# Advanced Mathematics

## Senior High

### Grade 11

# Teacher Guide

## Standards-Based

Papua New Guinea

Department of Education

'FREE ISSUE
NOT FOR SALE'

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## Grade 11

## Teacher Guide

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Papua New Guinea
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**Issued free to schools by the Department of Education**

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# Contents

# Acknowledgements

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# Acronyms

| | |
|---|---|
| AAL | Assessment As Learning |
| AFL | Assessment For Learning |
| AOL | Assessment of Learning |
| SSBoS | Secondary School Board of Studies |
| CDD | Curriculum Development Division |
| CRS | Classroom Response System |
| DA | Diagnostic Assessment |
| HOD | Head of Department |
| IHD | Integral Human Development |
| MTDG | Medium Term Development Goal |
| NGO | Non-Government Organisations |
| PBA | Performance Based Assessments |
| PNG | Papua New Guinea |
| SAC | Subject Advisory Committee |
| SBC | Standards Based Curriculum |
| SBE | Standards Based Education |
| SCG | Subject Curriculum Group |
| STEAM | Science, Technology, Engineering, Arts and Mathematics |
| STEM | Science, Technology, Engineering, and Mathematics |

# Secretary's Message

The aims and goals of SBC is to identify the important knowledge, skills, values and attitudes that all students are expected to acquire and master in order to effectively function in society and actively function in society and actively contribute to its development, student's welfare and enable them to acquire and apply 21$^{st}$ Century.

The aims of teaching and learning mathematics are to encourage and enable students to recognise that mathematics permeates the world around us. Students should be encouraged to appreciate the usefulness, power and beauty of mathematics and become confident in using mathematics to analyse and solve problems both in school and in real-life situations.

A variety of teaching and learning activities provides students with ideas to motivate students to learn, and make learning relevant, interesting and enjoyable. Teachers should provide students opportunity to develop mathematical curiosity and use inductive and deductive reasoning when solving problems and develop the knowledge, skills and attitudes necessary to pursue further studies in Mathematics.

Learning Mathematics enable students to develop abstract, logical and critical thinking and the ability to reflect critically upon their work and the work of others, develop a critical appreciation of the use of information and communication technology in mathematics appreciate the international dimension of mathematics and its multicultural and historical perspectives.

Teachers are encouraged to integrate Mathematics activities with other subjects, where appropriate, so that students can see the interrelationship between subjects and that the course they are studying provides a holistic education and a pathway for the future.

I commend and approve this Grade 11 Advanced Mathematics Teacher Guide to be used in all Senior High Schools throughout Papua New Guinea.

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**UKE W. KOMBRA, PhD**
Secretary for Education

# Introduction

The aims of teaching and learning mathematics are to encourage and enable students to recognise that mathematics permeates the world around us. Students should be encouraged to appreciate the usefulness, power and beauty of mathematics and become confident in using mathematics to analyse and solve problems both in school and in real-life situations.

The curriculum is designed to ensure that students build a solid foundation in mathematics by connecting and applying mathematical concepts in a variety of ways and situations. To support this process, teachers should provide students opportunity to develop mathematical curiosity and use inductive and deductive reasoning when solving problems and develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics.

Mathematics aims to provide a meaningful pedagogical framework for teaching and learning essential and in demand knowledge, skills, values, and attitudes that are required for the preparation of students for careers, higher education and citizenship in the 21st century.

Students should be prepared to gather and understand information, analyse issues critically, learn independently or collaboratively, organize and communicate information, draw and justify conclusions, create new knowledge, and act ethically.

Students' employability will be enhanced through the study and application of STEAM principles. STEAM is an integral component of the core curriculum. All students are expected to study STEAM and use STEAM related skills to solve problems relating to both the natural and the physical environments. The aim of STEAM education is to create a STEAM literate society. It is envisioned that the study of STEAM will motivate students to pursue and take up academic programs and careers in STEAM related fields. STEAM has been embedded in the Mathematics curriculum. Equal opportunities should be provided for all students to learn, apply and master STEAM principles and skills.

Time allocation for Advanced Mathematics is **400** minutes for Grade 11.

# Structure of the Teacher Guide

There are four main components to this teacher guide. They provide essential information on what all teachers should know and do to effectively implement the Mathematics curriculum.

**Part 1** provides generic information to help the teachers to effectively use the teacher guide and the syllabus to plan, teach and assess students' performance and proficiency on the national content standards and grade-level benchmarks. The purpose of the teacher guide, syllabus and teacher guide alignment, and the four pillars of PNG SBC, which are, morals and values education, cognitive and high level thinking, and 21st Century thinking skills, STEAM, and core curriculum. These are explained to inform as well as guide the teachers so that they align SBE/SBC aims and goals, overarching and SBC principles, content standards, grade-level benchmarks, learning objectives and best practice when planning lessons, teaching, and assessing students.

**Part 2** provides information on the strands, units, topics and learning objectives. How topics and learning objectives are derived is explained to the teachers to guide them to use the learning objectives provided for planning, instruction and assessment. Teachers are encouraged to develop additional topics and learning objectives to meet the learning needs of their students and communities where necessary.

**Part 3** provides information on SBC planning to help guide the teachers when planning SBC lessons. Elements and standards of SBC lesson plans are described as well as how to plan for underachievers, use evidence to plan lessons, and use differentiated instruction, amongst other teaching and learning strategies.

**Part 4** provides information on standards-based assessment, inclusive of performance assessment and standards, standards-based evaluation, standards-based reporting, and standards-based monitoring. This information should help the teachers to effectively assess, evaluate, report and monitor demonstration of significant aspects of a benchmark.

The above components are linked and closely aligned. They should be connected to ensure that the intended learning outcomes and the expected quality of education standards are achieved. The close alignment of planning, instruction and assessment is to the attainment of learning standards.

# Purpose of the Teacher Guide

This teacher guide describes what all teachers should know and do to effectively plan, teach, and assess the Grade 11 Advanced Mathematics content to attain the required learning and proficiency standards. The overarching purpose of this teacher guide is to help teachers to effectively plan, teach, assess, evaluate, report and monitor students' learning and mastery of national and grade-level expectations.
That is, the essential knowledge, skills, values and attitudes (KSVAs) described in the content standards and grade-level benchmarks, and their achievement of the national and grade-level proficiency standards.

Ample information with thorough guidelines is provided for the teacher.

