100

46.3 ÷ 100 = 0.463

3) Divide 6.43 by 1000

6.43 ÷ 1000 = 0.00643



REMEMBER: **When we divide by 10, 100 or 1000, we move the decimal point one, two or three places to the left.**

Dividing decimals by whole numbers

Example 1

15.5 ÷ 5 = $\square$

Set it out like this: → $5\overline{)15.5}$

Now follow the steps.

<u>STEP 1</u>: Put the decimal point first in the answer. It <u>always</u> goes exactly above the decimal point in the question.

$$\begin{array}{r} . \\ 5\overline{)15.5} \end{array}$$

<u>STEP2</u>: Divide exactly like dividing whole numbers:

(a) 15 . 5 ÷ 5 = 3

$$\begin{array}{r} 3\ . \\ 5\overline{)15.5} \\ \underline{15} \end{array}$$

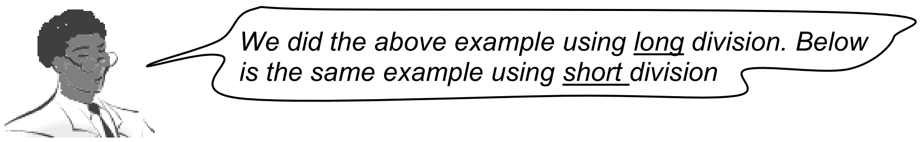
(b) bring down the 5.

$$\begin{array}{r} 3\ .\ 1 \\ 5\overline{)15.5} \\ \underline{15}\downarrow \\ 5 \end{array}$$

(c) 5 ÷ 5 = 1

no remainder

ANSWER: 3 .1



Here is the same example using short division. $5\,\underline{)\ 15.5}$

<u>STEP 1</u>: Put the decimal point in the answer.

$$\begin{array}{l} 5\,\underline{)\ 15.5} \\ . \end{array}$$

<u>STEP 2</u>: (a) 15 ÷ 5 = 3

no remainder

$$\begin{array}{l} 5\,\underline{)\ 15.5} \\ 3\,. \end{array}$$

(b) 5 ÷ 5 = 1

no remainder

$$\begin{array}{l} 5\,\underline{)\ 15.5} \\ 3\,.1 \end{array}$$

ANSWER: 3.1

If the divisor is a single digit whole number, we use the short division method. The decimal point in the quotient must be in line over the decimal point in the dividend.

Here are two more examples using short division.

1. Divide 0.018 by 6

   $0.018 \div 6 = 0.003$

WORKING OUT
Divisor → 6 ⟌ 0.018 ← Dividend
0.003 ← Quotient

2. Divide 2.63 by 4

   $2.63 \div 4 = 0.6575$

WORKING OUT
Divisor → 4 ⟌ 2.6300 ← Dividend
0.6575 ← Quotient

When the divisor is a two-digit whole number, we will use long division method. The same rule applies that the decimal point in the quotient has to be aligned directly over the decimal point in the dividend.

Here is an example showing division of two-digit whole numbers.

Divide 211.12 by 13

Set it out like this. ⟶ 13 ⟌ 211.12

Now follow the steps.

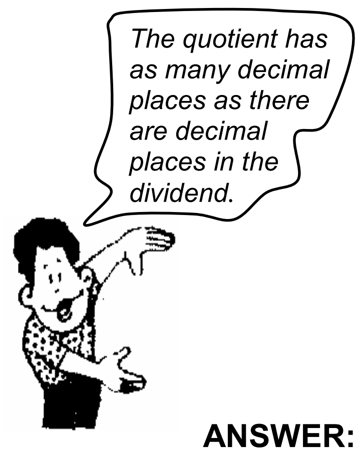
STEP 1: Put the decimal point in the answer. It always goes exactly above the decimal point in the question.

```
      .
13 ⟌ 211.12
```

STEP 2: Divide exactly like dividing whole numbers.

a) $211.12 \div 13 = 1$
b) bring down the 1.
c) $81 \div 13 = 6$
d) bring down the 1.
e) $31 \div 13 = 2$
f) bring down 2
g) $52 \div 13 = 4$
   no remainder

```
      16.24
13 ⟌ 211.12
    -13
      81
     -78
      31
     -26
       52
      -52
       00
```



ANSWER: 16.24

When the dividend contains decimal fractions and the divisor is a whole number, the decimal point of the quotient is aligned vertically above the decimal point of the dividend.

Dividing decimals by decimals

If you know how to divide decimals by whole numbers, now you are ready to learn how to divide decimals together.

First look at the following examples.

1. Divide 0.4 by 0.2

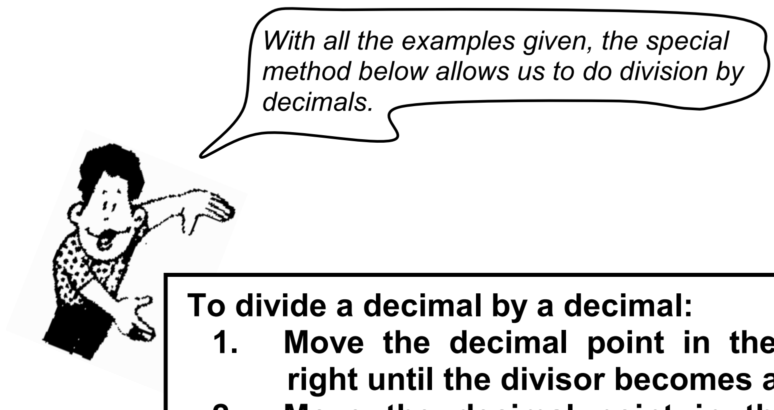
0.4 ÷ 0.2	Move the decimal point one place to the right
= 4 ÷ 2	Divide
= 2	**ANSWER**

2. Divide 0.175 by 0.5

0.175 ÷ 0.5	Move the decimal point one place to the right
= 1.75 ÷ 5	Divide
= 0.35	**ANSWER**

3. Divide 9.8 by 0.08

= 9.8 ÷ 0.08	Move the decimal point two places to the right
= 980 ÷ 8	Divide
= **122.5**	**ANSWER**



To divide a decimal by a decimal:
1. **Move the decimal point in the divisor to the right until the divisor becomes a whole number.**
2. **Move the decimal point in the dividend the same number of places to the right.**
3. **Do the division.**

Here are the examples.

Example 1 Divide 19.512 by 2.4

Set it out like this: ⟶ $2.4 \overline{)19.512}$

Now follow the steps

STEP 1: Move the decimal point one place to the right making the divisor a whole number and move the decimal point in the dividend the same number of decimal places to the right.

Divisor

$2.4 \overline{)19.512}$ ⟵ Dividend

STEP 2: Using long division:

Divide exactly like with whole numbers:

a) 195.12 ÷ 2.4 = 8

b) bring down the 1.

c) 31 ÷ 2.4 = 1

d) bring down the 2

e) 72 ÷ 2.4 = 3
no remainder

ANSWER: 8.13

8.13 ⟵ Quotient
Divisor ⟶ $24 \overline{)195.12}$ ⟵ Dividend
<u>192</u>
31
<u>24</u>
72
<u>72</u>
<u>00</u>

Example 2 Divide 42.9156 by 0.234

Move the decimal points three places in the divisor and three places in the dividend to the right, we get 234 and 42915.6 respectively.

same column
183.4 ⟵ Quotient
Divisor ⟶ $0.234 \overline{)42.9156}$ ⟵ Dividend
3 places 3 places
<u>234</u>
1951
<u>1872</u>
795
<u>702</u>
936
<u>936</u>
000 no remainder

ANSWER: 183.4

Example 3 Divide 125.646 by 8.33

Move the decimal points two places in the divisor and two places in the dividend to the right, we get 833 and 12564.6 respectively.

```
                          same column
                    15.08      ← Quotient
Divisor → 8.33. | 125.64.6     ← Dividend
    2 places         833              2 places
                     4234
                     4165
                      696
                      000
                      6960
                      6664
                       296   remainder
```

ANSWER: 15.08 r 296

NOW DO PRACTICE EXERCISE 11

Practice Exercise 11

1. Fill in the blanks to complete each statement below.

(a) To divide by 10, move the decimal point __________ place to the __________.

(b) To divide by 100, move the decimal point __________ places to the __________.

(c) To divide by 1000, move the decimal point __________places to the __________.

2. Evaluate using the methods you learnt.

(a)	46.3 ÷ 10 =	(f)	81.348 ÷ 1000 =
(b)	10.64 ÷ 100 =	(g)	24.9 ÷ 100 =
(c)	5.98 ÷ 1000 =	(h)	0.81 ÷ 10 =
(d)	0.0592 ÷ 100 =	(i)	0.61 ÷ 100 =
(e)	36.28 ÷ 1000 =	(j)	847. 612 ÷ 100 =

3. Find the value of the following divisions.

(a)	0.056 ÷7 =	(d)	0.0042 ÷ 14 =
(b)	19.25 ÷ 50 =	(e)	12.62 ÷ 4 =
(c)	0.8127÷ 9 =		

4. Using the special division method, find the value of the following.

(a)	11.07 ÷ 0.09 =	(d)	0.39426 ÷ 0.006 =
(b)	196.98 ÷ 0.98 =	(e)	0.0042 ÷ 0.03 =
(c)	4.224 ÷ 1.2 =		

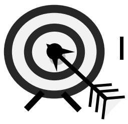
5. Solve the problem below.

Daniel has 95.7 kilograms of vegetables and 136.5 kilograms of fruits to deliver to three customers in equal amounts. How many kilograms of vegetables and kilograms of fruits will each costumer get?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2.

Lesson 12: Changing Fractions to Decimals

In lesson 8, you learnt to write common fractions which have denominators of 10, 100 or 1000 into decimals.



In this lesson you will:

- convert fractions to decimals using equivalent fractions and by division.
- determine terminating and repeating decimals

First, you will look at different ways of changing common fractions to decimal fractions.

$\frac{7}{10}$ $\frac{2}{100}$ $\frac{3}{5}$

$\frac{6}{1000}$ $\frac{1}{4}$ $\frac{5}{8}$

← THESE ARE COMMON FRACTIONS

THESE ARE DECIMAL FRACTIONS →

0.4 0.012 0.7

0.833

0.012 2.34

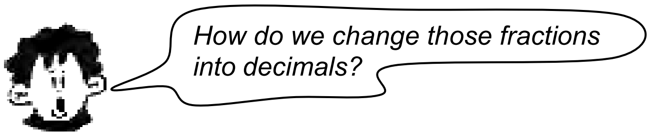
Here are some examples showing the relationships between common fractions and decimal fractions.

Common Fractions		Decimal Fractions
$\frac{7}{10}$	=	0.7
$\frac{12}{100}$	=	0.12
$\frac{3}{1000}$	=	0.003
$\frac{7}{100}$	=	0.07
$1\frac{42}{100}$	=	1.42
$3\frac{9}{10}$	=	3.9
$5\frac{27}{1000}$	=	5. 027

There are many fractions which do not have a denominator of 10, 100 or 1000.

Here are some examples:

$$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{4}{30}, \frac{9}{16}, \frac{25}{200}$$



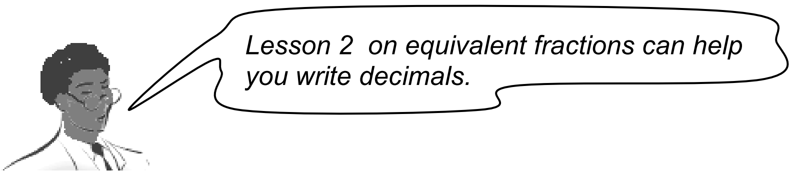
There are two ways to change fractions to decimals.

(a) by using equivalent fractions

(b) by division

A. Changing fractions to decimals by using equivalent fractions

In this method our aim is to change fractions into equivalent fractions which have a denominator of 10, 100 or 1000.



Example 1

Write $\frac{2}{5}$ as a decimal fraction.

STEP 1: Change $\frac{2}{5}$ to an equivalent fraction with denominator 10.

$$\frac{2}{5} \overset{\times 2}{\underset{\times 2}{=}} \frac{\boxed{}}{10} = \frac{4}{10}$$

STEP 2: Write $\frac{4}{10}$ as a decimal.

$\frac{4}{10}$ = **0.4** **ANSWER**

Example 2

Write $\frac{40}{50}$ as a decimal fraction.

STEP 1: Change $\frac{40}{50}$ to an equivalent fraction with denominator 100

x 2

$$\frac{40}{50} = \frac{\boxed{}}{100} = \frac{80}{100}$$

x 2

STEP 2: Write $\frac{80}{100}$ as a decimal fraction.

$$\frac{80}{100} = 0.80 \text{ or } 0.8$$

So, $\frac{80}{100}$ = 0.80 or 0.8 ANSWER

Always change the fraction into an equivalent fraction with a denominator of 10, 100 or 1000.

Example 3

Write $\frac{9}{20}$ as a decimal fraction.

x 5

$$\frac{9}{20} = \frac{45}{100}$$

x 5

$$\frac{45}{100} = 0.45$$

So, $\frac{9}{20}$ = 0.45 ANSWER

Example 4

Write $\frac{39}{200}$ as a decimal fraction.

x 5

$$\frac{39}{200} = \frac{195}{1000}$$

x 5

$$\frac{195}{1000} = 0.195$$

So, $\frac{39}{200}$ = 0.195 ANSWER

Example 5

Write $\frac{118}{500}$ as a decimal fraction.

$$\frac{118}{500} \xrightarrow{\times 2} = \frac{236}{1000}$$

$$\frac{236}{1000} = 0.236$$

So, $\frac{118}{500}$ = 0.236 ANSWER

B. Changing fractions to decimals by division

A fraction is made up of a numerator and a denominator.

Numerator → 1
Denominator → 2

$\frac{1}{2}$ is the same as $1 \div 2$.

So we can change $\frac{1}{2}$ into a decimal by dividing the numerator (1) by the denominator (2).

1 ÷ 2

$2\overline{)1.000}$ — We put a decimal point in our answer first. We put a decimal point and some zeros after 1.

STEP 1: $2\overline{)1.000}$ with 0. above — 2 into 1 cannot go so we put zero followed by a decimal point

STEP 2 : $2\overline{)1.000}$ with 0.5 above — 2 into 10 goes 5 times

STEP3 : $2\overline{)1.000}$ with 0.5 above

- 1 0 ← 5 times 2 equals 10

00 no remainder

$\frac{1}{2}$ = 0.5 ANSWER

Example 2

Write $\frac{1}{4}$ as a decimal fraction.

$$\begin{array}{r} 0.25 \\ 4 \overline{)\,1.000} \\ -\underline{8} \\ 20 \\ \underline{20} \\ 00 \end{array}$$

$\mathbf{\frac{1}{4}}$ **= 0.25 ANSWER**

Example 3

Write $\frac{5}{6}$ as a decimal fraction.

$$\begin{array}{r} 0.8333\ldots\ldots \\ 6 \overline{)\,5.0000} \\ -\underline{4\ 8} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \text{ remainder} \end{array}$$

$\mathbf{\frac{5}{6}}$ **= $0.8\overline{3}$ ANSWER**

Fractions can be written as either terminating or repeating decimals.

1. **A terminating decimal comes to an end. It has a definite number of decimal places.**

   Examples: 3.6, 0.00125, 0.75

2. **A repeating or recurring decimal does not come to an end, but forms a pattern that is repeated indefinitely (forever). To show that a set of digit is repeated, we place a bar or a dot over the first and last of the repeating digits.**

   Examples: 0.666… = $0.\overline{6}$, 1.272727… = $1.\overline{27}$

NOW DO PRACTICE EXERCISE 12

Practice Exercise 12

(1) Change these common fractions to decimal fractions.
The first one has been done for you.

(a) $\frac{3}{10} = 0.3$ (b) $\frac{68}{100} =$

(c) $\frac{15}{100} =$ (d) $\frac{361}{1000} =$

(e) $1\frac{4}{10} =$ (f) $6\frac{69}{100} =$

---

(2) Change the following fractions to decimal fractions.

The first one has been done for you.

(a) $\frac{4}{5} \overset{\times 2}{\underset{\times 2}{=}} \frac{8}{10}$ $\frac{8}{10} = 0.8$ So, $\frac{4}{5} =$ **0.8** **ANSWER**	(b) $\frac{3}{20} =$
(c) $\frac{17}{25} =$	(d) $\frac{43}{50} =$
(e) $\frac{117}{500} =$	(f) $\frac{12}{200} =$

(g) $3\frac{3}{10}$,	(h) $\frac{7}{20}$,

(3) Change the following fractions to decimal fractions by the division method. The first one has been done for you.

(a) $\frac{3}{4}$, $$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ \underline{2\ 8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$ $\mathbf{\frac{3}{4}}$ **= 0. 75 ANSWER**	(b) $\frac{5}{8}$,
(c) $\frac{1}{8}$,	(d) $\frac{2}{5}$,

4. Without dividing, indicate which of the following fractions are terminating and repeating decimals.

(a) $\frac{9}{5}$

(b) $\frac{7}{12}$

(c) $\frac{5}{16}$

(d) $\frac{3}{25}$

(e) $\frac{1}{7}$

(f) $\frac{2}{5}$

(g) $\frac{9}{11}$

(h) $\frac{3}{20}$

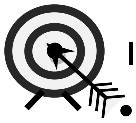
(i) $\frac{11}{15}$

(j) $\frac{7}{18}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2.

Lesson 13: Application of Decimals

In the previous lessons, you learnt to add, subtract, multiply and divide decimals.



In this lesson, you will:

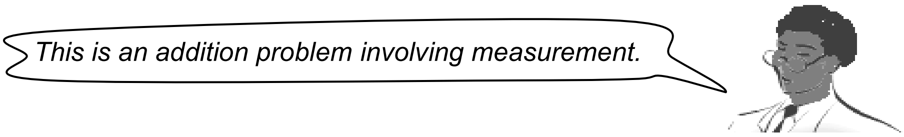
- work out problems involving application of decimals using the four operations skills.

Decimals are used in almost every aspect of life. Measurement, money and almost every area where numbers are used will require an understanding of decimals.

Look at the following examples.

Example 1

Katu bought 6 dressed chickens. Their weights are as follow: 1.4 kg, 1.9 kg, 2.05 kg, 0.94 kg, 1.1kg and 1.45 kg. What is the total weight of the six chickens?



As you learnt in Lesson 9 the steps for adding or subtracting decimals numbers are as follow:

1. Write the weights with the decimal falling in one column.

2. Add or subtract as if they are whole numbers

3. Place the decimal point of the result in the same column as the other numbers.

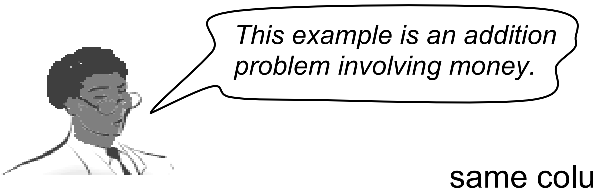
Solution:

$$
\begin{array}{rl}
 & 1.4 \text{ kg} \\
 & 1.9 \text{ kg} \\
 & 2.05 \text{ kg} \\
+ & 0.94 \text{ kg} \\
 & 1.1 \text{ kg} \\
 & \underline{1.45 \text{ kg}} \\
 & \mathbf{8.84 \text{ kg}} \quad \textbf{ANSWER}
\end{array}
$$

Example 2:

Five groups of people contributed the following amounts for a feast. K195.25, K405.75, K298.80, K277.05 and K654.10.

What is the total amount?



same column

Solution:

	Kina	Toea
	K195.	25
	K405.	75
	K298.	80
+	K277.	05
	K654.	10
	K1830.	**95**

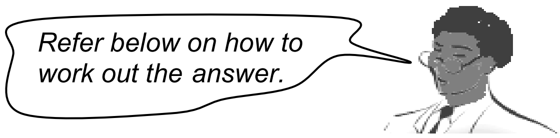
ANSWER

Therefore, the total amount of contribution is K1830.95.

Example 3

Kila rode his bicycle 5.6 km in the morning and 10.4 km in the afternoon.

How much farther did he travel in the afternoon than in the morning?



In the morning, Kila rode 5.6 km
In the afternoon, Kila rode10.4 Km

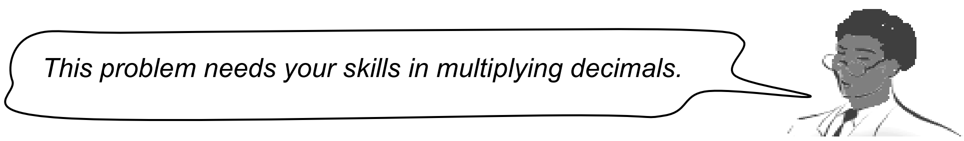
The difference is: 10.4 km
- 5.6 km

4.8 km ANSWER

Therefore, Kila traveled 4.8 km farther in the afternoon than in the morning.

Example 4

Vagi wants K325.68 converted to Australian dollars. If K1 = A$0.43, how much is this in Australian dollar?



In Lesson 10, you learnt that in multiplying decimals, we multiply them as if both are whole numbers and the number of decimal places in the product equals the sum of the number of decimal places in the factors.

METHOD 1

Solution:

$$\begin{array}{rll} 325.68 & \leftarrow & \text{(2 decimal places)} \\ \underline{\times\ 0.43} & \leftarrow & \text{(2 decimal places)} \\ 97704 & \leftarrow & (32568 \times 3) \\ \underline{1302720} & \leftarrow & (32568 \times 40) \\ \mathbf{140.0424} & \leftarrow & \textbf{(4 decimal places) ANSWER} \end{array}$$

METHOD 2

We know that $325.68 = \frac{32568}{100}$ and $0.43 = \frac{43}{100}$

So, $325.68 \times 0.43 = \frac{32568}{100} \times \frac{43}{100}$

$$= \frac{32568 \times 43}{100 \times 100}$$

$$= \frac{1400424}{10000}$$

$$= \mathbf{140.0424} \quad \textbf{ANSWER}$$

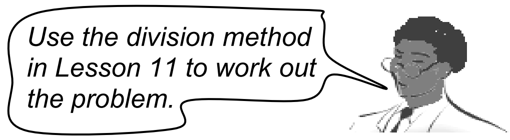
Therefore, K325.68 is equal to AU$140.0424.

In the problems below you will apply division skills.

Example 5

A basket of pawpaws weighed 10.5 kilograms. If each pawpaw weighs 0.70 kilograms, how many pieces are in the basket?

Solution:



$$\text{No. of pawpaws} = \frac{\text{Total weight}}{\text{weight of one piece}}$$

$$= \frac{10.5\text{kg}}{0.70\text{kg}}$$

$$= 15 \text{ pawpaws}$$

WORKING:

10.5 . ÷ 0.70 . (Move the decimal point 2 places to the right)

= 1050 ÷ 700 (Divide)

= **15** **Answer**

Example 6

A lottery prize of K23, 742.75 is to be divided equally among 3 winners.

How much will each get?

Solution:

Amount each gets = Total Prize ÷ 3 winners

= K23,742.75 ÷ 3

= K7,914.25 ANSWER

NOW DO PRACTICE EXERCISE 13

PRACTICE EXERCISE 13

Solve the following problems.

1. Jazelle bought 12.5 kilograms of sugar at K18.20 per kilogram, 10.75 kilogram of flour at K9.50 per kilogram and 8 tins of milk at K17.25 each.

   How much did she pay for all the items?

__________ **ANSWER**

2. Mrs. Kakas needs 2.75 and 3.55 metres of cloth for her two daughters.

   How many metres of cloth does she need?

__________ **ANSWER**

3. At a service station, Apa gave the attendant K300 for oil which costs K37.85 and fuel worth K255.65.

   What was his change?

__________ **ANSWER**

4. During their vacation, the Pogla family spent two nights at a lodge. If the rate for a double room was K122.75 per night and they had two double rooms, how much was their total bill?

__________ **ANSWER**

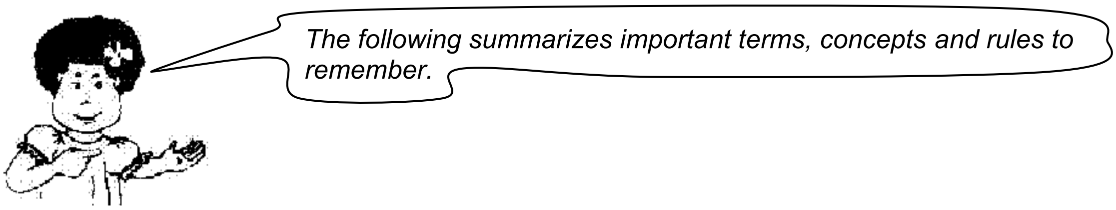
---

5. A container of coconuts weighs 200.75 kilograms. If it contains 125 coconuts, how much does each coconut weigh?

__________ **ANSWER**

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2.

SUB-STRAND 2: SUMMARY



- A decimal point separates the whole number from the fractional part.
- In reading decimals, the decimal point is read as "and" or "point" which indicates fractions less than one.
- To add or subtract decimals, use the PUP rule: place **P**oints **U**nder **P**oints. An empty space may be filled by zero.
- When multiplying or dividing decimals by 10, 100 or 1000, we move the decimal point to the right (x) or left (÷) depending on the number of zeros.
- All numbers ending in zero are multiples of 10 and all numbers divisible by 10 are multiples of 10
- In multiplying decimals, multiply as if there were no decimal points. The number of decimal places of the product is equal to the number of decimal places (total digit numbers to the right of decimal points) of the factors.
- In dividing decimals
  - ❖ When the dividend contains decimal fractions and the divisor is a whole number, the decimal point of the quotient is aligned vertically above the decimal point of the dividend.
  - ❖ When both the dividend and divisor contain decimal fractions, the divisor should be made a whole number by moving the decimal point to the right of the last digit. Then, the decimal point of the dividend should also be moved to the right for the same number of places as done with the divisor. The decimal point of the quotient should be placed directly above the decimal point of the dividend.
- To change a fraction into a decimal, divide the numerator by the denominator.
- Fractions can be written as either **terminating** or **repeating** decimals.
- A terminating decimal comes to a definite end.
- A repeating decimal does not come to an end, but forms a pattern that is repeated indefinitely.

REVISE LESSONS 8 – 13. THEN DO SUB-STRAND TEST 2 IN ASSIGNMENT 1.

ANSWERS TO PRACTICE EXERCISES 8-13

Practice Exercise 8

1. (a) $5 + \frac{8}{10} + \frac{5}{100}$ (b) $1 + \frac{8}{1000}$

(c) $20 + 1 + \frac{8}{10}$ (d) $30 + 2 + \frac{6}{10\,000}$

(e) $3 + \frac{3}{100} + \frac{8}{1000}$

2. (a) hundreds (d) thousandths
(b) tenths (e) ten-thousandths
(c) units

3. (a) 0.37 (b) 0.85 (c) 0.64 (d) 0.55

4. (a) 0.64 (d) 12.846
(b) 0.238 (e) 2.509
(c) 8.31

5. (b) 1 paper clip and 1 lined notepaper
(c) 2 cheques, 1 lined notepaper and 5 pins
(d) 3 lined notepaper, 2 cheques, 1 stamp, 1 staple and 1 paper clip

---

Practice Exercise 9

1. (a) 45.549 (b) 559.899 (c) 980.705

(d) 208.737 (e) 105. 562

2. (b) 14.565 (c) 245.644 (d) 223.512

(e) 1599.22

3. (a) 46.36 (b) 111.624 (c) 43.26

(d) 82.56 (e) 48.89

4. (a) 5.10 (b) 22.368 (c) 79.02

(d) 223.899 (e) 56.014

Practice Exercise 10

1. Any number that ends with zero is correct.

2.
(a)	80 = 10 x 8	(f)	900 = 100 x 9
(b)	30 = 10 x 3	(g)	70 = 10 x 7
(c)	200 = 100 x 2	(h)	150 = 10 x 15
(d)	700 = 100 x 7	(i)	5000 = 1000 x 5
(e)	110 = 10 x 11	(j)	120 = 10 x 12

3.
(a) $2.2 = \frac{22}{10}$ (b) $8.7 = \frac{87}{10}$ (c) $32.1 = \frac{321}{10}$

(d) $1.21 = \frac{121}{100}$ (e) $7.4 = \frac{74}{10}$ (f) $0.36 = \frac{36}{100}$

(g) $0.103 = \frac{103}{1000}$ (h) $12.92 = \frac{1292}{100}$

4. a) 91 (b) 194.7 (c) 421.4 (d) 83.2

(e) 195.8

5. (a) 3.64 (b) 32.37 (c) 3.292 (d) 38.958

(e) 24.4089

6.
(a)	0.52	(b)	0.7	(c)	1.26	(d)	1.26	(e)	2.88
(f)	2.38	(g)	9.72	(h)	0.332	(i)	24.8	(j)	2.832
(k)	0.375	(l)	48.6	(m)	0.4992	(n)	0.52	(o)	15.102
(p)	0.2678	(q)	2.618	(r)	438.98	(s)	3.822	(t)	13.15
(u)	0.162	(v)	38.43	(w)	3.276	(x)	6.706		

Practice Exercise 11

1. (a) one, left
(b) two, left
(c) three, left

2.
(a)	4.63	(f)	0.081348
(b)	0.1064	(g)	0.249
(c)	0.00598	(h)	0.081
(d)	0.000592	(i)	0.0061
(e)	0.03628	(j)	8.47612

3. (a) 0.008 (d) 0.0003
(b) 0.385 (e) 3.155
(c) 0.0903

4. (a) 123 (d) 65.71
(b) 201 (e) 0.14
(c) 3.52

5. Each customer gets 31.9 kg of vegetables and 45.5 kg of fruits.

Practice Exercise 12

1. (b) 0.68 (c) 0.15 (d) 0.361 (e) 1.4 (f) 6.69

2. (b) $\frac{3}{20}$ = 0.15 (c) $\frac{17}{25}$ = 0.68 (d) $\frac{43}{50}$ = 0.86 (e) $\frac{117}{500}$ = 0.234

(f) $\frac{12}{200}$ = 0. 060 (g) $3\frac{3}{10}$ = 3.3 (h) $\frac{7}{10}$ = 0.35

3. (a) $\frac{3}{4}$ = 0.75 (b) $\frac{5}{8}$ = 0.625 (c) $\frac{1}{8}$ = 0.125 (d) $\frac{2}{5}$ = 0.4

4. (a) terminating (f) terminating
(b) repeating (g) repeating
(c) terminating (h) terminating
(d) terminating (i) repeating
(e) repeating (j) repeating

Practice exercise 13

1. K467.625
2. 6.3 m of cloth
3. K6.50
4. K245.50
5. 1.606 kg

END OF SUB-STRAND 2

SUB-STRAND 3

PERCENTAGES

SUB-STRAND 3: PERCENTAGES

Introduction

The term **percent** is widely used by consumers and in business, statistics, accounting, finance and advertising as a tool to organize information or data so that it is easier to understand. Relationships of parts to a whole are expressed in percent.

The following statements and diagram show some uses of percents and percentages.

1. The enrolment in a high school increased by 20% over the previous year.

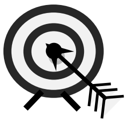
2. About 50% of Miro‟s income was spent on food.

In this Sub-strand, you will:

- define the meaning of percents and percentages
- express percent into fractions and decimals and vice versa
- solve different types of problems on percentages
- compute discount, sale price and marked price
- define the meaning of interest and apply the knowledge of percents in situations involving borrowing and lending money.

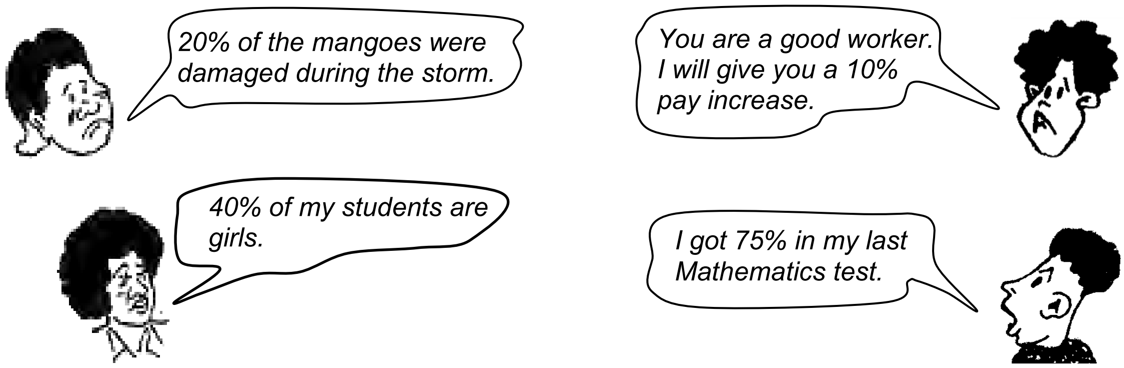
Lesson 14: Meaning of Percent

In Grade 6, you learnt about percentages.



In this lesson you will:

- define percent or percentage
- express percent as a fraction
- express percent as decimals and vice versa.



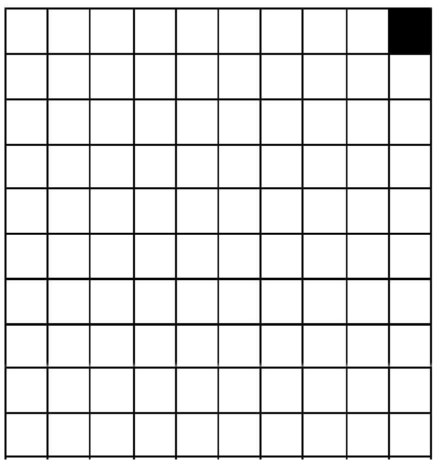
First, we will revise percentages.

Look at the figure below.

THERE ARE 100 SQUARES ALTOGETHER.

One square out of 100 is shaded.

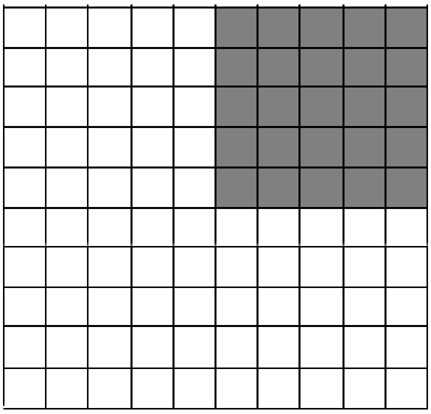
We say, 1% of the squares is shaded.



REMEMBER:

Percent comes from the Latin word "per centum" meaning "for every one hundred", "by the hundred", "out of a hundred", or "$\frac{1}{100}$ ".

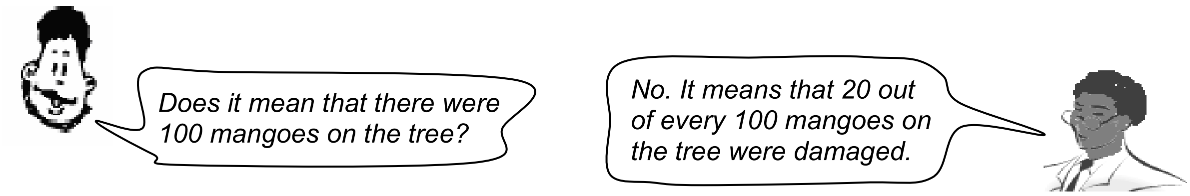
Here is another set of 100 squares:



In this diagram 25 small squares are shaded.

We say that 25 out of 100 of the squares are shaded.

Earlier we said that 20 out of 100 of the mangoes were damaged during the storm.



Percent is another way of writing a fraction whose denominator is 100. The percent sign (%) is used instead of the denominator 100. Therefore, 25 out of 100 or $\frac{25}{100}$ is 25% and 20 out of 100 or $\frac{20}{100}$ is 20%.

Here are some examples.

(a) 14% means $\frac{14}{100}$ (b) 9% means $\frac{9}{100}$

(c) 100% means $\frac{100}{100}$ (d) 150% means $\frac{150}{100}$

In the study of decimals, you learnt that $\frac{25}{100}$ is 0.25 (or 25 hundredths).

Therefore, $\frac{25}{100}$ = 0.25 = 25 %

A fraction or a decimal may be written as percent.

Below are examples of the use of percentages.

Example 1

Of 100 workers, 35 went to a convention. What part of the number of workers went to the convention? Express the number as: (a) a fraction, (b) a decimal and (c) a percent.

Solution:

Since 35 out of 100 workers went to a convention:

(a) the fractional representation is $\frac{35}{100}$.

(b) the decimal representation is 0.35

(c) as a percent, 35 out of 100 is 35%

Example 2

Rose collected 50 roses from her garden. She gave 35 roses to the church. What part of the number of roses went to the church?

Express the number as: (a) a fraction, (b) a decimal, and (c) a percent.

Solution:

Since 35 out of 50 roses were given to the church:

(a) as a fraction it is $\frac{35}{50}$.

(b) as a decimal, first change $\frac{35}{50}$ to a fraction with a denominator of 100.

$$\frac{35}{50} \times \frac{2}{2} = \frac{70}{100}$$

Therefore , $\frac{70}{100} = 0.70$ or 0.7.

(c) as a percent, since $\frac{70}{100} = 0.70$ or 70 hundredths

Therefore , 0.70 = 70%

Percent	Decimal	Fraction
35%	0.35	$\frac{35}{100}$
70%	0.70 or 0.7	$\frac{70}{100}$

Here are some examples of fractions whose denominators are not 100.

Example 1

Write $\frac{2}{5}$ as a percentage.

$$\frac{2}{5} \text{ of } 100\% = \frac{2}{\cancel{5}} \times \overset{20}{\cancel{100}}\ \% \quad \text{(Cancel by 5)}$$

$$= 2 \times 20\%$$

$$= 40\%$$

$$\boldsymbol{\frac{2}{5} = 40\%} \qquad \textbf{ANSWER}$$

Example 2

Write $\frac{3}{20}$ as a percentage.

$$\frac{3}{20} \text{ of } 100\% = \frac{3}{\cancel{20}} \times \overset{5}{\cancel{100}}\% \qquad \text{(Cancel by 20)}$$

$$= 3 \times 5\%$$

$$= 15\%$$

$$\frac{3}{20} = 15\% \qquad \textbf{ANSWER}$$

To write a fraction as a percentage, we multiply by 100%.