Thus, the teacher is expected to;

- understand the significance of aligning all the elements of Standards-Based Curriculum (SBC) as the basis for achieving the expected level of education quality,
- effectively align all the components of SBC when planning, teaching, and assessing students' learning and levels of proficiency,
- effectively translate and align the Biology syllabi and teacher guide to plan, teach and assess different Biology units and topics, and the KSVAs described in the grade-level benchmarks,
- understand the Biology national content standards, grade-level benchmarks, and evidence outcomes,
- effectively make sense of the content (KSVAs) described in the Biology national content standards and the essential components of the content de scribed in the grade-level benchmarks,
- effectively guide students to progressively learn and demonstrate proficiency on a range of Scientific skills, processes, concepts, ideas, principles, practices, values and attitudes,
- confidently interpret, translate and use Biology content standards and benchmarks to determine the learning objectives and performance standards, and plan appropriately to enable all students to achieve these standards,
- embed the core curriculum in their Biology lesson planning, instruction, and assessment to permit all students to learn and master the core KSVAs required of all students,
- provide opportunities for all students to understand how STEAM has and continues to shape the social, political, economic, cultural, and environment contexts and the consequences, and use STEAM principles, skills, processes, ideas and concepts to inquire into and solve problems relating to both the natural and physical (man-made) worlds as well as problems created by STEAM,
- integrate cognitive skills (critical, creative, reasoning, decision-making, and problem-solving skills), high level thinking skills (analysis, synthesis and evaluation skills), values (personal, social, work, health, peace, relationship, sustaining values), and attitudes in lesson planning, instruction and assessment,

- meaningfully connect what students learn in Biology with what is learnt in other subjects to add value and enhance students' learning so that they can integrate what they learn and develop in-depth vertical and horizontal understanding of subject content,
- formulate effective SBC lesson plans using learning objectives identified for each of the topics,
- employ SBC assessment approaches to develop performance assessments to assess students' proficiency on a content standard or a component of the content standard described in the grade-level benchmark,
- effectively score and evaluate students' performance in relation to a core set of learning standards or criteria, and make sense of the data to ascertain students' expected proficiency status of progress towards me standards, and use evidence from the assessment of students' performance to develop effective evidence-based intervention strategies to help students' making inadequate or slow progress towards meeting the grade-level and national expectations to improve their learning and performance.

# How to use the Teacher Guide

Teacher Guide provides essential information about what the teacher needs to know and do to effectively plan, teach and assess students learning and proficiency on learning and performance standards. The different components of the teacher guide are closely aligned with SBC principles and practice, and all the other components of PNG SBC. It should be read in conjunction with the syllabus in order to understand what is expected of teachers and students to achieve the envisaged quality of education outcomes.

The first thing teachers should do is to read and understand each of the sections of the teacher guide to help them understand the key SBC concepts and ideas, alignment of PNG SBC components, alignment of the syllabus and teacher guide, setting of content standards and grade-level benchmarks, core curriculum, STEAM, curriculum integration, essential knowledge, skills, values and attitudes, strands, units and topics, learning objectives, SBC lesson planning, and SBC assessment. A thorough understanding of these components will help teachers meet the teacher expectations for implementing the SBC curriculum, and therefore the effective implementation of the Grade 11 Advanced Mathematics Curriculum. Based on this understanding, teachers should be able to effectively use the teacher guide to do the following

## Identifying topics from benchmarks

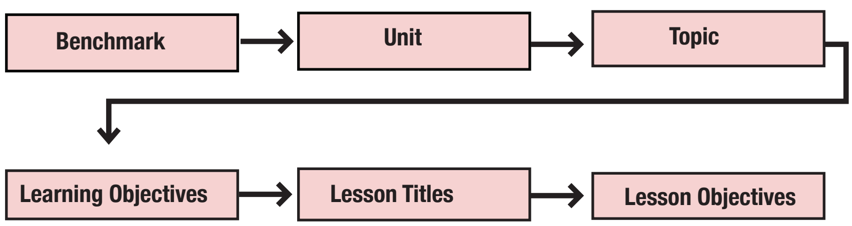
In order to identify the topic from the benchmark, the benchmark needs to be unpack. When unpacking a benchmark, identify what students will be able to know and do in order to mastered the benchmark.

Below is a description of how topics and learning objectives were derived from the grade-level benchmarks.

1. Write out the benchmark that you want to unpack.
2. Write the verbs (skills/actions) – Higher order thinking skills.
3. Underline or highlight the big idea (content) in the benchmark. The big idea (content) is the topic derived from the benchmark.
4. Write essential questions that would be engaging for students.
5. Develop sub-topics from the big idea (topic).
6. Write learning objectives according to the sub-topics.
7. Write lesson topics from the learning objectives.

## Determine Lesson Objectives and Lesson Titles

Topics and learning objectives have been identified and described in the Teacher Guide. Learning objectives are derived from topics that are extracted from the grade-level benchmarks. Lesson titles are deduced from the learning objectives. Teachers should familiarise themselves with this process as it is essential for lesson planning, instruction and assessment. However, depending on the context and students' learning abilities, teachers would be required to determine additional lesson objectives and lesson titles. Teachers should use the examples provided in this teacher guide to formulate additional lesson objectives and lesson titles to meet the educational or learning needs of their students.



### Identify and Teach Grade Appropriate Content

Grade appropriate content has been identified and scoped and sequenced using appropriate content organisation principles. The content is sequenced using the spiraling sequence principles. This sequencing of content will enable students to progressively learn the essential knowledge, skills, values and attitudes as they progress further into their schooling. What students learn in previous grades is reinforced and deepens in scope with an increase in the level of complexity and difficulty in the content and learning activities. It is important to understand how the content is organised so that grade appropriate content and learning activities can be selected, if not already embedded in the benchmarks and learning objectives, to not only help students learn and master the content, but ensure that what is taught is rigorous, challenging, and comparable.

### Integrate the Core Curriculum in Lesson Planning, Instruction and Assessment

Teachers should use this teacher guide to help them integrate the core curriculum – values, cognitive and high level skills, 21st Century skills, STEAM principles and skills, and reading, writing, and communication skills in their lesson planning, instruction and assessment. All students in all subjects are required to learn and master these skills progressively through the education system.