Now you will learn to change percentages to fractions in their simplest form.

Here are some examples.

Example 1

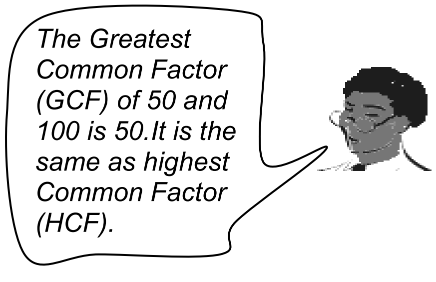
Write 50% as a fraction in its simplest form.

Remember: Percent means out of 100.

$$\text{So,} \quad 50\% = \frac{\overset{1}{\cancel{50}}}{\underset{2}{\cancel{100}}} \quad \text{(Cancel)}$$

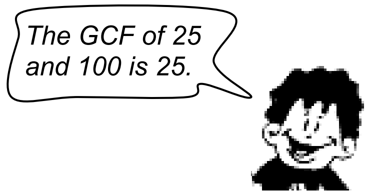
$$= \frac{1}{2}$$

$$\boldsymbol{50\% = \frac{1}{2}} \qquad \textbf{ANSWER}$$



Example 2

Write 25% as a fraction in its simplest form.

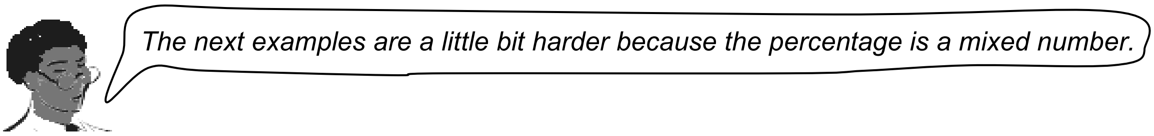


$$25\% = \frac{\overset{1}{\cancel{25}}}{\underset{4}{\cancel{100}}} \qquad \text{(Cancel)}$$

$$= \frac{1}{4}$$

$$\mathbf{25\% = \frac{1}{4}} \qquad \textbf{ANSWER}$$

To express percentage as a fraction in its simplest form, first write as a fraction with a denominator of 100, then simplify.



Example 3

Write $12\frac{1}{2}$% as a fraction in its simplest form.

Follow the steps:

STEP 1: Change $12\frac{1}{2}$% to an improper fraction.

$$12\frac{1}{2}\% = \frac{25}{2}\%$$

STEP 2: Multiply by $\frac{1}{100}$ (This is the same as dividing by 100.)

$$\frac{25}{2} \times \frac{1}{100} = \frac{25}{200}$$

STEP 3: Cancel by the GCF of 25 and 200 which is 25.

$$\frac{\overset{1}{\cancel{25}}}{\underset{8}{\cancel{200}}} \quad \text{(Divide both by 25.)}$$

$$\mathbf{12\frac{1}{2}\% = \frac{1}{8}} \qquad \textbf{ANSWER}$$

Example 4

Write $6\frac{1}{4}$% as a fraction it its simplest form.

STEP 1: Change $6\frac{1}{4}$% to an improper fraction.

$$6\frac{1}{4}\% = \frac{25}{4}\%$$

STEP 2: Multiply by $\frac{1}{100}$ $\quad \frac{25}{4} \times \frac{1}{100} = \frac{25}{400}$

STEP 3: Cancel $\quad \dfrac{\overset{1}{\cancel{25}}}{\underset{16}{\cancel{400}}}$

$$\mathbf{6\frac{1}{4}\% = \frac{1}{16}} \quad \textbf{ANSWER}$$

Since you know how to change percentages to fractions and vice versa, now you will learn to write percentages to decimals and vice versa.

Percentage to Decimal

The basic rule in converting or changing percentages to decimals is to move the decimal point two places to the left and remove the percentage sign (%).

Example 1

Change 80% to a decimal fraction.

80% means 80 out of 100 or $\frac{80}{100}$

$\frac{80}{100} = \frac{80.}{100} = .80$ (Move decimal point 2 places to the left.)

So, $80\% = \frac{80}{100} = 0.8$

Leave out the zero when it is last after a decimal point.

Write zero before the decimal.

80% = 0.80 or 0.8 ANSWER

Example 2

Change 25% to a decimal fraction.

Remember: 25% means 25 out of 100 or $\frac{25}{100}$

and : $\frac{25}{100} = \frac{25.}{100} = .25$

Move the decimal point 2 places left when dividing by 100.

so : $25\% = \frac{25}{100} = 0.25$

We always write a zero before a decimal point.

25% = 0.25 ANSWER

Example 3

Change 3% to a decimal fraction.

3% means $\frac{3}{100}$

$\frac{3}{100} = \frac{3.}{100} = .03$

Put a zero in the space 2 places to the left

3% = 0.03 ANSWER

Decimal to Percentage

The fastest way to change decimals to percents is to shift the decimal point two places to the right. This method is actually the reverse order of converting percent to its decimal equivalent as discussed in the previous lessons.

Example 1

Change 0.82 to a percentage.

0.82 x 100% = 082. %

= 82%

Rewrite the decimal number as 0.82 and multiply by 100. After determining the product attach the percent (%) sign.

0.82 = 82% ANSWER

Example 2

Change 0.1 to a percentage.

0.1 x 100 % = 010. %

= 10%

Put a zero in the space.

0.1 = 10% ANSWER

In all the examples given so far, you learnt more about the relationship between percents and fractions and between percents and decimals.

For the next examples, we will look at percents, fractions and decimals together and learn how to write any one of them in the other two forms.

Example 1

Write 25 % as a fraction and as a decimal fraction.

As a fraction: $25\% = \dfrac{\overset{1}{\cancel{25}}}{\underset{4}{\cancel{100}}}$ OR $\dfrac{1}{4}$ (Always cancel if possible.)

As a decimal: $\dfrac{25}{100}$ = .25 or 0.25 (Move decimal point 2 places to the left.)

So, $25\% = \dfrac{1}{4} = 0.25$

PERCENT	FRACTION	DECIMAL
25%	$\frac{1}{4}$	0.25

Example 2

Write 0.5 as a percent and as a fraction.

As a percent: 0.5 = 0.5 x 100%

= 50. % (Move decimal point 2 places to the right.)

= 50%

As a fraction: $50\% = \dfrac{\overset{1}{\cancel{50}}}{\underset{2}{\cancel{100}}}$ or $\dfrac{1}{2}$ (Cancel or simplify.)

So, $0.5 = 50\% = \dfrac{1}{2}$

Example 3

Write $\dfrac{7}{10}$ as a percent and as a decimal fraction.

As a percent: $\dfrac{7}{10} \times 100\% = \dfrac{700}{10}\%$

$= 70\%$

As a decimal: $70\% = \frac{70}{100}$

$= .70$

$= 0.7$

Example 4

Express 160% as a decimal and as a fraction.

As a decimal: 160% = 1.60 or 1.6

As a fraction: $160\% = \frac{\overset{8}{\cancel{160}}}{\underset{5}{\cancel{100}}} = \frac{8}{5}$

$= 1\frac{3}{5}$

In changing percent to fraction, a percent greater than 100 will result in a mixed number as in Example 4.

NOW DO PRACTICE EXERCISE 14

Practice Exercise 14

1. Students in a class sat a test which was out of 100.

   Here are some marks from the test.

NAME	MARK	PERCENTAGE
Bill	15	__________%
Pita	80	__________%
Ann	73	__________%
Robin	50	__________%
Ora	92	__________%
Martha	47	__________%

   (a) Write their marks as percentages in the third column.

   (b) The students had to score 50% or more to pass the test.

   Write down the names of the students who failed the test.

   ____________ ____________

2. Write the following fractions as percentages.

   The first one has been started for you.

(a) $\frac{2}{5}$, $\frac{2}{\cancel{5}} \times \overset{20}{\cancel{100}}\% = 2 \times 20\%$ $= \_\_\_\_\_ \%$	(b) $\frac{7}{20}$,
(c) $\frac{7}{12}$	(d) $\frac{3}{8}$
(e) $\frac{3}{25}$	(f) $\frac{3}{4}$
(g) $\frac{7}{10}$	(h) $\frac{21}{50}$
(i) $\frac{1}{4}$	(j) $\frac{9}{10}$

3. Write these percentages as fractions in their lowest form.
The first one has been started for you.

(a) 20% = $\frac{20}{100}$
= ______

(b) 5% = ______
= ______

(c) 15% = ______
= ______

(d) 40% = ______
= ______

(e) 75% = ______
= ______

(f) 12% = ______
= ______

(g) 19% = ______
= ______

(h) $66\frac{2}{3}$% = ______ x ______
= ______

---

4. Change the following percentages to decimal fractions.
The first one has been started for you.

(a) 15% = $\frac{15}{100}$ = ______	(b) 50% = $\frac{}{100}$ = ______
(c) 9% = ______ = ______	(d) 17% = ______ = ______
(e) 20% = ______ = ______	(f) 2% = ______ = ______
(g) 28% = ______ = ______	(h) 36% = ______ = ______
(i) 75% = ______ = ______	(j) 1% = ______ = ______

5. Change the following decimal fractions to percentages.

(a) 0.3 = 0.3 x 100 % = _______ %	(b) 0.14 = _______ x 100% = _______ %
(c) 0.25 = _______ x ______ = _______ %	(d) 0.97 = _______ x _______ = _______ %
(e) 0.7 = _______ x ______ = _______ %	(f) 0.04 = _______ x ______ = _______ %
(g) 0.75 = _______ x ______ = _______ %	(h) 0.15 = _______ x ______ = _______ %
(i) 0.49 = _______ x ______ = _______ %	(j) 0.01 = _______ x ______ = _______ %

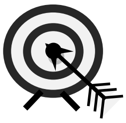
6. Complete the table by filling in the spaces. The first one has been done for you.

PERCENT	FRACTION	DECIMAL
13%	$\frac{13}{100}$	0.13
		0.03
	$\frac{1}{50}$	
	$\frac{3}{10}$	
90%		
	$\frac{3}{20}$	
		0.89
92%		
		0.12
31%		

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 15: Problems on Percentages 1

In the previous lessons, you learnt how to express percent as a decimal and as a fraction.



In this lesson you will:

- identify the three types of percentage problems
- solve different types of problems on percentage

This lesson will enable you to solve many different types of percentage problems.

There are three types of simple problems involving percentage. These are given in the following examples where **n** represents the missing number.

A. 50% of 20 = **n**

B. **n%** of 250 = 50

C. 20**%** of **n** = 100

A problem involving percent is best understood by thinking of the relationship between factors and their product.

In 50% of 20 = n, n is a product, use multiplication.

In n% of 250 = 50, n is a factor, use division.

In 20% of n = 100, n is a factor, use division.

In the preceding examples, three quantities are involved: two factors and a product**.**

TYPE A Finding a Percent of a Number

50% of K20 = n

What is missing, the product or factor? What operation is used to find the missing number?

50% of K20 = **n**

50% x K20 = **n**

$\frac{50}{100}$ x K20 = **n**

0.5 x K20 = **n**

10 = **n**

Therefore, the missing number is 10.

To find a percent of a number, write the percent as a fraction or a decimal and multiply by the number.

Here is another example.

What is 5% of K40?

5% of K40 = **n**

5% x K40 = **n**

$\frac{5}{100}$ x K40 = **n**

0.05 x K40 = **n**

K2.00 = **n**

Therefore, 5% of K40 is K2.

TYPE B Finding what Percent of One Number is Another

n% of 250 = 50

What is missing? Is it a product or a factor? What operation is used to find the missing percent?

n% of 250 = 50

n% x 250 = 50

n% = 50 ÷ 250

n% = 0.20

n = 20 %

Therefore, 20% of 250 = 50 or 50 is 20% of 250.

Here is another example.

What percent of 50 is 20?

n% of 50 = 20

n% x 50 = 20

n% = 20 ÷ 50

n% = 0.40

n% = 40

n = 40%

Therefore, 20 is 40 % of 50.

TYPE C: Finding the Number when a Percent of it is known

20% of **n** = 100

What is missing, a product or one of the factors? What operation is used to solve for a factor? What is the product in the statement, 20% of **n** = 100?

$20\% \text{ of } \mathbf{n} = 100$

$20\% \times \mathbf{n} = 100$

$\frac{20}{100} \times \mathbf{n} = 100$

$0.2 \times \mathbf{n} = 100$

$\mathbf{n} = 100 \div 0.2$

$\mathbf{n} = 500$

The missing number is 500.

Therefore, 100 is 20% of 500.

Here is another example.

60 is 30% of what number?

$30\% \text{ of } \mathbf{n} = 60$

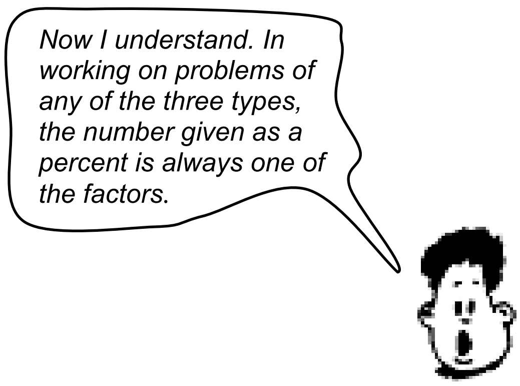
$30\% \times \mathbf{n} = 60$

$\frac{30}{100} \times \mathbf{n} = 60$

$0.3 \times \mathbf{n} = 60$

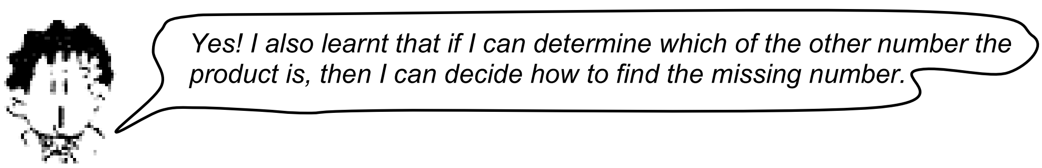
$\mathbf{n} = 60 \div 0.3$

$\mathbf{n} = 200$



The missing number is 200.

Therefore, 60 is 30% of 200.



NOW DO PRACTICE EXERCISE 15

Practice Exercise 15

Find the missing number.

(a) 20% of 60 = **n**

(b) 5% of K500 = __________

(c) __________ = 40% of 200L

(d) 10% of 30 = __________

(e) **n** % of 80 = 20

(f) **n** % of K240 = K60

(g) **n** % of 100 = 10

(h) What percent of 500 is 125?

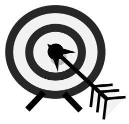
(i) 12 is 30% of what number?

(j) 20% of **n** = 200

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 16: Problems on Percentage 2

You learnt in the previous lesson how to work out the types of simple problems involving percentages.



In this lesson you will;

- solve different types of percentage problems and apply the skills in solving practical situations.

First, we will look at examples which show application of percentages in real life situations.

A percent problem may be stated in the three different ways.

Examples:

1) John sold 5 of his 10 chickens. What percent of his chickens did he sell?
2) John sold 50% of his 10 chickens. How many chickens did he sell?
3) John sold five chickens. This was 50% of his chickens. How many chickens did he have originally?

We can solve each of these simple percent problems by translating them into the types of percent problems we learnt in Lesson 15.

The first problem may be translated into **n** % of 10 = 5.

The second problem may be translated into 50% of 10 = **n.**

The third problem may be translated into 50% of **n** = 5.

Example 1

Tom scored 8 out of 10 marks in a test.

What was his percentage mark?

The problem asked, what percent of 10 is 8?

$$\mathbf{n}\ \%\ \text{of}\ 10 = 8$$

$$\mathbf{n}\ \% \times 10 = 8$$

$$\mathbf{n}\ \% = \frac{8}{10}$$

$$\mathbf{n}\ \% = 0.8$$

$$\mathbf{n}\ \% = 80\%$$

Therefore, 80% of 10 is 8.

Tom scored 80%. ANSWER

Example 2

Garo bought 500 day-old chicks for his chicken farm. 6% of the chicks died.

How many chicks died?

The problem states, 6% of the 500 chicks died.

6% of 500 = **n**

6% x 500 = **n**

0.06 x 500 = **n**

30 = **n**

Therefore, 30 chicks died. ANSWER

Example 3

By buying a set of notebooks instead of buying them individually, Peter saved K3.00. This is 10% of the price for a set.

What is the price of a set of notebooks?

The problem states, K3.00 is 10% of the price.

10% of the price is K3.00

10% of **n** = K3.00

n = K 3.00 ÷ 10%

n = K3.00 ÷ 0.1 $\left(\frac{3.00}{0.1} = \frac{30.0}{1}\right)$

n = K30.00

Therefore, the price of the set of notebooks is K30.00. ANSWER

NOW DO PRACTICE EXERCISE 16

Practice Exercise 16

Solve the following:

1) 20 out of 30 students came to school.

What percentage of the students came to school?

__________**ANSWER**

---

2) Paru receives K120 every fortnight and spends 32% of what he receives.

How much is Paru‟s fortnightly expenses?

__________**ANSWER**

---

3) In a canoe race, only 9 canoes were able to finish the race. This is 45% of the total number of canoes that participated.

How many canoes actually participated in the race?

__________**ANSWER**

4) A man was walking from his village to the next village which was a distance of 8 km. When he had walked 5 km, he rested.

What percentage of the total distance had he walked?

__________**ANSWER**

---

5) Of the 365 students in Mathematics at FODE, 80% passed the end-of-year examination.

How many students passed the examination?

__________**ANSWER**

---

6) Miro received 10% increase in his salary.

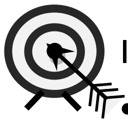
If his increase is K120.00, what was his previous salary?

__________**ANSWER**

CORRECT YOUR WORKS. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 17: Problems on Percentage 3

You learnt to work out different types of simple percentage problems with context in Lesson 16.



In this lesson you will:

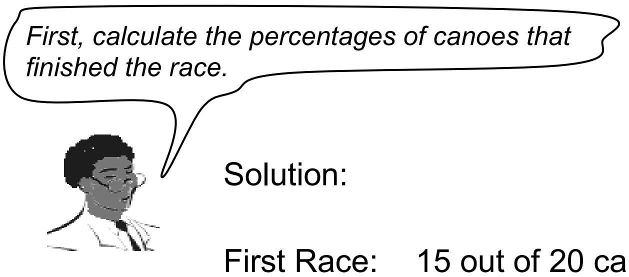
- solve different types of percentage problems requiring more than one calculation.

Examples of percentage problems with more than one calculation.

Example 1

There were two canoe races at a village. In the first race, 20 canoes participated and 15 finished the race. In the second race, 25 canoes participated and 18 finished the race.

Which race had the bigger percentage of canoes to finish?



Solution:

First Race: 15 out of 20 canoes or $\frac{15}{20}$ finished the race

$\mathbf{n}\ \% = \frac{15}{20}$

$\mathbf{n}\ \% = 0.75$

$\mathbf{n} = 75\%$

Second Race: 18 out of 25 canoes or $\frac{18}{25}$ finished the race

$\mathbf{n}\ \% = \frac{18}{25}$

$\mathbf{n}\ \% = 0.72$

$\mathbf{n} = 72\%$

The first race had the bigger percentage. ANSWER

Example 2

A trade store owner buys a carton of noodles for K12.00. He increased the price by 25% before he sells the noodles.

(a) What is his profit?

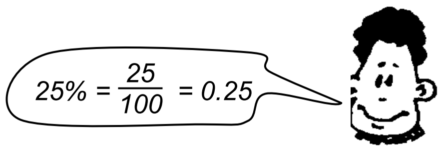
(b) How much does he sell the noodles?

Solution:

(a) He increases the price of K12.00 by 25%.

So, increase **n** = 25% of K12.00

n = $\frac{25}{100}$ x K12.00



n = 0.25 x K12.00

n = K3.00

Profit is K3.00 ANSWER

(b) Selling Price = Cost Price + Profit

= K12.00 + K3.00

= K15.00 ANSWER

Example 3

Wardstrip Primary School has 400 students of which 35% are girls.

(a) How many are girls?

(b) What percentage of the students are boys?

Solution:

(a) Number of girls **n** = 35% of 400

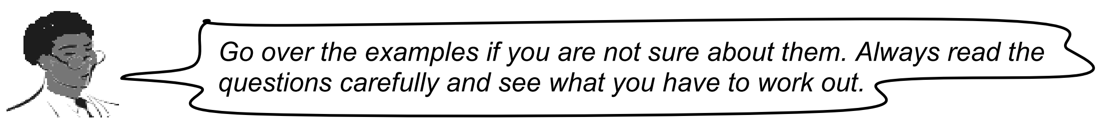
n = $\frac{35}{100}$ x 400

n = 0.35 x 400

n = 140 girls ANSWER

(b) Percentage of boys = 100% - 35%

= 65% ANSWER



NOW DO PRACTICE EXERCISE 17

Practice Exercise 17

Solve the following problems.

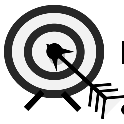
PROBLEM	WORKING OUT
1) Peter sat the English and Mathematics tests. In English he scored 35 out of 50 and in Mathematics, 40 out of 60. In which test did he score the better percentage mark? **ANSWER:** He scored a better percentage mark in the ___________ test.	ENGLISH: 35 out of 50 = **n**% **n**% = $\frac{35}{50}$ **n**% = **n** **=** ______% MATHEMATICS: 40 out of 60 = **n**% **n**% = $\frac{40}{60}$ **n**% = **n** **=** ______%
2) Toua bought an old car for K3000. He wants to increase the price by 10% and sell it. (a) What is his profit? (b) How much will he sell the car for? **ANSWERS** (a) Profit = K______ (b) Selling Price = K______	(a) Increase: **n** = 10% of K3000 **n** = **n** = **n** = K_____ (b) Selling Price = Cost Price + Profit = =

3) In a group of 60 girls, 25% play netball and the rest, play softball. (a) How many play netball? (b) What percentage of the girls play softball? **ANSWERS** (a) ______ play netball. (b) ______% of the girls play softball.	Netball: 25% of 60 = = = Sofball: Percentage = =
4) John planted 80 cabbage seeds in his garden. After one week, he counted 15 seedlings growing. (a) What percentage of seeds had grown? (b) What percentage of the seeds were yet to grow? **ANSWERS** (a) ______% of the seeds had grown. (b) ______% of seeds were yet to grow	Percentage(**n**%) = 15 out of 80 or $\frac{15}{80}$ **n**% = **n**% = **n** = _______ % Percentage left = 100% - _____% = ________%
5) A man buys a carton of tinned fish for K36.00 and sells it for K40.00 (a) How much profit does he make? (b) What is his percentage profit? **ANSWERS** (a) His profit was _K______ (b) His percentage profit was _________%.	Profit = Selling Price - Cost Price = K40.00 - K36.00 = __K_______ Percentage Profit = $\frac{\text{Profit}}{\text{Cost Price}}$ x 100% = __________ x 100 = _______ = _______%

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 18: Computing the Discount, Sale Price and Marked Price

You learnt to solve different types of percentage problems requiring more than one calculation in Lesson 17.



In this lesson you will:

- define discount, marked price, sale price and rate of discount
- identify discount, marked price, sale price and rate of discount
- work out problems involving discount.

When you go to a store, you sometimes see these signs or advertisements.

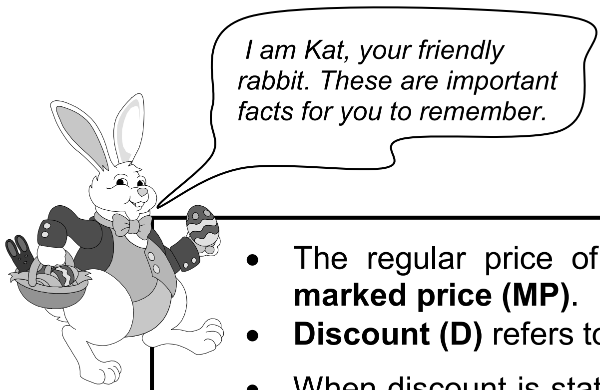
“10% reduction on each item bought”

“20% Discount” or 12% Off”

„Reduction Sale by 15%”

Advertisements are used to attract the consumers to buy quality goods at lower prices. Discounts come as part of a store‟s clearance and promotional sales. Housewives especially, take advantage of this opportunity to save.

For better understanding of these signs, look at what our friend Kat has for you.



- The regular price of an article is called its **list price** or **marked price (MP)**.
- **Discount (D)** refers to the reduction or decrease in prices.
- When discount is stated as a percentage, it is called **rate of discount (r %)** or **percent of discount**.
- The price of an article after the discount has been deducted is called its **net price or sale price (SP)**.

Below are examples of problems on discount.

Example 1

The marked price of a book is K40.00.

How much is the selling price if 20% discount is given to those who buy more than 10 copies?

Given in the problem: Marked price (MP) = K40

Rate of discount (r%) = 20%

Unknown in the problem: Discount (D) = __________

Sale price (SP) = __________

Solution: We have to find the amount of discount first to get the sale price.

Discount = 20% of K40 = 0.2 x K40 = K8

Therefore, Sale price (SP) = K40 – K8

SP = K32

The selling price of the book is K32. ANSWER

Example 2

What is the rate of discount if a lady‟s bag selling for K80 is sold at K60?

Given: Marked price (MP) = K80

Sale price (SP) = K60

Find: Discount (D) = __________

Rate of discount (r%) = __________

Solution: D = K80 – K60

= K20

Therefore, rate of discount is;

r% of K80 = 20

$$r\% = \frac{20}{80}$$

r% = 0.25 or r = 25%

The rate of discount given is 25%. ANSWER

Example 3

Find the regular price of an article that is sold for K90 at 20% reduction.

Solution: Given: Sale Price = K90

Rate of discount = 20%

Find: Regular price or marked price (MP)

If the rate of marked price = 100%, then 100% - 20% = 80% is the rate of sale price.

Therefore: 80% of Marked price = Sale price

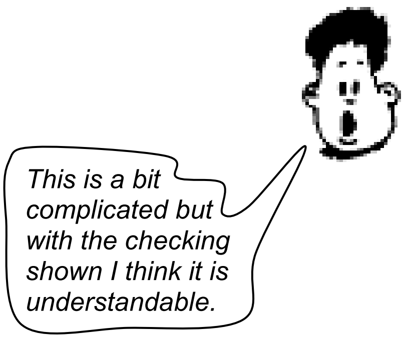
80% of n = K90

80% x n = K90

n = K90 ÷ 80%

n = K90 ÷ 0.8

n = K112.50



Marked Price = K112.50 ANSWER

To check: Sale price = K90

Rate of discount = 20%

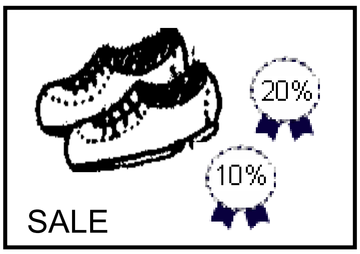
Marked Price = K112.50

Discount = 20% of K112.50 = K22.50

K112.50 – K22.50 = K90

Example 4

What is the sale price of a pair of shoes marked K200 when the rates of discount are 20% and 10%?



Given: Marked Price = K200

Rates of discount = 20% and 10%

Find: Sale price (SP)

Solution: Amount of first discount = 20% of K200 = K40

Sale price after fist discount = K200 – K40 = K160

Amount of second discount = 10% of K160 = K16

Therefore: Sale price = K160 – K16 = K144

Sale price = K144 ANSWER

REMEMBER:

To find the discount (D) when the marked price (MP) and the rate of discount (r %) are given, multiply the marked price by the rate of discount.

$$\mathbf{D = MP \times r\%}$$

To find the sale price (SP), subtract the discount from the marked price.

$$\mathbf{SP = MP - D}$$

To find the rate of discount (r%), find what percent of the marked price the discount is.

$$\mathbf{r\% \text{ of } MP = D \quad \text{or} \quad r\% = \frac{D}{MP}}$$

To find the marked price (MP) when the sale price (SP) and rate of discount (r%) are given, subtract the given rate from 100%. Then divide the sale price by the result.

$$\mathbf{MP = \frac{SP}{100\% - r\%}}$$

When two or more successive discount rates are given, do not add the rates but use them one at a time.

NOW DO PRACTICE EXERCISE 18

Practice Exercise 18

1) Use this advertisement to answer the questions that follow.

(a) Ata purchased the Challenge Master while it was on special sale.

How much did he save?

ANSWER__________

(b) Shaun purchased the Sonic Rally Race game and the Formula 1 Raceway before the sale.

How much would he have saved if he bought them during the special sale?

ANSWER__________

(c) Simon purchased Stock Challenge, Sonic Speedway and Challenge Master at the special sale.

How much change did he receive from K70?

ANSWER__________

(d) What is the total cost of 1 LCD Golf game, 2 Sonic Speedway, 2 Hot Wheel Grippers and 1 Sea Battle game bought at the special sale?

ANSWER__________

2) An advertisement is marked as follows: "Originally K45, now only K31.50".

a) What is the amount of reduction?

ANSWER__________

b) What is the percent of reduction?

ANSWER__________

---

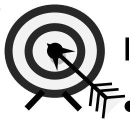
3) On which is the rate of discount greater and by how much: a bag of goods from K150 to K120 or one reduced from K100 to K75?

ANSWER__________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 19: Borrowing and Lending Money

You learnt to apply your knowledge about percentages to situations in real life involving discounts.



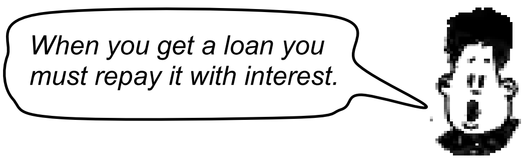
In this lesson you will:

- define interest, principal and rate of interest
- apply the knowledge and skills about percent, base, and rate to situations involving borrowing and lending money.

Sometimes, a person may need a big amount of money to buy a car or a truck or to start a business. Businessmen need capital to survive. However, instances arise when they need to borrow from banks or to get a loan to finance a project.

It is often possible to borrow money from banks, investment houses, savings and loans societies or associations, cooperatives, credit unions, and other financing companies or even from a friend.

When you borrow money it is called a LOAN. Therefore, a loan is money borrowed while interest is extra money paid when you repay a loan.



The formula for computing the interest is:

INTEREST = AMOUNT REPAID – AMOUNT BORROWED
or
AMOUNT BORROWED = AMOUNT REPAID – INTEREST
or
AMOUNT REPAID = AMOUNT BORROWED + INTEREST

Look at the following examples about loans.

Example 1

Alex secured a loan of K2500 from a bank to start a business. The bank charged Alex K750 interest on his loan.

How much was his loan repayment?

Loan repayment = Amount Borrowed + Interest

= K2500 + K750

= K3250

Therefore, Alex had to repay K3250 to the bank. ANSWER

Example 2

Bona borrowed K1980 from a bank for his business. He paid back K2543 to the bank.

How much interest did the bank charge him?

Interest = Amount Repaid – Amount Borrowed

= K2543 – K1980

= K563

Therefore, the bank charged Bona an interest of K563. ANSWER

Example 3

A man borrowed from the bank to build a house. The bank charged him K695 interest and he repaid K2985 to the bank.

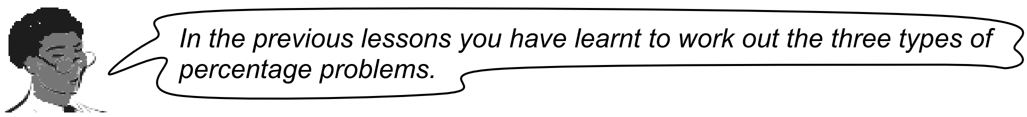
How much money did he borrow?

Amount Borrowed = Amount Repaid – Interest

= K2985 – K695

= K2290

Therefore, the man borrowed K2290. ANSWER



We will now look at problems on interest related to the three types of percentage problems.

Usually, when money is deposited in or borrowed from the bank, interest is paid.

Interest is the major source of income for banks. Three factors determine how much interest is charged for a loan which are; the **principal,** the **time** and the **rate of interest.**

- The **principal** is the amount of money on which interest is calculated. It also refers to the amount of loan.
- The **interest rate** is the percent of the principal to be paid each unit of time.
- The **time** is the number of units expressed in days, months and years for which the money is borrowed. It is usually expressed in years
- **Interest** is calculated as a percentage of the amount of money borrowed (principal) per annum (p.a) or per year.

Now look at Table 19.1

TABLE 19.1

Principal	**Rate**	**Time**	**Interest**
K100	8% p.a.	1 year	K8
K100	8% p.a.	2 years	K16
K100	8% p.a.	3 years	K24
K100	8% p.a.	4 years	K32

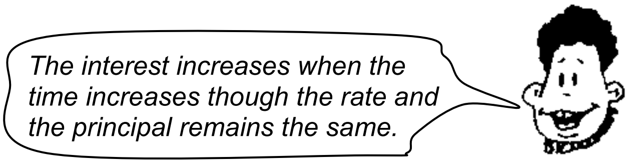
The first entry is computed this way:

8% of K100 = 0.08 x K100

= K8.00

Since the rate is for a period of one year, and the time is one year, the interest is K8.

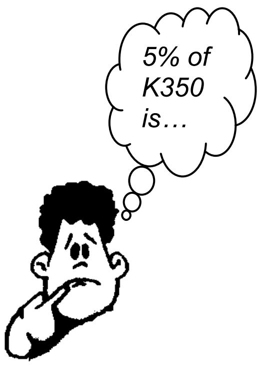
Notice what happens to the interest if the rate and the principal remain the same while the time in years increases.



Here are some examples.

Example 1

Find the interest of K350 for one year at 5% interest?



Given: principal = K350

rate = 5% or 0.5

time = 1 year

What is asked: Interest for 1 year

Solution:

Interest = 5% of K350

= K350 x 0.05 x 1

= K17.50

Therefore, K17.50 is the interest for 1 year. ANSWER

Example 2

Mrs. Aisi borrowed K1000 at 10% interest per annum for 6 months.

How much did she pay back?

Given: principal = K1500

time = 6 months

rate = 10%

What is asked: a) interest for 6 months or $\frac{6}{12}$ year

b) amount to be paid back

Solution:

a) 10% x K1500 = interest for 1 year

$$10\% \times K1500 \times \frac{6}{12} = \text{interest for 6 months}$$

$$0.1 \times K1500 \times 0.5 = K75, \text{ interest for 6 months}$$

b) principal + interest = amount to be paid back

K1500 + K75 = K1575

Therefore, K1575 is the amount to be paid back by Mrs. Aisi ANSWER

NOW DO PRACTICE EXERCISE 19.

Practice Exercise 19

(1) Write short answers to the following questions.

(a) What is a loan?

____

____

(b) What is interest?

____

____

(c) What is principal?

____

____

(2) Tau borrowed K1500 to start his tradestore. The interest charged on this loan was K590.

Calculate the loan repayment.

____________ **ANSWER**

(3) Vagi borrowed K1890 from the bank to start his piggery business. The bank asked him to repay K2750.

What was the interest charged?

____________ **ANSWER**

(4) A retailer wished to buy a utility to transport his goods to his store so he got a loan from the bank. The interest charged was K975. The retailer repaid K3475 to the bank.

Calculate the amount borrowed.

____________ **ANSWER**

---

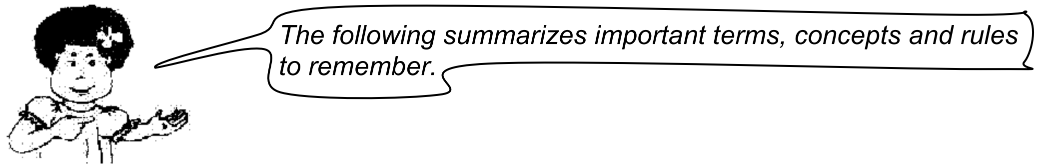
(5) At the end of each year, Hanna gets 9% interest to her bank account.

What interest is added if the balance is K516.24?

____________ **ANSWER**

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

SUB-STRAND 3: SUMMARY



- A percentage is a convinient way of writing fractions that have a denominator of 100. “Per cent”, written % means “per 100” or “for every 100”.

  Example: $7\% = \frac{7}{100}$

- To write a percentage as a fraction or a mixed number, first write it as a fraction with denominator of 100, then simplify.

  Example: $125\% = \frac{125}{100} = 1\frac{1}{4}$

- To change fractions to percentages, first change the denominator of the fraction to 100.

  Example: $\frac{3}{20} = \frac{15}{100} = 15\%$

- To change a percentage to a decimal, we can write it first as a fraction and then as a decimal.

  Example: $93\% = \frac{93}{100} = 0.93$

- To change a decimal to a percentage, we can write it as a fraction first, then change it to a percentage or we can multiply the decimal by 100% or simply move the decimal point 2 places to the right and put the symbol %.

  Example: $0.93 = \frac{93}{100} = 93\%$ or $0.93 \times 100\% = 93\%$

- A problem involving percentages is best understood by thinking of the relationship between factors and their product.

  Type A: 10% of 30 = n, n is a product; use multiplication.

  Type B: n% of 250 = 50, n is a factor, use division.

  Type C: 20% of n = 100, n is a factor, use division.

- Discount refers to the reduction in prices.
- Interest is the extra amount paid when money is deposited or borrowed from the bank.
- The **principal** is the amount of money on which interest is calculated. It also refers to the amount of loan.
- The **interest rate** is the percent of the principal to be paid each unit of time.
- The **time** is the number of units expressed in days, months and years for which the money is borrowed. It is usually expressed in years

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REVISE LESSONS 14-19. THEN DO SUB-STRAND TEST 3 IN ASSIGNMENT 1.