### Integrate Cognitive, High Level, and 21st Century Skills in Lesson Planning, Instruction and Assessment

Teachers should integrate the cognitive, high level and 21st Century skills in their annual teaching programs, and give prominence to these skills in their lesson preparation,teaching and learning activities, performance assessment, and performance standards for measuring students' proficiency on these skills.
Mathematics addresses the skills and processes of solving problems arising in everyday life, society and the workplace. Thus,students will be able to make informed decisions, problem-solving and management knowledge, skills, values and attitudes in Mathematics. This enables them to function effectively in the work and higher education environments as productive and useful citizens of a culturally diverse and democratic society in an interdependent world.

In addition, it envisaged all students attaining expected proficiency levels in these skills and will be ready to pursue careers and higher education academic programs that demand these skills, and use them in their everyday life after they leave school at the end of Grade 12. Teachers should use the teacher guide to help them to effectively embed these skills, particularly in their lesson planning and in the teaching and learning activities as well as in the assessment of students' application of the skills.

### Integrate Mathematics Values and Attitudes in Lesson Planning, Instruction and Assessment

In Mathematics, students are expected to learn, promote and use work, relationship, peace, health, social, personal, family, community, national and global values in the work and study environments as well as in their conduct as community, national and global citizens. Teachers should draw from the information and suggestions provided in the syllabus and teacher guide to integrate values and attitudes in their lesson planning, instruction, and assessment. They should report on students' progression towards internalizing different values and attitudes and provide additional support to students who are yet to reach the internalization stage to make positive progress towards this level.

### Integrate Science, Technology, Engineering, Arts and Mathematics (STEAM)

Teachers should draw from both the syllabus and teacher guide in order to help them integrate STEAM principles and skills, and methodologies in their lesson planning, instruction and assessment. STEAM teaching and learning happens both inside and outside of the classroom. Effective STEAM teaching and learning requires both the teacher and the student to participate as core investigators and learners, and to work in partnership and collaboration with relevant stakeholders to achieve maximum results. Teachers should use the syllabus, teacher guides and other resources to guide them to plan and implement this and other innovative and creative approaches to STEAM teaching and learning to make STEAM principles and skills learning fun and enjoyable and, at the same time, attain the intended quality of learning outcomes.

### Identify and Use Grade and Context Appropriate, Innovative, Differentiated and Creative Teaching and Learning Methodologies

SBC is an eclectic curriculum model. It is an amalgam of strengths of different curriculum types, including behavioural objectives, outcomes, and competency. Its emphasis is on students attaining clearly defined, measurable, observable and attainable learning standards, i.e., the expected level of education quality. Proficiency (competency) standards are expressed as performance standards/criteria and evidence outcomes, in real life or related situations) to indicate that they are meeting, have met or exceeded the learning standards. The selection of grade and contextually appropriate teaching and learning methodologies is critical to enabling all students to achieve the expected standard or quality of education. Teaching and learning methodologies must be aligned to the content, learning objective, and performance standard in order for the teacher to effectively teach and guide students towards meeting the performance standard for the lesson. They should be equitable and socially inclusive, differentiate, student-centred, and lifelong. They should enable STEAM principles and skills to be effectively taught and learned by students. Teachers should use the teacher guide to help them make informed decisions when selecting the types of teaching and learning methodologies to use in their teaching of the subject content, including STEAM principles and skills.

### Plan Standards-Based Lessons

SBC lesson planning is quite difficult to do. However, this will be easier with more practice and experience over time. Effective SBC lesson plans must meet the required standards or criteria so that the learning objectives and performance standards are closely aligned to attain the expected learning outcomes. Teachers should use the guidelines and standards for SBC lesson planning and examples of SBC lesson plans provided in the teacher guide to plan their lessons. When planning lessons, it is important for teachers to ensure that all SBC lesson planning standards or criteria are met. If standards are not met, instruction will not lead to the attainment of intended performance and proficiency standards. Therefore, students will not attain the national content standards and grade-level benchmarks.

### Use Standards-Based Assessment

Standards-Based Assessment has a number of components. These components are intertwined and serve to measure evaluate, report, and monitor students' achievement of the national and grade-level expectations, i.e., the essential knowledge, skills, values and attitudes they are expected to master and demonstrate proficiency on.

Teachers should use the information and examples on standards-based assessment assess, record, evaluate, report and monitor students' performance in relation to the learning standards.

### Make informed Judgments About Students' Learning and Progress Towards Meeting Learning Standards

Teachers should use the teacher guide to effectively evaluate students' performance and use the evidence to help students to continuously improve their learning as well as their classroom practice.

It is important that teachers evaluate the performance of students in relation to the performance standards and progressively the grade-level benchmarks and content standards to make informed judgments and decisions about the quality of their work and their progress towards meeting the content standards or components of the standards. Evaluation should not focus on only one aspect of students' performance. It should aim to provide a complete picture of each student's performance. The context, inputs, processes, including teaching and learning processes, and the outcomes should be evaluated to make an informed judgment about each student's performance, Teachers should identify the causal factors for poor performance, gaps in students learning, gaps in teaching, teaching and learning resource constraints, and general attitude towards learning. Evidence-based decisions can then be made regarding the interventions for closing the gaps to allow students to make the required progress towards meeting grade-level and national expectations.

### Prepare Students' Performance Reports

Reporting of students' performance and progress towards the attainment of learning standards is an essential part of SBC assessment. Results of students' performance should be communicated to particularly the students and their parents to keep them informed of students' academic achievements and learning challenges as well as what needs to be done to enable the students' make positive progress towards meeting the proficiency standards and achieve the desired level of education quality. Teachers should use the information on the reporting of students' assessment results and the templates provided to report the results of students' learning.

### Monitor Students' Progress Towards Meeting the National Content Standards and Grade-Level Benchmarks

Monitoring of student's progress towards the attainment of learning standards is an essential component of standards-based assessment. It is an evidence-based process that involves the use of data from students' performance assessments to make informed judgements about students' learning and proficiency on the learning standards or their components, identify gaps in students' learning and the causal factors, set clear learning improvement targets, and develop effective evidence-based strategies (including pre-planning and re-teaching of topics), set clear time-frames, and identify measures for measuring students' progress towards achieving the learning targets.

Teachers should use the teacher guide to help them use data from students' performance assessments to identify individual students' learning weaknesses and develop interventions, in collaboration with each student and his/her parents or guardians, to address the weaknesses and monitor their progress towards meeting the agreed learning goals.

### Develop additional Benchmarks

Teachers can develop additional benchmarks using the examples in the teacher guide to meet the learning needs of their students and local communities. However, these benchmarks will not be nationally assessed as these are not comparable. They are not allowed to set their own content standards or manipulate the existing ones. The setting of national content standards is done at the national level to ensure that required learning standards are maintained and monitored to sustain the required level of education quality.