ANSWERS TO PRACTICE EXERCISES 14-19

Practice Exercise 14

1. (a)

Name	Mark	Percentage
Bill	15	15%
Pita	80	80%
Ann	73	73%
Robin	50	50%
Ora	92	92%
Martha	47	47%

(b) Bill , Martha

---

2. (a) 40% (b) 35% (c) $58\frac{1}{3}\%$ (d) $37\frac{1}{2}\%$

(e) 12% (f) 75% (g) 70% (h) 42%

(i) 25% (j) 90%

---

3. (a) $\frac{1}{5}$ (b) $\frac{1}{20}$ (c) $\frac{3}{20}$ (d) $\frac{2}{5}$

(e) $\frac{3}{4}$ (f) $\frac{3}{25}$ (g) $\frac{19}{100}$ (h) $\frac{2}{3}$

---

4. (a) 0.15 (b) 0.5 (c) 0.09 (d) 0.17

(e) 0.2 (f) 0.02 (9) 0.28 (h) 0.36

(i) 0.75 (j) 0.01

---

5. (a) 30% (b) 14% (c) 25% (d) 97%

(e) 70% (f) 4% (9) 75% (h) 15%

(i) 49% (j) 1%

6.

Percent	Fraction	Decimal
13%	$\frac{13}{100}$	0.13
3%	$\frac{3}{100}$	0.03
2%	$\frac{2}{100}$ or $\frac{1}{50}$	0.02
30%	$\frac{30}{100}$ or $\frac{3}{10}$	0.3
90%	$\frac{90}{100}$ or $\frac{9}{10}$	0.9
15%	$\frac{15}{100}$ or $\frac{3}{20}$	0.15
89%	$\frac{89}{100}$	0.89
92%	$\frac{92}{100}$ or $\frac{23}{20}$	0.92
12%	$\frac{12}{100}$ or $\frac{3}{25}$	0.12
31%	$\frac{31}{100}$	0.31

Practice Exercise 15

1. a) 12 b) 25 c) 80L d) 3 e) 25%

f) 25% g) 10% h) 25% i) 40 j) 1000

Practice Exercise 16

1) 66.7% or $66\frac{2}{3}$ % 2) K38.40 3) 20 canoes

4) 62.5% or $62\frac{1}{2}$ % 5) 292 6) K1 200

Practice Exercise 17

1) English 70%, Mathematics $66\frac{2}{3}$ %. He scored better in the English test.

2) (a) Profit = K300 (b) Selling Price = K3300

3) (a) 15 girls play netball (b) 75% of girls play softball

4) (a) 18.75% of the seeds had grown
(b) 81.25% of the seeds were yet to grow.

5) (a) Profit = K4.00 (b) Percentage Profit = 11.11% or 11 $\frac{1}{9}$ %t to

---

Practice Exercise 18

1) a) K18.00 b) K20.00 c) K2.00 d) K106.00

2) a) k13.50 b) 30%

3) The bag reduced from K100 to K75 sells at a discount rate of 25%, the other at 20%.

Practice Exercise 19

1) (a) A loan is money borrowed.
(b) Interest is extra money paid when a loan is repaid.
(c) Principal is the amount of money on which interest is calculated. it refers to the amount of loan.

2) Loan repayment = Amount borrowed + interest
= K1 500 + K590
= <u>K2 090.00</u>

3) Interest charged = Amount repaid - Amount borrowed
= K2 750 - K 1890
= <u>K860.00</u>

4) Amount borrowed = Amount repaid - Interest
= K3 475 - K975
= <u>K2 500</u>

5) Interest = 9% of account balance
= 9% of K516.24
= 0.09 x K516.24
= K46.4616
= K46.46

END OF SUB-STRAND 3

SUB-STRAND 4

RATIOS AND RATES

SUB-STRAND 4: RATIOS AND RATES

Introduction

The previous Sub-strand introduced you to different ways of understanding of fractions. In this Sub-strand, this understanding is extended to include important concepts in mathematics.

The terms fraction, percent and ratio are used to make comparisons and suggest ways of expressing numerical relationships.

The following statements and the diagram above show some uses of ratios.

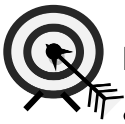
1. The ratio of adults to children watching a certain show is 2 is to 3.
2. To make 8 pieces of pizza, Eope uses 2 cups of flour. If David wants to make a dozen, he has to use 3 cups of flour.

In this Sub-strand, you will:

- learn about ratios
- use ratio and rates
- determine equivalent ratios
- write ratios in their simplest form
- solve problems using ratio
- determine the relationship between Ratio and Scale for length.

Lesson 20: What is Ratio?

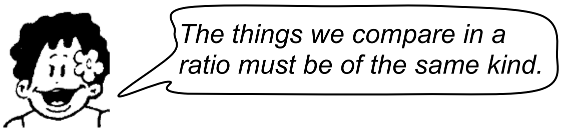
In the previous lessons, you learnt to convert fractions to decimals and percentages. You also learnt to calculate discount, marked price, sale price and rate of discount



In this lesson you will:

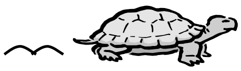
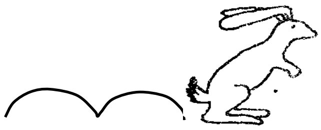
- define ratio
- compare and order quantities using ratio

First you will learn the meaning of “RATIO”. (you say Ray – she – oh).



Ratio is a way of comparing quantities of the same kind, in order.

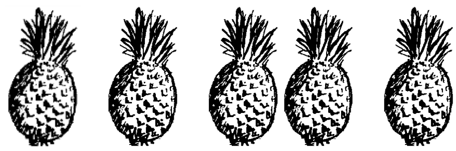
The pictures below will give you ideas on ratio.

 Speed of a tortoise = 1 km per hr	 Speed of a rabbit = 20 km per hr

The ratio of the two speeds is 1 is to 20 or 1: 20

 Number of coins in Ibon‟s pocket = 5	 Number of coins in Ara‟s pocket = 2

The ratio of the two lots of 20t coins is 5 is to 2 or 5: 2

 Number of apples = 4	 Number of pineapples = 5

The ratio of the two lots of fruits is 4 is to 5 or 4: 5.

The ratio also shows the ORDER in which we compare the things.

Use the ratio sign (**:**) *between the quantities we compare.*

You learnt that the speed of the tortoise is 1 km/hr while the speed of the rabbit is 20 km/hr.

We write the ratio of the two speeds this way:

Speed of tortoise **:** Speed of rabbit = 1 **:** 20

RATIO SIGN RATIO SIGN

There are two ways to say these in ratio sign.

(1) The speed of the tortoise <u>to</u> the speed of the rabbit <u>equals</u> 1 <u>is to</u> 20.

(2) The <u>ratio of</u> the speed of the tortoise <u>to</u> the speed of the rabbit is 1 <u>is to</u> 20.

Here is another example.

Walking along the beach, Sarah collected 12 shells and Jane collected 7 shells.

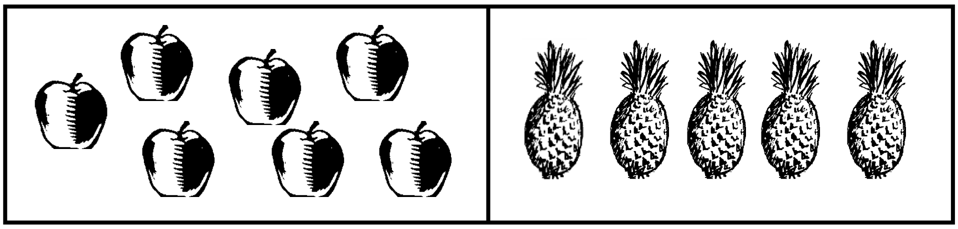
So, Sarah‟s shells : Jane‟s shells = 12 : 7

We say: Sarah‟s shells to Jane‟s shells ratio equals 12 is to 7 or
The ratio of Sarah‟s shells to Jane‟s shells is 12 is to 7.

It is important that we write the numbers of a ratio in order.

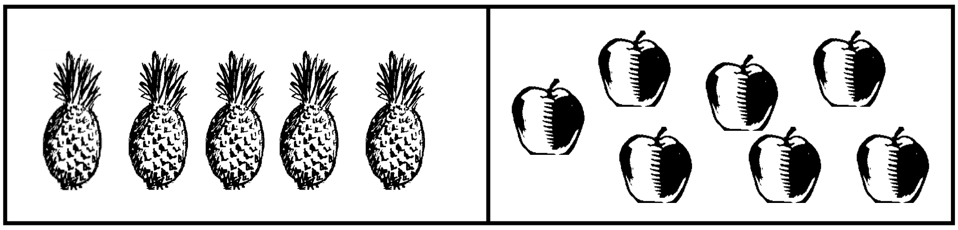
Look at the diagrams below.

DIAGRAM 1



Apples : Pineapples = 7 : 5

DIAGRAM 2



Pineapples : Apples = 5 : 7

Both ways of writing the ratios on the 2 diagrams are correct.

The order of the numbers in the ratio must be the same as the order of the quantities we are comparing.

Here is an example.

Example

FIRST PLACE

Apples : Pineapples = 7 : 5

SECOND PLACE

The number 7 refers to APPLES so it must be in the <u>same</u> place (FIRST).

The number 5 refers to PINEAPPLES so it must be in the <u>same</u> place (SECOND).

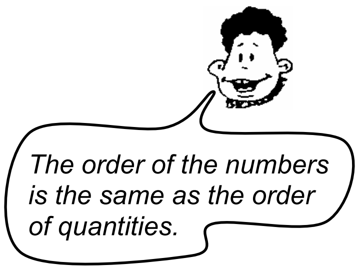
We can change the order of the numbers if we also change the order of the quantities.

Example

FIRST PLACE

Pineapples : Apples 5 : 7

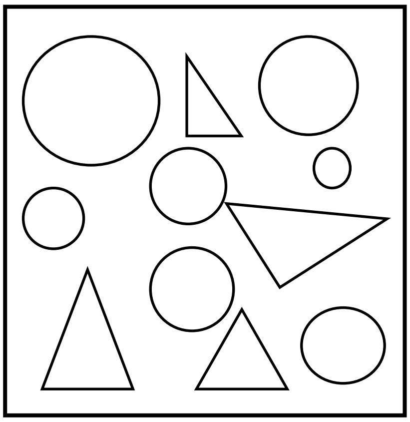
SECOND



The order of the numbers in a ratio is the same as the order of the quantities.

Sometimes, the order of the quantities does not matter.

Look at the diagram below.



In the diagram there are some circles and some triangles. We can write the ratio of these two different types of shapes in 2 ways:-

(a) The ratio of circles to triangle is 7 is to 4.

Circles : Triangles = 7 : 4

(b) The ratio of triangles to circles is 4 is to 7.

Triangles : Circles = 4 : 7

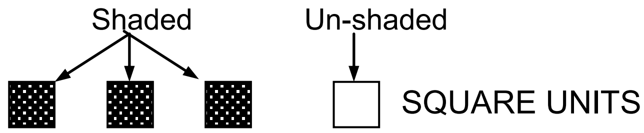
Here is another example.

The diagram shows two types of square units: shaded and un-shaded.

Write the ratio shown in the diagram in two ways:

(a) Shaded : Un-Shaded

(b) Un-shaded : Shaded



ANSWER: There are 3 shaded units and 1 un-shaded unit, so

(a) Shaded : Un-shaded = 3 : 1

(b) Un-shaded : Shaded = 1 : 3

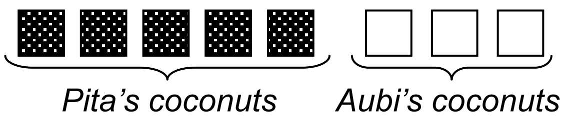
We can use diagrams of shaded and un-shaded units to show ratios.

Example 1

Pita sold 5 coconuts and Aubi sold 3 coconuts. Draw a diagram to show the ratio of Pita‟s coconuts to Aubi‟s coconuts.

RATIO DIAGRAM

Pita‟s coconuts : Aubi‟s coconuts = 5 : 3

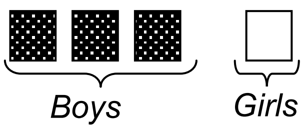


Example 2

There were 3 boys and 1 girl in a family. So the ratio of boys to girls is 3 is to 1. Use a diagram to show the ratio of boys to girls.

RATIO DIAGRAM

boys : girls = 3 : 1



NOW DO PRACTICE EXERCISE 20.

Practice Exercise 20

1. Write these ratio sentences in words. The first one is done for you.

   a) There are 6 glasses in one serving bowl, so

   Glasses : Serving bowl = 6 : 1

   (i) *Glasses to serving bowl equals 6 is to 1.*

   (ii) *The ratio of glasses to serving bowl is 6 is to 1.*

   b) In the Mathematics class, there are 18 boys and 12 girls, so

   Boys : Girls = 18 : 12

   (i) ______________________________

   (ii) ______________________________

   c) Ororo collected 18 mangoes and Tau collected 24 mangoes, so

   Ororo‟s mangoes : Tau‟s Mangoes = 18 : 24

   (i) ______________________________

   (ii) ______________________________

2. Here are some sentences. Write the sentences using the ratio sign.

   a) The ratio of adults to children watching a certain show is 3 is to 2.

   ____

   b) In a class of 45 students, there are 24 boys and 21 girls.

   (i) The ratio of boys to girls

   ____

   (ii) The ratio of girls to boys

   ____

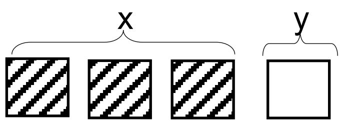
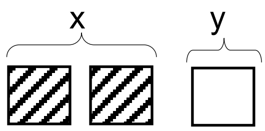
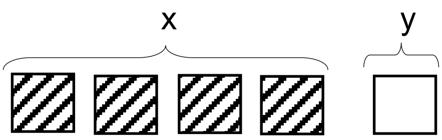
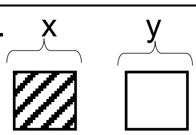
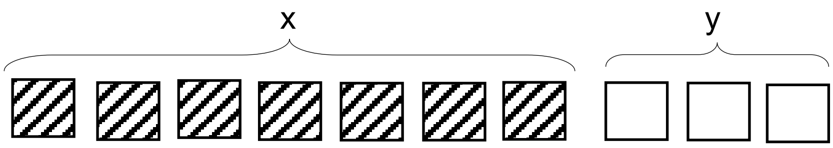
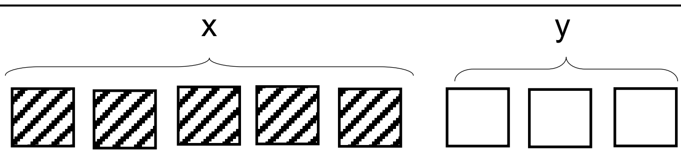
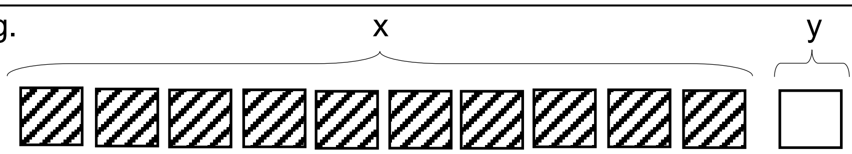
   (iii) The ratio of boys to class

   ____

   (iv) The ratio of girls to class

   ____

3. Complete the table by writing down the values of the ratios x : y and y : x.
   The first one is done for you.

DIAGRAM	x : y	y : x
a. 	**3 : 1**	**1 : 3**
b. 		
c. 		
d. 		
e. 		
f. 		
g. 		

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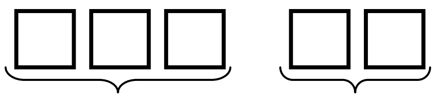
4. Use diagrams of shaded and un-shaded units to show the following ratios.

a) A business owned 3 cars and 2 trucks, so the ratio of cars to trucks is 3 : 2.

RATIO

_______ : ________ = 3 : 2

DIAGRAM



____________ _________

b) The ratio of chickens to pigs in a mumu is 7 **:** 2.

RATIO DIAGRAM

_______ **:** ________ = ___ **:** _____

c) A dog had 8 pups. 5 were black and 3 were brown. What is the ratio of the two different colored pups. (Choose the two correct answers).

(a) Black : Brown = 5 **:** 3

(b) Brown : Black = 5 **:** 3

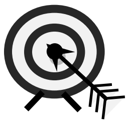
(c) Black : Brown = 3 **:** 5

(d) Brown : Black = 3 **:** 5

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 21: Using Ratio and Rates

In Lesson 20, you defined ratio and learnt to compare and order quantities using ratio.



In this lesson you will:

- work out the ratio of quantities of the same kind
- work out the ratio of quantities of different kinds.

This lesson consists of two parts. In the first part, you will learn how to work out the ratio when quantities of the same kind are measured in the same and in different units. In the second, you will work out the ratio when the quantities are of different kinds.

Comparing the Same Kind of Quantities in the Same Unit

Example 1

The width of a rectangle is 8 cm and its length is 17 cm. The ratio of its width to length is 8 cm to 17 cm, or

Length : Width = 8cm : 17 cm

= 8 : 17

We compare centimetres with centimetres, we do not need to write the units.

Example 2

Oala and Christine sell fruits to earn extra money. Oala earns K40 and Christine, K33.

What is the ratio of Oala‟s earnings to Christine‟s earnings?

Solution: Oala‟s earnings : Christine earnings

K40 : K33

40 : 33

We compare kina with kina, so we do not need to write the units.

Therefore, the ratio of Oala‟s earnings to Christine‟s earnings is 40 is to 33 or 40:33.

When comparing quantities of the same unit, do not write the unit in the answer.

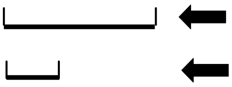
Comparing the Same Kind of Quantities in Different Units

Sometimes quantities are expressed in different units. So, the conversion to common units is performed to simplify a ratio.

Consider the following examples.

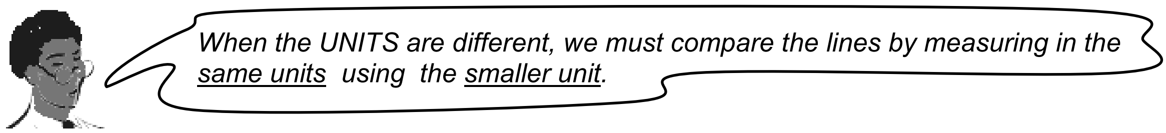
Example 1

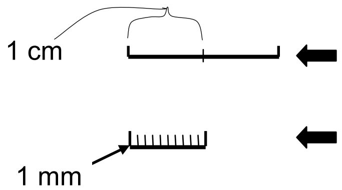
Work out the ratio of 2 cm to 7 mm.



This line is 2 CENTIMETRES long.

This line is 7 MILLIMETRES long.



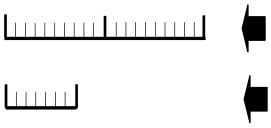


The 2 cm line has been measured with LARGER units.

The 7 mm line has been measured with SMALLER units.

STEP 1 Change the quantities to the smaller unit.

The smaller unit is millimetres.



2 cm = 20 mm (because there are 10 mm in 1 cm)

7 mm stays the same; 7 mm.