### Avoid Standardisation

The implementation of Grade 11 Advanced Mathematics curriculum must not be standardised. SBC does not mean that the content, lesson objectives, teaching and learning strategies, and assessment are standardised. This is a misconception and any attempt to standardise the components of curriculum without due consideration of the teaching and learning contexts, student's backgrounds and experiences, and different abilities and learning styles of students will be counterproductive. It will hinder students from achieving the expected proficiency standards and hence, high academic standards and the desired level of education quality. That is, they should not be applied across all contexts and with all students, without considering the educational needs and the characteristics of each context.

Teachers must use innovative, creative, culturally relevant, and differentiated teaching and learning approaches to teach the curriculum and enable their students to achieve the national content standards and grade-level benchmarks. And enable all students to experience success in learning the curriculum and achieve high academic standards.

What is provided in the syllabus and teacher guide are not fixed and can be changed. Teachers should use the information and examples provided in the syllabus and the teacher guide to guide them to develop, select, and use grade, context, and learner appropriate content, learning objectives, teaching and learning strategies, and performance assessment and standards. SBC is evidence-based hence decisions about the content, learning outcomes, teaching and learning strategies, students' performance, and learning interventions should be based on evidence. Teaching and learning should be continuously improved and effectively targeted using evidence from students' assessment and other sources.

# Syllabus and Teacher Guide Alignment

A teacher guide is a framework that describes how to translate the content standards and benchmarks (learning standards) outlined in the syllabus into units and topics, learning objectives, lesson plans, teaching and learning strategies, performance assessment, and measures for measuring students' performance (performance standards). It expands the content overview and describes how this content identified in the content standards and their components (essential KSVAs) can be translated into meaningful and evidence-based teaching topics and learning objectives for lesson planning, instruction and assessment. It also describes and provides examples of how to evaluate and report on students' attainment of the learning standards, and use evidence from the assessment of students' performance to develop evidence-based interventions to assist students who are making slow progress towards meeting the expected proficiency levels to improve their performance.

Grade 11 Advanced Mathematics comprises of the Syllabus and Teacher Guide. These two documents are closely aligned, complimentary and mutually beneficial. They are the essential focal points for teaching and learning the essential Mathematics knowledge, skills, values and attitudes.

| Syllabus and Teacher Guide Alignment | |
|---|---|
| **Syllabus** | **Teacher Guide** |
| Outlines the ultimate aim and goals, and what to teach and why teach it | Describes how to plan, teach, and assess students' performance |
| • Overarching and SBC principles<br>• Content overview<br>• Core curriculum<br>• Essential knowledge, skills, values and attitudes<br>• Strands and units<br>• Evidence outcomes<br>• Content standards and grade-level<br>• benchmark<br>• Overview of assessment, evaluation, and Reporting | • Determine topics for lesson planning, instruction and assessment<br>• Formulate learning objectives<br>• Plan SBC lesson plans<br>• Select teaching and learning strategies<br>• Implement SBC assessment and evaluation<br>• Implement SBC reporting and monitoring |

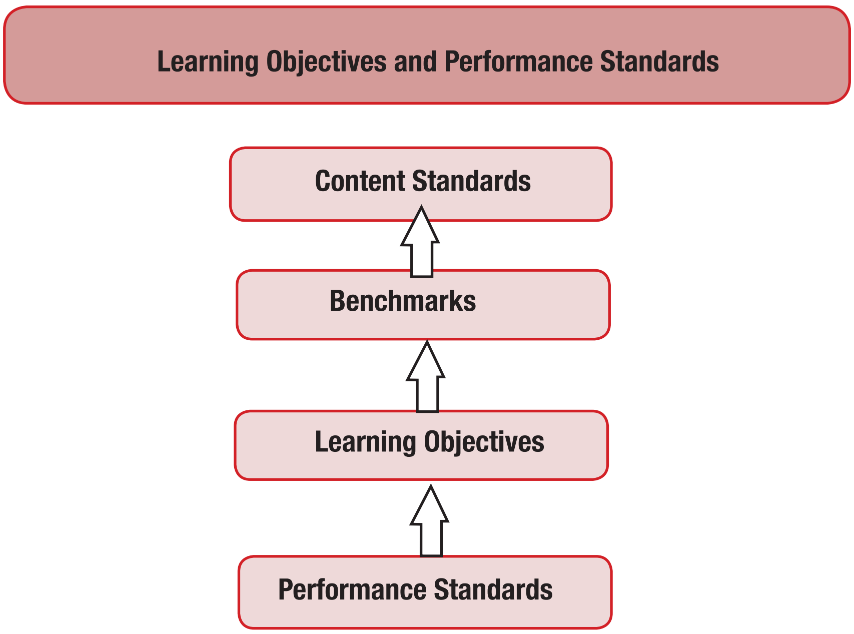
The syllabus outlines the ultimate aim and goals of SBE and SBC, what is to be taught and why it should be learned by students, the underlying principles and articulates the learning and proficiency standards that all students are expected to attain. On the other hand, the teacher guide expands on what is outlined in the syllabus by describing the approaches or the how of planning, teaching, learning, and assessing the content so that the intended learning outcomes are achieved.

This teacher guide should be used in conjunction with the syllabus. Teachers should use these documents hen planning, teaching and assessing Grade 11 Advanced Mathematics content.

Teachers will extract information from the syllabus (e.g., content standards and grade-level benchmarks) for lesson planning, instruction and is for measuring students' attainment a content standard as well as progress to the next grade of schooling.

## Learning and Performance Standards Alignment

Content Standards, Benchmarks, Learning Objectives, and Performance Standards are very closely linked and aligned (see below). There is a close linear relationship between these standards. Students' performance on a significant aspect of a benchmark (KSVA) is measured against a set of performance standards or criteria to determine their level of proficiency using performance assessment. Using the evidence from the performance assessment, individual student's proficiency on the aspect of the benchmark assessed and progression towards meeting the benchmark and hence the content standard are then determined.




Effective alignment of these learning standards and all the other components of PNG SBE and SBC (ultimate aim and goals, overarching, SBC and subject-based principles, core curriculum, STEAM, and cognitive, high level, and 21st Century skills) is not only critical but is also key to the achievement of high academic standards by all students and the intended level of education quality. It is essential that teachers know and can do standards alignment when planning, teaching, and assessing students' performance so that they can effectively guide their students towards meeting the grade-level benchmarks (grade expectations) and subsequently the content standards (national expectations).

# Learning and Performance Standards

Standards-Based Education (SBE) and SBC are underpinned by the notion of quality. Standards define the expected level of education quality that all students should achieve at a particular point in their schooling. Students' progression and achievement of education standard (s) are measured using performance standards or criteria to determine their demonstration or performance on significant aspects of the standards and therefore their levels of proficiency or competency. When they are judged to have attained proficiency on a content standard or benchmark or components of these standards, they are then deemed to have met the standard(s) that is, achieved the intend level of education quality.

Content standards, benchmarks, and learning objectives are called learning standards while performance and proficiency standards (evidence outcomes) can be categorised as performance standards. These standards are used to measure students' performance, proficiency, progression and achievement of the desired level of education quality. Teachers are expected to understand and use these standards for lesson planning, instruction and assessment.

## Content Standards

Content standards are evidence-based, rigorous and comparable regionally and globally. They have been formulated to target critical social, economic, political, cultural, environment, and employable skills gaps identified from a situational analysis. They were developed using examples and experiences from other countries and best practice, and contextualized to PNG contexts.

Content standards describe what (content - knowledge, skills, values, and attitudes) all students are expected to know and do (how well students must learn and apply what is set out in the content standards) at each grade-level before proceeding to the next grade. These standards are set at the national level and thus cannot be edited or changed by anyone except the National Subject-Based Standards Councils.

Content Standards;

- are evidenced-based,
- are rigorous and compards,
- are set at the national level,
- state or describe the expected levels of quality or achievement,
- are clear, measurable and attainable,
- are linked to and aligned with the ultimate aim and goals of SBE and SBC and overarching and SBC principles,
- delineate what matters, provide clear expectations of what students should progressively learn and achieve in school, and guide lesson planning, instruction,assessment,
- comprise knowledge, skills, values, and attitudes that are the basis for quality education,
- provide teachers a clear basis for planning, teaching, and assessing lessons, and
- provide provinces, districts, and schools with a clear focus on how to develop and organise their instruction and assessment programs as well as the content that will include in their curriculum.

## Benchmarks

Benchmarks are derived from the content standards and benchmarked at the grade-level. Benchmarks are specific statements of what students should know (i.e., essential knowledge, skills, values or attitudes) at a specific grade-level or school level. They provide the basis for measuring students' attainment of a content standard as well as progress to the next grade of schooling.

Grade-level benchmarks;

- are evidenced-based,
- are rigorous and comparable to regional and global standards;
- are set at the grade level,
- are linked to the national content standards,
- are clear, measurable, observable and attainable,
- articulate grade level expectations of what students are able to demonstrate to indicate that they are making progress towards attaining the national content standards,
- provide teachers a clear basis for planning, teaching, and assessing lessons,
- state clearly what students should do with what they have learned at the end of each school-level,
- enable students' progress towards the attainment of national content standards to be measured, and
- enable PNG students' performance to be compared with the performance of PNG students with students in other countries.

## Approach for Setting National Content Standards and Grade-Level Benchmarks

**Strand** ⇨ **Content Standards** ⇨ **Grade- Level Benchmarks**

## Development of Additional Benchmarks

Teachers should develop additional benchmarks to meet the learning needs of their students. They should engage their students to learn about local, provincial, national and global issues that have not been catered for in the grade-level benchmarks but are important and can enhance students' understanding and application of the content. However, it is important to note that these benchmarks will not be nationally examined as they are not comparable. Only the benchmarks developed at the national level will be tested. This does not mean that teachers should not develop additional benchmarks. An innovative, reflect, creative and reflexive teacher will continuously reflect on his/her classroom practice and use evidence to provide challenging, relevant, and enjoyable learning opportunities for his/her students to build on the national expectations for students.

Teachers should follow the following process when developing additional grade-level benchmarks.

**Benchmark Development Process**

Determine what needs to be taught (essential knowledge, skills, values and attitudes) → Determine what needs to be taught (essential knowledge, skills, values and attitudes) → Determine what needs to be taught (essential knowledge, skills, values and attitudes)

## Learning Objectives

Learning or instructional Objectives are precise statements of educational intent. They are formulated using a significant aspect or a topic derived from the benchmark, and is aligned with the educational goals, content standards, benchmarks, and performance standards. Learning objectives are stated in outcomes language that describes the products or behaviours that will be provided by students. They are stated in terms of measurable and observable student behaviour. For example, students will be able to explore the idea of direct and indirect proportion.

## Performance Standards

Performance Standards are concrete statements of how well students must learn what is set out in the content standards, often called the "be able to do" of "what students should know and be able to do." Performance standards are the indicators of quality that specify how competent a students' demonstration or performance must be. They are explicit definitions of what students must do to demonstrate proficiency or competency at a specific level on the content standards.

Performance standards;

- measure students' performance and proficiency **(using performance indicators)** in the use of a specific knowledge, skill, value, or attitude in real life or related situations,
- provide the basis **(performance indicators)** for evaluating, reporting and monitoring students' level of proficiency in use of a specific knowledge, skills, value, or attitude,
- are used to plan for individual instruction to help students not yet meeting expectations **(desired level of mastery and proficiency)** to make adequate progress towards the full attainment of benchmarks and content standards, and
- are used as the basis for measuring students' progress towards meeting grade-level benchmarks and content standards.

## Proficiency Standards

Proficiency standards describe what all students in a particular grade or school level can do at the end of a strand, or unit. These standards are sometimes called evidence outcomes because they indicate if students can actually apply or use what they have learned in real life or similar situations. They are also categorized as benchmarks because that is what all students are expected to do before exiting a grade or are deemed ready for the next grade.

# Core Curriculum

A core set of common learnings (knowledge, skills, values, and attitudes) are integrated into the content standards and grade-level benchmarks for all subjects. This is to equip all students with the most essential and in-demand knowledge, skills, and dispositions they will need to be successful in modern/postmodern work places, higher-education programs and to be productive, responsible, considerate, and harmonious citizens. Common set of learning are spirally sequenced from Preparatory-Grade 12 to deepen the scope and increase the level of difficulty in the learning activities so that what is learned is reinforced at different grade levels.

The core curriculum includes:

- cognitive (thinking) skills (refer to the syllabus for a list of these skills),
- reasoning, decision-making and problem-solving skills,
- high level thinking skills (analysis, synthesis and evaluation skills),
- 21st Century skills (refer to illustrative list in the appendix 2),
- reading, writing and communication skills,
- STEAM principles and skills,
- essential values and attitudes (core personal and social values, and sustaining values), and
- spiritual values and virtues.