STEP 2 Write the quantities as a ratio.

The ratio of 2 cm to 7 mm = the ratio of 20 mm to 7 mm.

= 20mm : 7mm

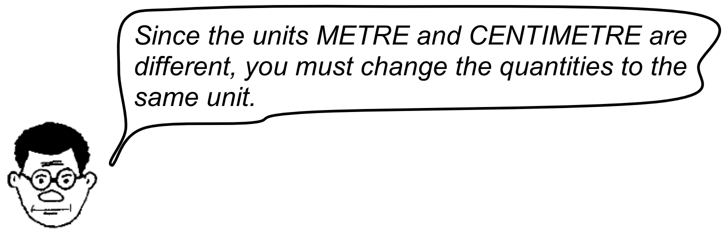
= 20 : 7 ANSWER

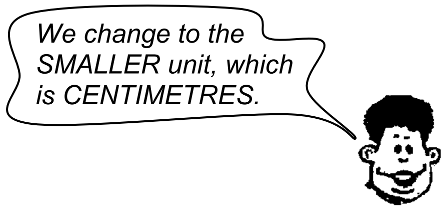
REMEMBER:

- **Change different units to the SAME UNIT.**
- **Write the final ratio without the unit.**

Example 2

Work out the ratio of 1 metre to 1 centimetre.





STEP 1 Change the quantities to the smaller unit.
The smaller unit is centimetres.
1 m = 100 cm (There are 100 cm in one metre.)
1 cm stays the same.

STEP 2 Write the quantities as a ratio.
The ratio of 1 m to 1 cm = the ratio of 100 cm to 1 cm
= 100 cm : 1 cm
= 100 : 1 ANSWER

Example 3

Work out the ratio of 1 hour 40 minutes to 3 minutes.

STEP 1 Change the quantities to the smaller unit.
The smaller unit is minutes.
1 h 40 min = 60 min + 40 min (There are 60 minutes in one hour.)
3 min stays the same.

STEP 2 Write the quantities as a ratio.
The ratio of 1 h 40 min to 3 min = the ratio of 100 min to 3 min
= 100 : 3
= 100 : 3 ANSWER

NOW DO PRACTICE EXERCISE 21A

Practice Exercise 21A

1) Work out the ratio of the length of LINE A to the length of LINE B.

LINE A 3 cm

LINE B 13 mm

STEP 1 Change the quantities to the ______________ unit.

The smaller unit is ____________________.

3 cm = ______mm (There are 10 mm in a centimetre.)

13 mm stays the same.

STEP 2 Write the quantities as a ratio.

Ratio of 3 cm to 13 mm = ratio of _____ mm to _____ mm

= _____ : _____

= _____ : _____ **ANSWER**

---

2) Work out the ratio of 1 kilogram to 7 grams.

STEP 1 Change the quantities to the ______________ unit.

The smaller unit is ____________________.

1 kg = ______g (There are 1000 g in 1 kilogram.)

____ g stays the same.

STEP 2 Write the quantities as a ratio.

Ratio of 1 kg to 7 g = ratio of _____ g to _____ g

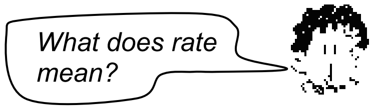
= _____ : _____

= _____ : _____ **ANSWER**

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

You have learnt to work out the ratio of two quantities of the same kind.

Now you are going to learn using ratio to express rate and distinguish rate from ratios.



Rates and Unit Rate

A rate is similar to a ratio, but in a rate, the two numbers compared are of two different kinds of measurements.

30 mL of water compared to 10 mL of cordial is an example of ratio. Both numbers are capacities measured in millilitres.

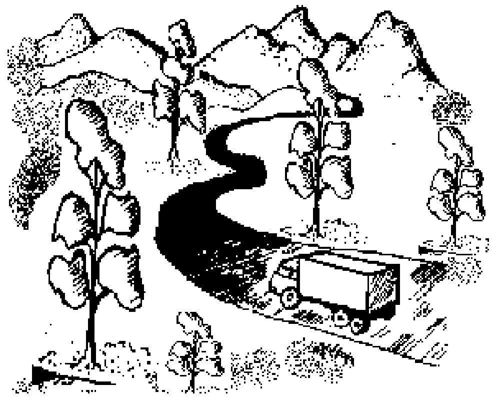
135 km in 1 hour is a comparison of distance and time. This is an example of rate. One number is a distance measurement (km) and the other is time measurement (hours).

Ratio is also used to express rate or unit rate. A unit rate is the simplified form of rate.

Example 1

A truck travels 554 kilometres on 62 litres of petrol.

What is the rate?



STEP 1 Write the ratio of 554 km to 62 litres in fraction form.

$$\frac{554 \text{ km}}{62 \text{ litres}}$$

STEP 2 Write the ratio in simplest form.

$$\frac{554 \text{ km}}{62 \text{ litres}} = \frac{277\text{km}}{31 \text{ litres}}$$ (Simplify by dividing the fraction by the HCF of 2)

STEP 3 Divide 277 by 31.

$277 \div 31 = 8.935$ km/L

= 8.9 km/L rounded off to one decimal place.

Therefore, the rate is 8.9 km/L ANSWER

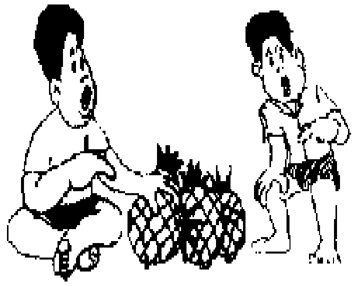
Km/L is stated as kilometres per Litre. When rate is stated this way, it is referred to as a unit rate.

Example 2

Three pineapples are sold for K36. What is the rate?

STEP 1 Write the ratio of K36 to three pineapples.

$$\frac{\text{K36}}{\text{3 pineapples}}$$



STEP 2 Express in simplest form.

$$= \frac{\text{K12}}{\text{1pineapple}}$$

= K12

Therefore, the rate is K12 per pineapple. ANSWER

REMEMBER:

- **Rate is a comparison of two quantities which have different units. It is written as a fraction.**
- **A rate is similar to a ratio, but in a rate, the two numbers compared are two different kinds of measurements.**
- **A unit rate is the simplified form of rate.**

Example: 10 km in 2 hours is a rate while
5km per hour is the unit rate.

NOW DO PRACTICE EXERCISE 21B

Practice Exercise 21B

1) Write the following as a unit rate.

a) 70 kilometres in 5 hours ___________

b) 3960 words in 30 pages ___________

c) K455 in 7 hours ___________

d) 1365 wooden sticks in 13 bundles ___________

e) 5500 cocoa trees planted on 50 hectares ___________

2) Solve the following problems.

a) Fifteen litres of petrol cost K95.00. What is the rate?

______________ **ANSWER**

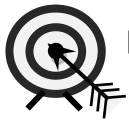
b) A jogger covers a distance of 36 km in a time of 4.5 hrs. what is the rate?

______________ **ANSWER**

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4

Lesson 22: Equivalent Ratios

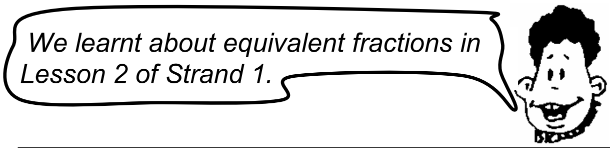
In Lesson 21 you learnt to work out ratios of quantities of the same kind and quantities of different kinds.



In this lesson you will:

- determine equivalent ratios
- write ratio of two numbers and quantities into their simplest form.

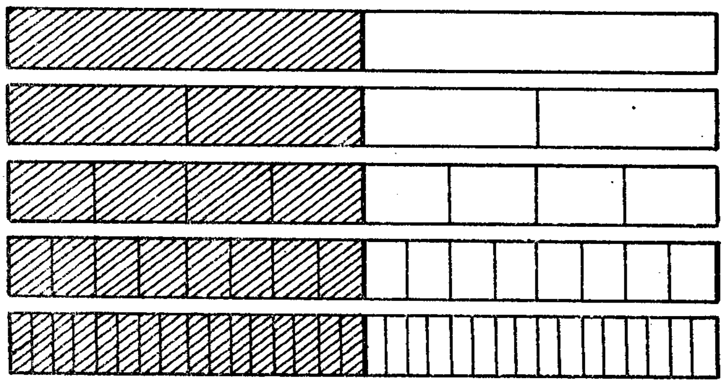
First you will learn to work out equivalent ratios.



Equivalent means "the same". (*You say ee-kwiv- a- lent)*

Look at the diagrams of these ratios.

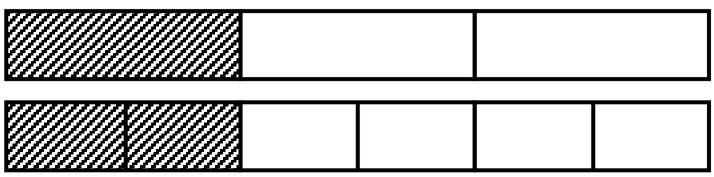
Example 1

DIAGRAM	RATIO Shaded **:** Un-shaded
	1 : 1 2 : 2 4 : 4 8 : 8 16 : 16

The diagrams above show EQUIVALENT RATIOS.

They are equivalent because they are equal and really mean the <u>same</u> thing.

Example 2

DIAGRAM	RATIO Shaded **:** Un-shaded
	1 : 2 2 : 4

The ratios 1: 2 and 2 : 4 are EQUIVALENT RATIOS.

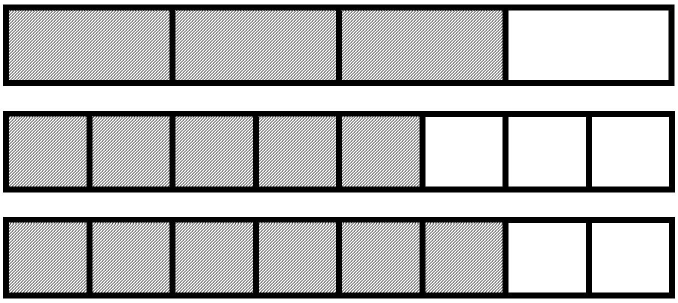
They are equivalent because they mean the <u>same</u> thing.

Here is another example.

Which of the following ratios are equivalent?

DIAGRAM

RATIO
Shaded :Un-shaded



3 : 1

5 : 3

6 : 2

ANSWER: 3 **:** 1 and 6 **:** 2 are equivalent ratios. They both have the same area shaded.

The diagrams show that 3 **:** 1 and 6 **:** 2 <u>mean the same thing</u>.

Equivalent or Equal ratios are ratios which mean the same thing.

NOW DO PRACTICE EXERCISE 22A.

Practice Exercise 22A

Which of the following ratios are equivalent ratios?

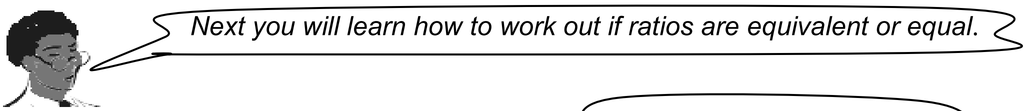
DIAGRAM	RATIO Shaded **:** Un-shaded
	2 **:** 3 1 **:** 1 8 **:** 12 3 **:** 2 4 **:** 6 2 **:** 2

ANSWER: (i) The ratio 2 : 3 is equivalent to the ratio _____ : _____

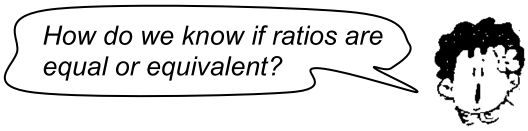
2 : 3 is also equivalent to the ratio _____ : _____

(ii) the ratio 1 : 1 is equivalent to the ratio _____ : _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.



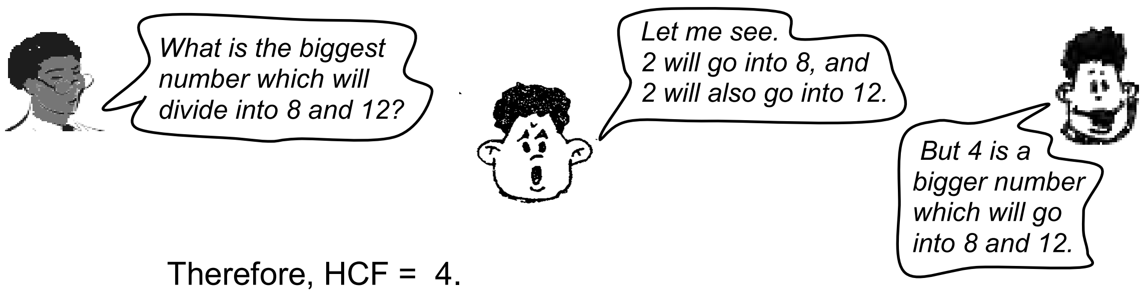
Example



Are the ratios 8 : 12 and 4 : 6 equivalent?

STEP 1 Write 8 : 12 in its SIMPLEST FORM.

We do this by finding the HIGHEST COMMON FACTOR of the numbers in the ratio. (You learnt about Highest Common Factor, H.C. F., in Lower Primary Mathematics). The H.C.F. is the biggest number which will divide into both numbers of the ratio.



Therefore, HCF = 4.

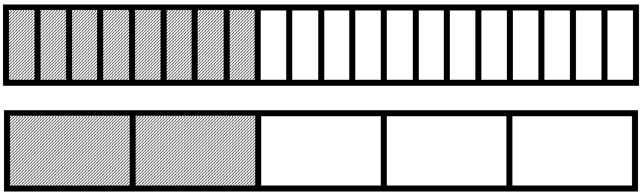
The Highest Common Factor is __4__ because 4 is the biggest number that will divide into 8 and 12.

Solution:

$$\begin{array}{r} 2 \\ 4\overline{)\ 8} \end{array} \qquad\qquad \begin{array}{r} 3 \\ 4\overline{)\ 12} \end{array}$$

Therefore, 8 : 12 = 2 : 3 in its simplest form.

The ratios 8 : 12 and 2 : 3 are equivalent fractions.



8 : 12

2 : 3

STEP 2 Write 4 : 6 in its simplest form.

What is the HCF for 4 and 6?

HCF is 2.

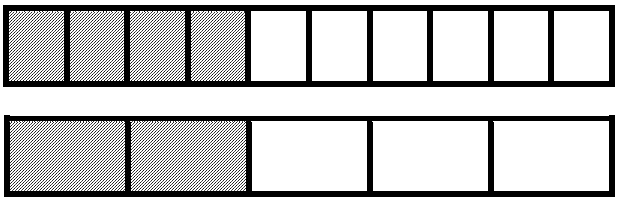
Divide both 4 and 6 by 2.

Solution:

$$\begin{array}{r} 2 \\ 2\overline{)\ 4} \end{array} \qquad\qquad \begin{array}{r} 3 \\ 2\overline{)\ 6} \end{array}$$

Therefore, 4 : 6 = 2 : 3 in its simplest form.

The ratios 4 : 6 and 2 : 3 are equivalent ratios.



4 : 6

2 : 3

STEP 3 Do the two ratios have the SAME simplest form?

8 : 12 = 2 : 3, and

4 : 6 = 2 : 3

Yes, they do have the same simplest form, 2 : 3.

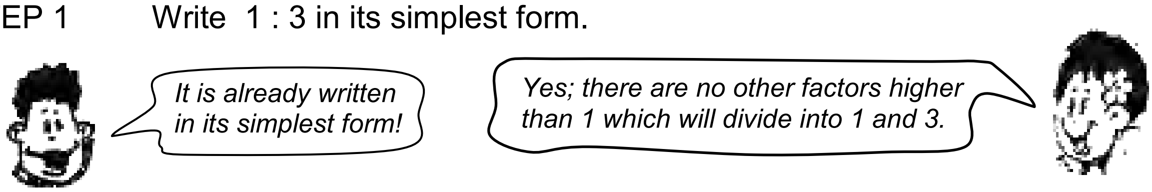
This means that the two ratios 8 : 12 and 4 : 6 are also equivalent.

Here is another example.

Example

Change the ratios 1 : 3 and 5 : 15 into their simplest form.
Are these two ratios equivalent?

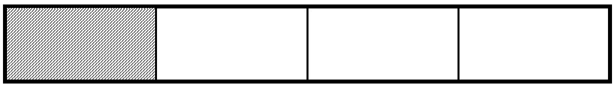
STEP 1 Write 1 : 3 in its simplest form.



Therefore, 1 : 3 in its simplest form stays the same.

We can check with diagram.

The simplest form of 1 : 3 is 1 : 3.



1 : 3

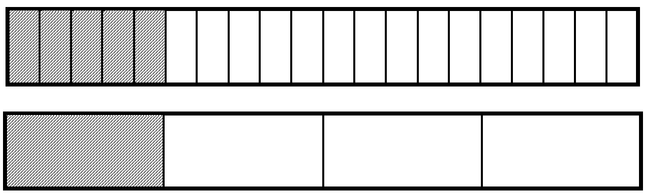
STEP 2 Write 5 : 15 in its simplest form.

HCF = 5

Solution:

$5 \overline{)\,5\,}$ = 1 $\qquad$ $5 \overline{)\,15\,}$ = 3

Therefore, 5 : 15 = 1 : 3 in its simplest form.



5 : 15

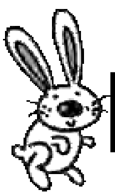
1 : 3

STEP 3 Do the two ratios have the SAME simplest form?

1 : 3 = 1 : 3
5 : 15 = 1 : 3

Yes, they do have the same simplest form, 1 : 3.

Therefore, the two ratios 1 : 3 and 5 : 15 are equivalent.

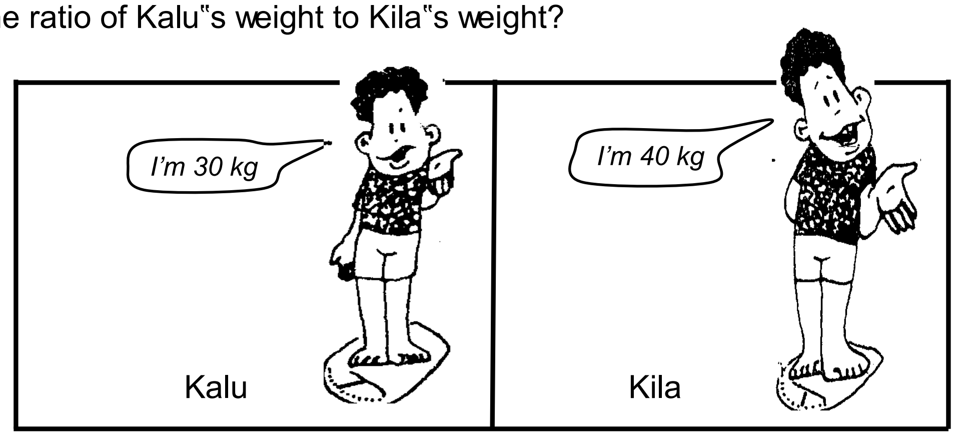


The simplest form of a ratio will also be an equivalent ratio.

Look at the two examples.