The essential knowledge, skills, values and attitudes comprising the core curriculum are interwoven and provide an essential and holistic framework for preparing all students for careers, higher education and citizenship.

All teachers are expected to include the core learnings in their lesson planning, teaching, and assessment of students in all their lessons. They are expected to foster, promote and model the essential values and attitudes as well as the spiritual values and virtues in their conduct, practice, appearance, and their relationships and in their professional and personal lives. In addition, teachers are expected to mentor, mould and shape each student to evolve and possess the qualities envisioned by society.

Core values and attitudes must not be taught in the classroom only; they must also be demonstrated by students in real life or related situations inside and outside of the classroom, at home, and in everyday life. Likewise, they must be promoted, fostered and modelled by the school community and its stakeholders, especially parents. A holistic of school approach to values and attitudes in teaching, promoting and modelling is critical to students and the whole school community to internalise the core values and attitudes and make them habitual in their work and school place, and in everyday life. Be it work values, relationship values, peace values, health values, personal and social values, or religious values, teachers should give equal prominence to all common learnings in their lesson planning, teaching, assessment, and learning interventions. Common learnings must be at the heart of all teaching and extra-curricular programs and activities.

# Science, Technology, Engineering, Arts and Mathematics

STEAM education is an integrated, multidisciplinary approach to learning that uses science, technology, engineering, arts and mathematics as the basis for inquiring about how STEAM has and continues to change and impact the social, political, economic, cultural and environmental contexts and identifying and solving authentic (real life) natural and physical environment problems by integrating STEAM-based principles, cognitive, high level and 21st Century skills and processes, and values and attitudes.

Mathematics is focused on both goals of STEAM rather than just the goal of problem-solving. This is to ensure that all students are provided opportunities to learn, integrate, and demonstrate proficiency on all essential STEAM principles, processes, skills, values and attitudes to prepare them for careers, higher education and citizenship.

Through STEAM education students will be able to:

(i) examine and use evidence to draw conclusions about how STEAM has and continues to change the social, political, economic, cultural and environmental contexts.

(ii) Investigate and draw conclusions on the impact of STEAM solutions to problems on the social, political, economic, cultural and environmental contexts.

(iii) Identify and solve problems using STEAM principles, skills, concepts, ideas and process.

(iv) Identify, analyse and select the best solution to address a problem.

(v) build prototypes or models of solutions to problems.

(vi) replicate a problem solution by building models and explaining how the problem was or could be solved.

(vii) test and reflect on the best solution chosen to solve a problem.

(viii) collaborate with others on a problem and provide a report on the process of problem solving used to solve the problem.

(ix) use skills and processes learnt from lessons to work on and complete STEAM projects.

(x) demonstrate STEAM principles, skills, processes, concepts and ideas through simulation and modelling.

(xi) explain the significance of values and attitudes in problem-solving.

STEAM is a multidisciplinary and integrated approach to understanding how science, technology, engineering, arts and mathematics shape and are shaped by our material, intellectual, cultural, economic, social, political and environmental contexts. And for teaching students the essential in demand cognitive, high level and 21st Century skills, values and attitudes, and empower them to effectively use these skills and predispositions to identify and solve problems relating to the natural and physical environments as well as the impact of STEAM-based solutions on human existence and livelihoods, and on the social, political, economic, cultural, and environmental systems.

STEAM disciplines have and continue to shape the way we perceive knowledge and reality, think and act, our values, attitudes, and behaviours, and the way we relate to each other and the environment. Most of the things we enjoy and consume are developed using STEAM principles, skills, process, concepts and ideas.

Things humans used and enjoyed in the past and at present are developed by scientists, technologists, engineers, artists and mathematicians to address particular human needs and wants. Overtime, more needs were identified and more products were developed to meet the ever changing and evolving human needs. What is produced and used is continuously reflected upon, evaluated, redesigned, and improved to make it more advanced, multipurpose, fit for purpose, and targeted towards not only improving the prevailing social, political, economic, cultural and environmental conditions but also to effectively respond to the evolving and changing dynamics of human needs and wants. And, at the same time, solutions to human problems and needs are being investigated and designed to address problems that are yet to be addressed and concurred. This is an evolving and ongoing problem-solving process that integrates cognitive, high level, and 21st Century skills, and appropriate values and attitudes.

STEAM is a significant framework and focal point for teaching and guiding students to learn, master and use a broad range of skills and processes required to meet the skills demands of PNG and the 21st Century. The skills that students will learn will reflect the demands that will be placed upon them in a complex, competitive, knowledge-based, information-age,technology- driven economy and society. These skills include cognitive (critical, synthetic, creative, reasoning, decision-making, and problem-solving) skills, high level (analysis, synthesis and evaluation) skills and 21st Century skills (see Appendix 4). Knowledge-based, information, and technology driven economies require knowledge workers not technicians. Knowledge workers are lifelong learners, are problem solvers, innovators, creators, critical and creative thinkers, reflective practitioners, researchers (knowledge producers rather than knowledge consumers), solutions seekers,outcomes oriented, evidence-based decision makers, and enablers of improved and better outcomes for all.

STEAM focuses on the skills and processes of problem solving. These skills and processes are at the heart of the STEAM movement and approach to not only problem solving and providing evidence-based solutions but also the development and use of other essential cognitive, high level and 21st Century skills. These skills are intertwined and used simultaneously to gain a broader understanding of the problems to enable creative, innovative, contextually relevant, and best solutions to be developed and implemented to solve the problems and attain the desired outcomes. It is assumed that by teaching students STEAM-based problem-solving skills and providing learning opportunities inside and outside the classroom will motivate more of them to pursue careers and academic programs in STEAM related fields thus, closing the skills gaps and providing a pool of cadre of workers required by technology, engineering, science, and mathematics-oriented industries.

Although, STEAM focuses on the development and application of skills in authentic (real life) contexts, for example the use of problem- solving skills to identify and solve problems relating to the natural and physical worlds, it does not take into account the significant influence values and attitudes have on the entire process of problem solving. Values and attitudes are intertwined with knowledge and skills. Knowledge, skills, values and attitudes are inseparable. Decisions about skills and processes of skills development and application are influenced by values and attitudes (mindset) that people hold. In the same light, the use of STEAM principles, processes and skills to solve problems in order to achieve the outcomes envisaged by society are influenced by values and the mindset of those who have identified and investigated the problem as well as those who are affected by the problem and will benefit from the outcome.