Example 1

What is the ratio of Kalu‟s weight to Kila‟s weight?



STEP 1 Write the ratio of the two weights.

Kalu‟s weight : Kila‟s weight = 30 : 40

STEP 2 Find the HCF for 30 and 40.

The biggest number that will divide into both 30 and 40 is _10_.

Therefore, HCF is 10.

STEP 3 Divide both numbers of the ratio by the HCF.

$\begin{array}{r} 3 \\ 10\overline{)\,30} \end{array}$ $\qquad$ $\begin{array}{r} 4 \\ 10\overline{)\,40} \end{array}$

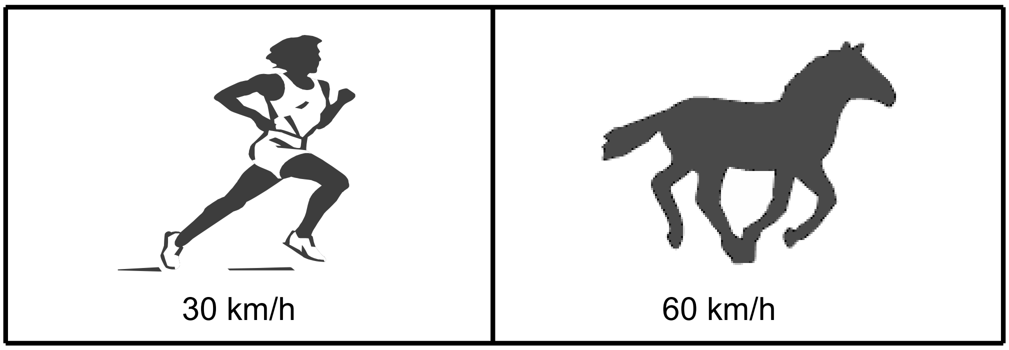
STEP 4 Write the ratio in simplest form.

30 : 40 = 3 : 4

Therefore, the ratio of Kalu's weight to Kila's weight is 3 : 4.

Example 2

What is the ratio of the speed of the runner to the speed of the horse below?



STEP 1 Write the ratio of the two speeds

Speed of runner : Speed of horse = 30 : 60

STEP 2 Find the HCF for 30 and 60.

The biggest number that will divide into both 30 and 60 is 30 .
Therefore, HCF is 30 .

STEP 3 Divide both numbers of the ratio by the HCF.

$$30 \overline{)\,30\,}^{\;1} \qquad\qquad 30 \overline{)\,60\,}^{\;2}$$

STEP 4 Write the ratio in simplest form.

30 : 60 = 1 : 2

Therefore, the ratio of the speed of the runner to the speed of the horse is 1: 2.

NOW DO PRACTICE EXERCISE 22B.

Practice Exercise 22B

---

1. Find the highest common factor of the numbers in each ratio, and change the ratio into its simplest form.

(a) 25 : 45 ___________ (d) 8 : 56 ___________

(b) 30 : 50 ___________ (e) 70 : 85 ___________

(c) 21 : 7 ___________ (f) 13 : 26 ___________

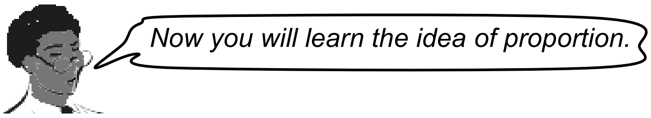
---

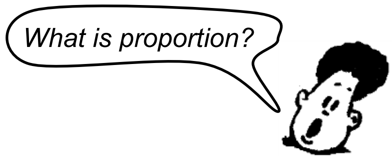
2. Find out whether the following ratios are equivalent.

(a) 6 : 15 and 10 : 25 ___________

(b) 12 : 16 and 28 : 21 ___________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.





Proportion

Consider the following ratios:

5 : 8 10 : 16 30 : 48 50 : 80 500 : 800

All the ratios indicate the same number when written in their simplest form which is 5 : 8 and therefore, they are equal ratios.

Equating any two equal ratios will make the statement a **proportion**.

5 : 8 = 50 : 80

- **Equal or equivalent ratios form a PROPORTION.**
- **In equivalent ratios, 4 numbers are involved, namely the end and the middle numbers.**
- **The end numbers are called EXTREMES, while the middle numbers are called MEANS.**

Look at the equivalent ratios below:

extremes

5 : 8 = 50 : 80

means

In the proportion given above, 5 and 80 are the extremes and 8 and 50 are the means.

REMEMBER: THE PROPORTION RULE

The product of the means is equal to the product of the extremes. This property will hold for any true proportion.

Since ratios can be written as fractions, this is sometimes called **Cross Multiplication**.

5 : 8 = 50 : 80

$$\frac{5}{8} = \frac{50}{80}$$

Note that the cross products of equal ratios are always equal.

5 x 80 = 400 product of the extremes

8 x 50 = 400 product of the means

Example 1

Since the ratio 6 to 7 is equal to the ratio 18 to 21, we can write the proportion as

$$6 : 7 = 18 : 21 \quad \text{or} \quad \frac{6}{7} = \frac{18}{21}$$

The proportion is read " 6 is to 7 as 18 is to 21."

Example 2

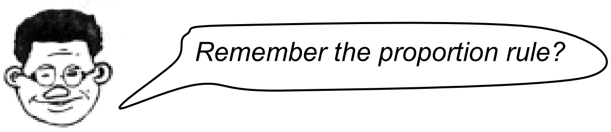
List the means and the extremes in the proportion $\frac{3}{7} = \frac{18}{42}$.

ANSWER:

means	extremes
7 and 18	3 and 42

Example 3

Is 5 : 7 = 30 : 42 a true proportion?



We write the ratios as fractions.

$$\frac{5}{7} \quad \frac{30}{42}$$

$7 \times 30 = 210$ The product of the means is 210.

$5 \times 42 = 210$ The product of the extremes is 210.

Since the products are equal, the statement is a true proportion.

NOW DO PRACTICE EXERCISE 22C

Practice Exercise 22C

1. Put a check (√) on the means and an asterisk (*) on the extremes for the following proportions.

a) $\frac{3}{7} = \frac{9}{21}$

b) $\frac{5}{11} = \frac{35}{77}$

c) $\frac{3}{8} = \frac{15}{40}$

d) $\frac{x}{30} = \frac{11}{15}$

e) $\frac{17}{19} = \frac{n}{57}$

f) $\frac{13}{8} = \frac{39}{24}$

g) 10 : 15 = 2 : 3

h) x : 10 = 40 : 25

2. Which of the following is a true proportion?

a) $\frac{7}{11} = \frac{35}{55}$

b) $\frac{5}{6} = \frac{8}{9}$

c) $\frac{7}{15} = \frac{84}{180}$

d) $\frac{3}{10} = \frac{50}{150}$

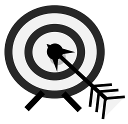
e) $\frac{15}{0.6} = \frac{75}{2}$

f) $\frac{24}{14} = \frac{42}{14}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 23: Problems using Ratio

In Lesson 22, you learnt to work out equivalent ratios and write ratios in their simplest form.



In this lesson you will:

- use ratios to work out and solve some problems in real life situations.

To solve problems using ratio, first you will have to recall how to find equivalent ratios.

Example 1

What number goes in the box to make these two ratios equivalent?

$$\frac{9}{\square} = \frac{3}{4}$$

STEP 1: To change 9 to 3 we must DIVIDE BY 3.

$$9 \xrightarrow{\div 3} 3$$

WORKING OUT

$$3 \overline{)\,9} \quad = 3$$

STEP 2: What number goes in the $\square$ so that, if we also divide by 3, we will get 4?

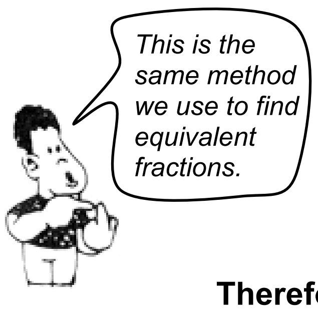
$$\square \xrightarrow{\div 3} 4$$

WORKING OUT

3 x 4 = 12

ANSWER: $\boxed{12} \div 3 = 4$

We can write STEP 1 and STEP 2 together like this:



$$\frac{9 \xrightarrow{\div 3}}{\square \xrightarrow{\div 3}} = \frac{3}{4}$$

WORKING OUT

$$3 \overline{)\,9} \quad = 3$$

3 x 4 = $\boxed{12}$

Therefore, the number in the $\square$ is 12.

Example 2

What number goes in the $\square$ to make these two ratios equivalent?

$$\frac{5}{2} = \frac{\square}{10}$$

Both STEP 1 and STEP 2 are shown together.

$$\frac{5}{2} \overset{\times 5}{=} \frac{\square}{10}$$

WORKING OUT

$$5 \overline{)\,10\,}\quad 5$$

$$5 \times 5 = \boxed{25}$$

Therefore, $\frac{5}{2} = \frac{\boxed{25}}{10}$

WORKING OUT

$$\frac{\cancel{25}^{\,5}}{\cancel{10}_{\,2}} = \frac{5}{2}$$

$$\frac{5}{2} = \frac{5}{2}$$

CHECK: Are the ratios equivalent?

Yes, they have the same simplest form.

ANSWER: The number in the $\square$ is 25.

Example 3

Find the missing number to make these two ratios equivalent?

$$\frac{18}{42} = \frac{3}{\square}$$

$$\frac{18}{42} \overset{\div 6}{=} \frac{3}{\square}$$

WORKING OUT

$$3 \overline{)\,18\,}\quad 6$$

$$42 \times 6 = \boxed{7}$$

Therefore, $\frac{18}{42} = \frac{3}{\boxed{7}}$

WORKING OUT

$$\frac{\cancel{18}^{\,3}}{\cancel{42}_{\,7}} = \frac{3}{7}$$

$$\frac{3}{7} = \frac{3}{7}$$

CHECK: Are the ratios equivalent?

Yes, they have the same simplest form.

ANSWER: The missing number is 7.

Example 4

Kua and Tau sold coffee and agreed to share the money from the sale in the ratio of 2 : 1, because Kua‟s coffee was twice as much as that of Tau‟s.

If Kua received K60, how much did Tau receive?

STEP 1 Write the ratio for the money.

Kua‟s money : Tau‟s money = 2 : 1

STEP 2 Write the ratio in fraction form.

$$\frac{\text{Kua's money}}{\text{Tau's money}} = \frac{2}{1}$$

STEP 3 We know that Kua‟s money = K60, so

$$\text{KUA‟S MONEY} \longrightarrow \frac{\text{K60}}{\text{Tau's Money}} = \frac{2}{1} \quad \text{(because Kua‟s money =K60)}$$

STEP 4 Write an equivalent ratio for $\frac{2}{1}$ which has a numerator of 60.

$$\begin{array}{l} \text{KUA‟S MONEY (K)} \longrightarrow \\ \text{TAU‟S MONEY (K)} \longrightarrow \end{array} \frac{60 \xrightarrow{\div 30}}{\square \xrightarrow{\div 30}} = \frac{2}{1}$$

$$\frac{60}{\boxed{30}} = \frac{2}{1}$$

ANSWER: Tau received K30 for his coffee.

Example 5

The ratio of Ata‟s age to Pita‟s age is 4 : 5. If Pita is 15 years old, how old is Ata?

STEP 1 Write the ratio of the ages.

Ata‟s age : Pita‟s age = 4 : 5

STEP 2 Write the ratio in fraction form.

$$\frac{\text{Ata's age}}{\text{Pita's age}} = \frac{4}{5}$$

STEP 3 We know that Pita‟s age is 15 years, so

$$\text{PITA‟S AGE} \longrightarrow \frac{\text{Ata's age}}{15} = \frac{4}{5}$$

STEP 4 Write an equivalent ratio for $\frac{4}{5}$ which has a denominator of 15.

ATA‟S AGE →
PITA‟S AGE →

$$\frac{\boxed{}}{15} \xrightarrow{\div 3} \;=\; \frac{4}{5}$$

WORKING OUT

$$5 \overline{)\,15\,} = \boxed{3}$$

ATA‟S AGE →

$$\frac{\boxed{12}}{15} = \frac{4}{5}$$

$$\boxed{3} \times 4 = \boxed{12}$$

ANSWER: **Ata's age is 12 years.**

NOW DO PRACTICE EXERCISE 23

Practice Exercise 23

1. Write the number that goes in the $\square$ to make the following ratios equivalent.

a) $\frac{\square}{33} = \frac{5}{3}$

b) $\frac{\square}{20} = \frac{7}{5}$

c) $\frac{5}{\square} = \frac{10}{56}$

d) $\frac{3}{4} = \frac{18}{\square}$

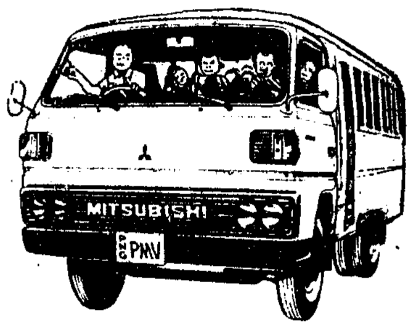
e) $\frac{3}{8} = \frac{\square}{56}$

f) $\frac{2}{\square} = \frac{8}{20}$

2. The ratio of children to adults traveling on a PMV Is 2 : 3.
If there were 18 adults, how many children were there?

STEP 1 Write the ratio for the passengers on PMV.

Children : __________ = 2 : 3



STEP 2 Write the ratio in fraction form.

$$\frac{\text{Children}}{\text{Adults}} = \frac{2}{3}$$

STEP 3 We know there are 18 adults, so

$$\text{Adults} \rightarrow \frac{\text{Children}}{\_\_\_\_} = \_\_\_$$

STEP 4 Write an equivalent ratio for $\frac{2}{3}$ which has a denominator of ________.

$$\begin{array}{l} \text{Children} \rightarrow \\ \text{Adults} \rightarrow \end{array} \frac{\square}{\_\_\_} \overset{\div}{=} \frac{\_\_\_}{\_\_\_}$$

$$\text{Children} \rightarrow \frac{\square}{\_\_\_} = \frac{\_\_\_}{\_\_\_}$$

WORKING OUT

$$\square \times \_\_\_ = \square$$

ANSWER: The number of children on the PMV is _________.

3. The ratio of banana trees to coconut trees in Morea"s garden is 3 : 10.

If Morea has 15 banana trees, how many coconut trees does he have?

STEP 1 Write the ratio for the trees

__________ : __________ = ____ : ____

STEP 2 Write the ratio in fraction form:-

$\frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_} = \frac{\_\_\_\_\_\_}{\_\_\_\_}$

STEP 3 We know there are _______ banana trees, so

Bananas → $\frac{\_\_\_\_\_\_\_\_}{\text{Coconuts}} = \frac{\_\_\_\_\_\_}{\_\_\_}$

STEP 4 Write an equivalent ratio for $\frac{2}{3}$ which has a denominator of _________.

Bananas → ______ (÷) = ______

Coconuts → $\square$ (÷) ______

WORKING OUT

$\square$

Coconuts → $\frac{\_\_\_\_\_\_}{\square} = \frac{\_\_\_\_\_\_}{\_\_\_\_}$

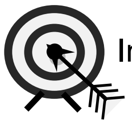
$\square$ x ____ = $\square$

ANSWER: Morea has __________coconut trees.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 24: Ratio and Scale for Length

In Lesson 23, you learnt to use ratio to solve problems in real life situations. .



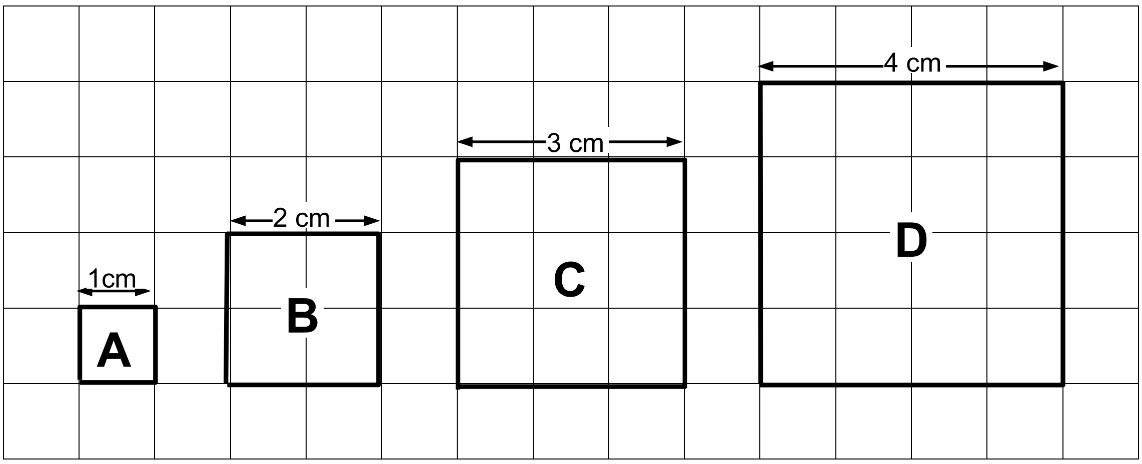
In this lesson you will:

- determine the relationship between ratio and scale for length.

Before determining the relationship between ratio and scale for length, you must first learn about a special number which is used to change one shape into another similar shape.

We call this special number a SCALE.

Look at the drawings of 4 squares below.



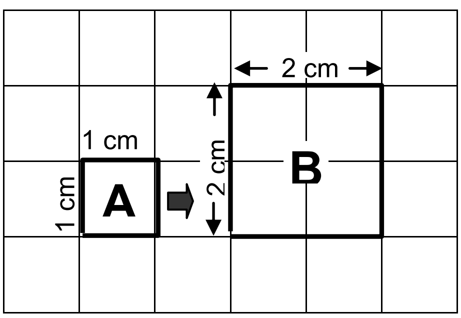
The sides of SQUARE A are 1 cm in length.

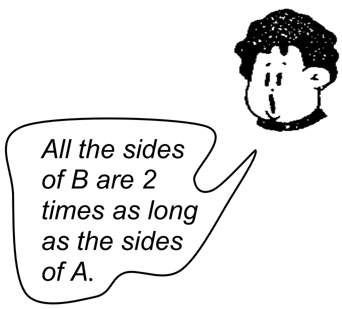
The sides of SQUARE B are 2 cm in length.

The sides of SQUARE C are 3 cm in length.

The sides of SQUARE D are 4 cm in length.

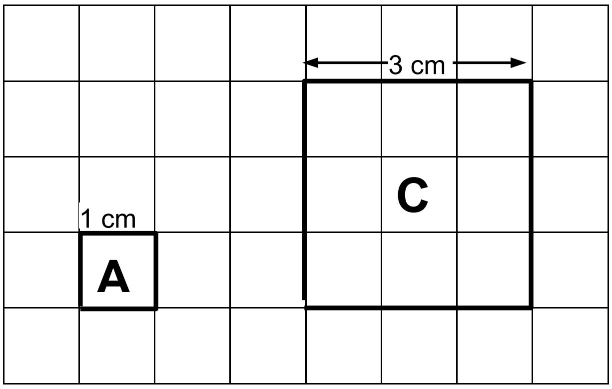
We can **enlarge** Square A to make Square B if we draw all the sides of the second shape 2 times as long as the first shape.