## STEAM Problem-Solving Methods and Approaches

Problem-solving involves the use of problem-solving methods and processes to identify and define a problem, gather information to understand its causes, draw conclusions, and use the evidence to design and implement solutions to address it. Even though there are many different problem-solving methods and approaches, they share some of the steps of problem-solving, such as;

- identifying the problem,
- understanding the problem by collecting data,
- analyse and interpret the data,
- draw conclusions,
- use data to consider possible solutions,
- select the best solution,
- test the effectiveness of the solution by trialling and evaluating it, and
- review and improve the solution.

STEAM problem solving processes go from simple and technical to advance and knowledge-based processes. However, regardless of the type of process used, students should be provided opportunities to learn the essential principles and processes of problem solving and, more significantly, to design and create a product that addressed a real problem and meets a human need.

The following are some of the STEAM problem solving processes.

### 1. Engineering and Technology Problem Solving Methods and Approaches

Engineering and technology problem-solving methods are used to identify and solve problems relating to the physical world using the design process. The following are some of the methods and approaches used to solve engineering and technology related problems.

**Parts Substitution**
It is the most basic of the problem-solving methods. It simply requires the parts to be substituted until the problem is solved.

**Diagnostics**
After identifying a problem, the technician would run tests to pinpoint the fault. The test results would be used either as a guide for further testing or for replacement of a part, which also need to be tested. This process continues until the solution is found and the device is operating properly.

**Troubleshooting**
Troubleshooting is a form of problem solving, often applied to repair failed products or processes.

**Reverse Engineering**
Reverse engineering is the process of discovering the technological principles underlying the design of a device by taking the device apart, or carefully tracing its workings or its circuitry. It is useful when students are attempting to build something for which they have no formal drawings or schematics.

**Divide and Conquer**
Divide and conquer is the technique of breaking down a problem into sub-problems, then breaking the sub-problems down even further until each of them is simple enough to be solved. Divide and conquer may be applied to all groups of students to tackle sub-problems of a larger problem, or when a problem is so large that its solution cannot be visualised without breaking it down into smaller components.

**Extreme Cases**
Considering "extreme cases" – envisioning the problem in a greatly exaggerated or greatly simplified form, or testing using extreme condition – can often help to pinpoint a problem. An example of the extreme-case method is purposely inputting an extremely high number to test a computer program.

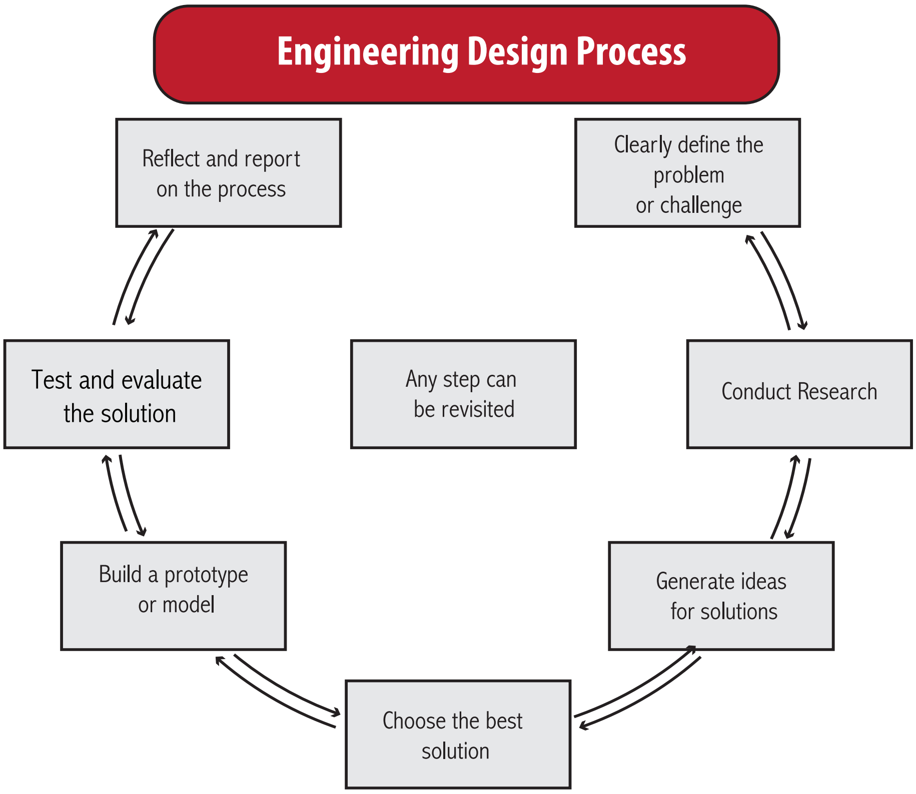
**Trial and Error**
The trial and error method involve trying different approaches until a solution is found. It is often used as a last resort when other methods have been exhausted.

- Test and evaluate the solution.
- Repeat steps as necessary to modify the design or correct faults.
- Reflect and report on the process.

**1. Engineering Design Process**

Technological fields use the engineering design process to identify and define the problem or challenge, investigate the problem, collect and analyse data, and use the data to formulate potential solutions to the problem, analyse each of the solutions in terms of its strengths and weaknesses, and choose the best solution to solve the problem. It is an open-ended problem-solving process that involves the full planning and development of products or services to meet identified needs. It involves a sequence of steps as illustrated.

1) Analyse the context and background, and clearly define the problem.
2) Conduct research to determine design criteria, financial or other constraints, and availability of materials.
3) Generate ideas for potential solutions, using processes such as brainstorming and sketching.
4) Choose the best solution.
5) Build a prototype or model.
6) Test and evaluate the solution.
7) Repeat steps as necessary to modify the design or correct faults.
8) Reflect and report on the process.




## STEAM-Based Lesson planning

Effective STEAM lesson planning is key to the achievement of expected STEAM outcomes. STEAM skills can be planed and taught using separate STEAM-based lesson plans or integrated into the standards-based lesson plans. To effectively do this, teachers should know how to write effective standards and STEAM-based lesson plans.

An example of a STEAM-based lesson plan is provided in appendix.
Teachers should use this to guide them to integrate STEAM content and teaching, learning and assessment strategies into their standards-based lesson plans.