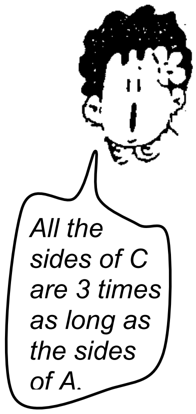




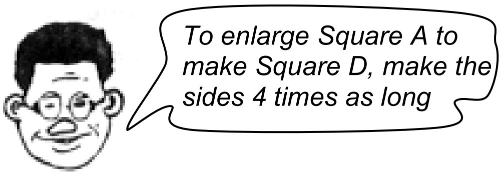
We say we have used a Scale for length of 2 to enlarge Square A. Our result is Square B.

We can **enlarge** Square A to make Square C if we draw all the sides of the second shape 3 times as long as the first shape.

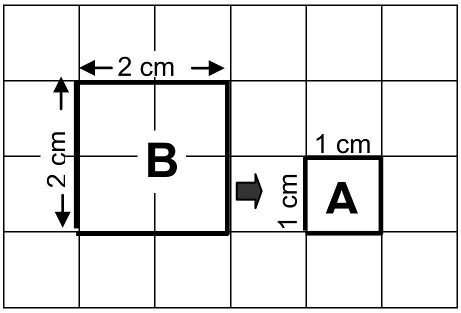


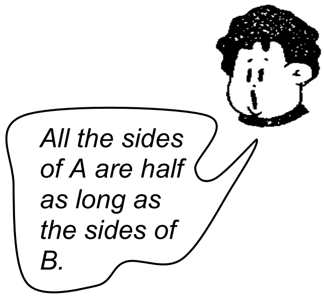


We say we have used a Scale for length of 2 to enlarge Square A. Our result is Square C.



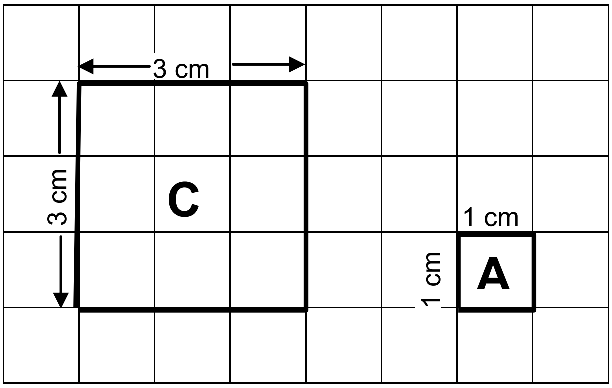
Also we can **reduce** Square B to make Square A if we draw all the sides of the second shape half the length of the sides of the first shape.

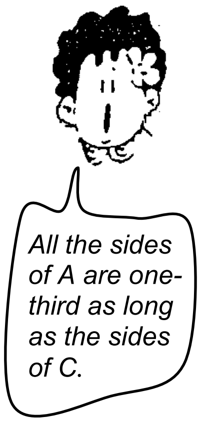




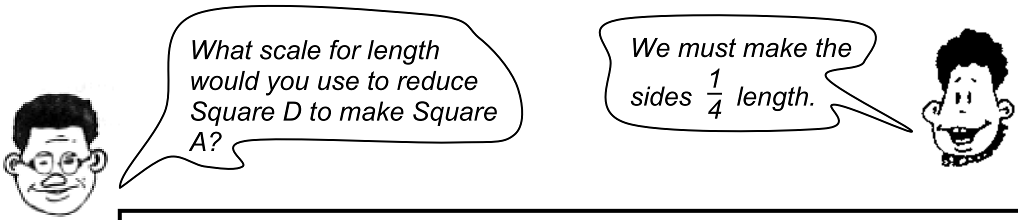
We say we have used a Scale for length of $\frac{1}{2}$ to reduce Square B. Our result is Square A.

We can **reduce** Square C to make Square A if we draw all the sides of the second shape one-third of the length of the sides of the first shape.





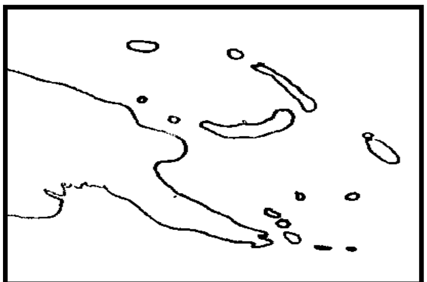
We say we have used a Scale for length of $\frac{1}{3}$ to enlarge Square C. Our result is Square A.



Scale for length is a number we use to change one shape into another shape.

So far, you have learned how to enlarge and to reduce shapes.

Scale for length is shown on drawings or diagrams such as maps, or plans of buildings and can be used to work out actual lengths and distances.



HERE IS THE SCALE FOR LENGTH marked on the map. ➡ SCALE: 1 cm = 1 km

Scale for length is also a ratio and we can write it as ratio.

Example

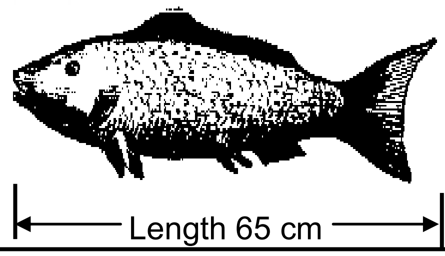
The diagram shows a drawing of a fish.

The actual fish is longer than its drawing.

The scale for length of the drawing is shown.

SCALE ; 1 mm = 1 cm

DRAWING OF A FISH



SCALE : 1 mm = 1 cm

This means that every **millimetre** of length on the drawing represents one **centimetre** of length on the **actual fish**.

We can work this out as a ratio.

What is the ratio of the length of the drawing to the length of the actual fish?

Drawing : Fish = 1 mm : 1 cm

= 1mm : _10_ mm (There are 10 mm in 1 cm)

= 1 : 10 (Same units; mm)

ANSWER: **Drawing : Fish = 1 : 10** ⬅ THIS IS THE RATIO

We can use this ratio to help us know more about the measurement of the ACTUAL FISH.

For example, what is the length of the actual fish?

STEP 1 Write the ratio.

Drawing : Fish = 1 : 10

STEP 2 Write the ratio in fraction form.

$$\frac{\text{Drawing}}{\text{Fish}} = \frac{1}{10}$$

STEP 3 We know that the length of the fish in the DRAWING is <u>65</u> mm (See diagram.) so,

LENGTH ON DRAWING (mm) $\frac{65}{\text{Fish}} = \frac{1}{10}$

STEP 4 Write an equivalent ratio for $\frac{1}{10}$ which has a numerator of 65.

LENGTH ON DRAWING (mm) → 65

LENGTH OF ACTUAL FISH (mm) → $\square$

$$\frac{65}{\square} = \frac{1}{10}$$

WORKING OUT

$$1\,\overline{)\,65}$$ = 65

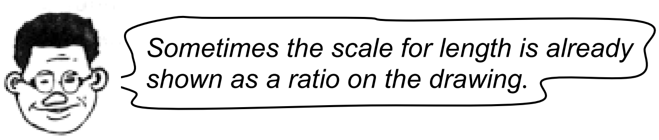
LENGTH OF ACTUAL FISH (mm) → $\square$

$$\frac{65}{\square} \overset{\div 65}{=} \frac{1}{10}$$

65 x 10 = $\boxed{650}$

LENGTH OF ACTUAL FISH (mm) $\frac{65}{650} = \frac{1}{10}$ (Check your answer.)

ANSWER: The length of the ACTUAL FISH is <u>650</u> mm (or <u>65</u> <u>cm</u>).

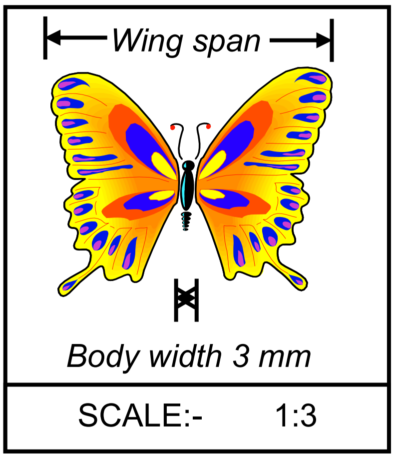


The scale for length of this drawing of a butterfly is shown as

SCALE 1 : 3

This is sometimes written in fraction form

SCALE $\frac{1}{3}$



We can use this ratio to help us know more about the measurements of the actual butterfly.

Example

1) What is the length of the wing span of the actual butterfly?

STEP 1 Write the ratio.

Drawing : Butterfly = 1 : 3

STEP 2 Write the ratio in fraction form (it may already be in fraction form.)

$$\frac{\text{Drawing}}{\text{Bitterfly}} = \frac{1}{3}$$

STEP 3 We know that the length of the wing span on the drawing is _4_cm, so

LENGTH ON DRAWING (cm) ⟶ $\dfrac{4}{\text{Butterfly}} = \dfrac{1}{3}$

STEP 4 Write an equivalent ratio for $\frac{1}{3}$ which has a numerator of 4.

LENGTH ON DRAWING (cm) ⟶ 4

LENGTH ON BUTTERFLY (cm) ⟶ $\square$

$$\frac{4}{\square} = \frac{1}{3}$$

WORKING OUT

$$1\,\overline{)\,4\,}\quad 4$$

LENGTH OF BUTTERFLY (cm) ⟶ $\square$

$$\frac{4}{\square} \xrightarrow{\div 4} \frac{1}{3}$$

$$4 \times 3 = \boxed{12}$$

LENGTH OF BUTTERFLY (cm) ⟶ $\dfrac{4}{\boxed{12}} = \dfrac{1}{3}$ (Check your answer.) $\dfrac{\cancel{4}}{\cancel{12}} = \dfrac{1}{3}$

ANSWER: The length of the wing span of the butterfly is 12 cm. $\dfrac{1}{3} = \dfrac{1}{3}$

In the same way, you can work out that the body width of the butterfly is 9 mm.

NOW DO PRACTICE EXERCISE 24

Practice Exercise 24

1) This drawing shows a sea shell.

Length 19 mm

SCALE: 1 mm = 1 cm

(a) What is the ratio of the length of the drawing to the length of the actual shell? The first step has been started for you. Continue and complete the solution.

Drawing : Shell = 1 mm : 1 : cm

= 1 mm : 10 mm (because there are 10 mm to 1 cm)

= 1 : 10 (same units, mm)

ANSWER: Drawing : Shell = 1 : 10

(b) Use the ratio you have worked out in (a) to find the length of the actual shell.

STEP 1 Write the ratio.

Drawing : Shell = 1 : 10

STEP 2 Write the ratio in fraction form.

$\frac{\text{Drawing}}{\_\_\_\_} = \frac{\_\_\_\_}{\_\_\_\_}$

STEP 3 We know that the length of the shell in the drawing is ___ mm

So, LENGTH ON DRAWING (mm) $\frac{\_\_\_\_}{\_\_\_\_} = \frac{\_\_\_\_}{\_\_\_\_}$

STEP 4 Write an equivalent ratio for $\frac{\_\_\_}{\_\_\_}$ which has a numerator of ____.

WORKING OUT

LENGTH ON DRAWING (mm) → $\frac{\_\_\_\_}{\square} = \frac{\_\_\_\_}{\_\_\_\_}$

LENGTH OF SHELL (mm) →

____ $\overline{)\ \_\_\_\_}$

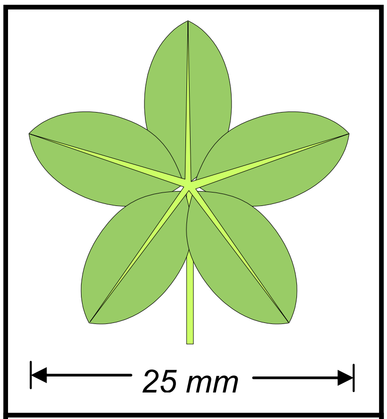
LENGTH OF SHELL (mm) → $\frac{\_\_\_\_}{\square} = \frac{\_\_\_\_}{\_\_\_\_}$

○ x ___ = ___

LENGTH OF SHELL (mm) → $\frac{\_\_\_\_}{\square} = \frac{\_\_\_\_}{\_\_\_\_}$ (Check your answer.)

ANSWER: The length of the real shell is ______ mm (or _____ cm).

2) The width of this drawing of a leaf is 25 mm.
What is the actual width of the leaf?



SCALE- 1 : 4

STEP 1 Write the ratio:

Drawing : Flower = _____ : _____

STEP 2 Write the ratio in fraction form:-

$\dfrac{\text{Drawing}}{\_\_\_\_\_\_} = \dfrac{\_\_\_\_\_\_}{\_\_\_\_\_}$

STEP 3 We know that the width of the drawing of the leaf is _____mm.

So

WIDTH ON DRAWING (mm) → $\dfrac{\_\_\_\_\_\_}{\_\_\_\_\_} = \dfrac{\_\_\_\_\_\_}{\_\_\_\_\_}$

STEP 4 Write an equivalent ratio for _____ which has a numerator of _____.

WORKING OUT

WIDTH ON DRAWING (mm) → ______ = ______

WIDTH OF LEAF (mm) → ☐ ______

WIDTH ON DRAWING (mm) → ______ = ______ ◯ x ____ = ____

WIDTH OF LEAF (mm) → ☐ ______

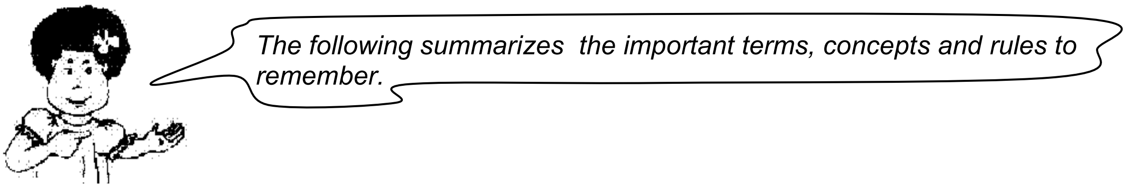
______ = ______ (Check your answer.)

WIDTH OF LEAF (mm) → ☐ ______

ANSWER: The actual width of the leaf is ______ mm (or ______cm).

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

SUB-STRAND 4: SUMMARY



- Ratio is a way of comparing two quantities or numbers with a definite order. It is usually represented by a pair of numerals. For example: 2 to 3 in the form 2 : 3 or $\frac{2}{3}$.

  When we speak of a ratio of 2 adults to 3 children, $\frac{2}{3}$, we mean the ratio of the numbers of adults,2, to the number of children, 3.
- A ratio is a relation between two numbers.
- In expressing a ratio, we write the numerals in the correct order. A ratio of 2 to 3 is $\frac{2}{3}$ and a ratio of 3 to 2 is $\frac{3}{2}$.
- The order of the numbers in a ratio is the same as the order of the quantities.
- A relation between two measures is called rate.
- A rate is involved whenever one measure is paired with another such as 45 kilometres per hour, K25 for 10 eggs, etc.
- A rate is a comparison of two quantities which have different units. It is written as a fraction.
- Equal ratios are two ratios which mean the same thing.
- A statement of equality between two ratios is called a proportion.
- Since ratio is a comparison by division and can be written as a fraction, given any ratio, we can form many proportions by either multiplying or dividing the numerator and denominator by the same number.
- Scale for Length is a number we use to change one shape into another shape.

REVISE LESSONS 20 - 24 THEN DO SUB-STRAND TEST 4 IN ASSIGNMENT 1.

ANSWERS TO PRACTICE EXERCISES 20 TO 24

Practice Exercise 20

1. b) i. Boys to girls equals 18 is to 12.
   ii. The ratio of boys to girls is 18 is to 12.

   c) i. Ororo‟s mangoes to Tau‟s mangoes equals 18 is to 24
   ii. The ratio of Ororo‟s mangoes to Tau‟s mangoes is 18 is to 24.

2. a) adults : children = 3 : 2

   b) i. Boys : Girls = 24 : 21
   ii. Girls : Boys = 21 : 24
   iii. Boys : Class = 24 : 45
   iv. Girls : Class = 21 : 45

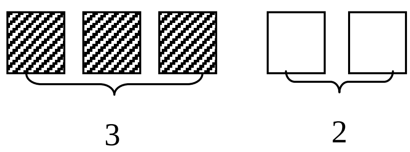
3.

x : y	x : y
b. 2 : 1	1 : 2
c. 4 : 1	1 : 4
d. 1 : 1	1 : 1
e. 7 : 3	3 : 7
f. 5 : 3	3 : 5
g. 10 : 1	1 : 10

4. a) RATIO DIAGRAM

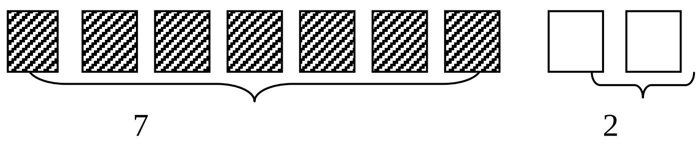
Cars : Trucks = 3 : 2

:



b) RATIO DIAGRAM

Chicken : Pig = 7 : 2



5. a and d

Practice Exercise 21A

1) 3 cm : 13 mm = 30 mm : 13 mm

2) 1 kg : 7 g = 1000g : 7 g

Practice Exercise 21B

1. a) 14 km/hr
   b) 132 words per page
   c) K65 per h
   d) 105 per bundle
   e) 110 cocoa trees per hectares

2) a) K6.33/L
   b) 8 km/h

Practice Exercise 22A

i. 2 : 3 = 4 : 6
   2 : 3 = 8 : 12

ii. I : 1 = 2 : 2

Practice Exercise 22B

1. a) H.C.F. = 5; Simplest Form = 5 : 9
   b) H.C.F. = 10; Simplest Form = 3 : 5
   c) H.C.F. = 7; Simplest Form = 3 : 1
   d) H.C.F. = 8; Simplest Form = 1 : 7
   e) H.C.F. = 5; Simplest Form = 14 : 17
   f) H.C.F. = 13; Simplest Form. = 1 : 2

2) a) 6 : 15 and 10 : 25 are equivalent
   b) 12 : 16 and 28 : 21 are not equivalent

Practice Exercise 22C

1. Put a check (√) on the means and an asterisk (*) on the extremes for the following proportions.

(a) means: 7 and 9
extremes: 3 and 21

(b) means: 11 and 35
extremes: 5 and 77

(c) means: 8 and 15
extremes: 3 and 40

(d) means: 30 and 11
extremes: x and 15

(e) means: 19 and n
extremes: 17 and 57

(f) means: 8 and 39
extremes: 13 and 24

(g) means: 15 and 2
extremes: 10 and 3

(h) means: 10 and 40
extremes: x and 25

2. (a) $\frac{7}{11} = \frac{35}{55}$ (c) $\frac{7}{15} = \frac{84}{180}$

Practice Exercise 23

1. (a) 55 (b) 28
(c) 28 (d) 24
(e) 21 (f) 5

2. 12 children

3. 50 coconut trees

Practice Exercise 24

1. 19 cm 2. 100 mm or 10 cm

END OF SUB-STRAND 4

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