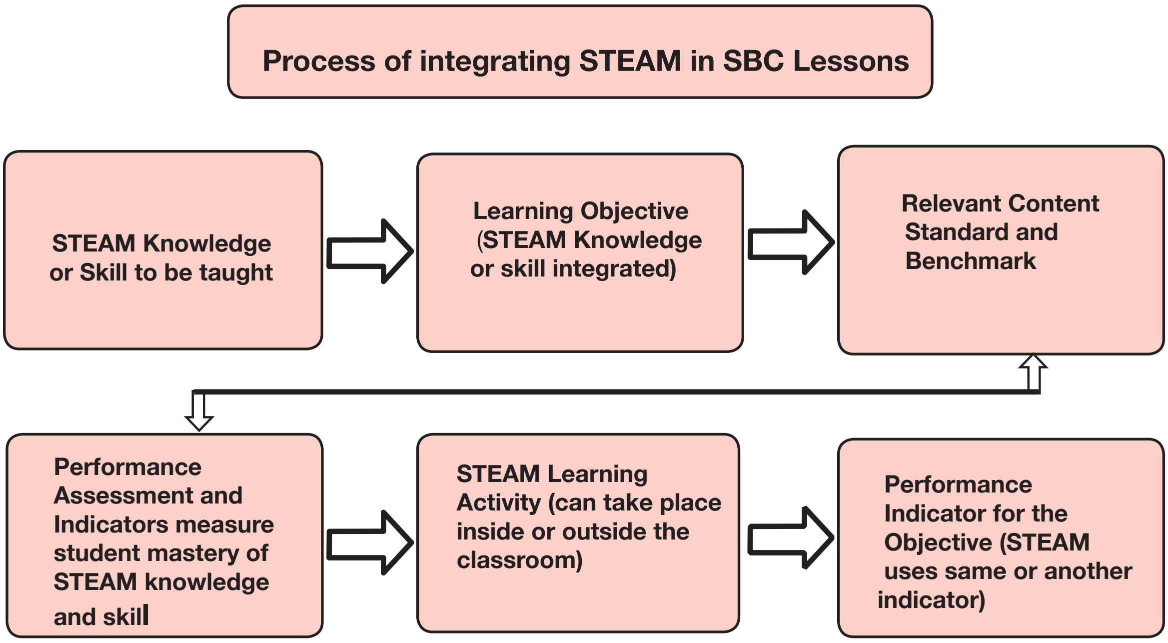
Integration of STEAM problem-solving skills into standards-based lesson plans

Knowing how to integrate STEAM problem-solving skills, principles, values and attitudes as well as STEAM teaching, learning, and assessment strategies into standards-based lesson plans is essential for achieving the desired STEAM learning outcomes. When integrating STEAM problem-solving skills into the standards-based lesson plans, teachers should ensure that these skills are not only effectively aligned to the learning objective and performance standards, they must also be effectively taught and assessed.

STEAM principles and problem-solving skills are integrated into the content standards and grade-level benchmarks. A list of these skills, including 21st Century skills, is provided in the syllabus. Teachers should ensure that these skills are integrated in their standards-based lesson plans, taught and assessed to determine students' level of proficiency on each skill or specific components of the skill.

Teachers are expected to integrate the essential STEAM principles, processes, skills, values and attitudes described in the Grade 11 benchmarks when formulating their standards-based lesson plans. Opportunities should be provided inside and outside of the classroom for students to learn, explore, model and apply what they learn in real life or related situations. These learning experiences will enable students to develop a deeper understanding of STEAM principles, processes, skills, values and attitudes and appreciate their application in real life to solve problems.

Teachers should use the following process as guide to integrate STEAM principles and problem-solving skills into the standards-based lesson plans.




Steps for integrating STEAM problem-solving principles and skills into standards-based lesson plans.

**Step 1**: Identify the STEAM knowledge or skill to be taught (from the table of KSVAs for each content standard and benchmark). This is captured in the learning objective stated in the standards-based lesson plan.

**Step 2:** Develop and include a performance standard or indicator for measuring student mastery of the STEAM knowledge or skill (e.g. level of acceptable competency or proficiency) if this is different from the one already stated in the lesson plan.

**Step 3:** Develop student learning activity (An activity that will provide students the opportunity to apply the STEAM knowledge or skill specified by the learning objective and appropriate statement of the standards). Activity can take place inside or outside of the classroom, and during or after school hours.

**Step 4:** Develop and use performance descriptors (standards or indicators) to analyse students' STEAM related behaviours and products (results or outcomes), which provide evidence that the student has acquired and mastered the knowledge or skill of the learning objective specified by the indicator (s) of the standard(s).

## STEAM Teaching Strategies

STEAM education takes place in both formal and informal classroom settings. It takes place during and after school hours. It is a continuous process of inquiry, data analysis, making decisions about interventions, and implementing and monitoring interventions for improvements.

There are a variety of STEAM teaching strategies. However, teaching strategies selected must enable teachers to guide students to use the engineering and artistic design processes to identify and solve natural and physical environment problems by designing prototypes and testing and refining them to effectively mitigate the problems identified. The following are some of the strategies that could be used to utilise the STEAM approach to solve problems and coming up with technological solutions.

1. Inquiry-Based Learning
2. Problem-Based Learning
3. Project-based learning
4. Collaborative Learning

Collaborative learning involves individuals from different STEAM disciplines and expertise in a variety of STEAM problem solving approaches working together and sharing their expertise and experiences to inquire into and solve a problem.Teachers should plan to provide students opportunities to work in collaboration and partnership with experts and practitioners engaged in STEAM related careers or disciplines to learn first-hand about how STEAM related skills, processes, concepts, and ideas are applied in real life to solve problems created by natural and physical environments. Collaborative learning experiences can be provided after school or during school holidays to enable students to work with STEAM experts and practitioners to inquiry and solve problems by developing creative, innovative and sustainable solutions. Providing real life experiences and lessons, e.g., by involving students to actually solve a scientific, technological, engineering, or mathematical, or Arts problem, would probably spark their interest in a STEAM career path.

Developing STEAM partnerships with external stakeholders e.g., high education institutions, private sector, research and development institutions, and volunteer and community development organizations can enhance students' learning and application of STEAM problem solving principles and skills.

Some examples of STEAM-related partnership experiences may include:

- Participatory Learning
- Group-Based Learning
- Task Oriented Learning
- Action Learning
- Experiential Learning
- Modeling
- Simulation

## STEAM Learning Strategies

Teachers should include in their lesson plans STEAM learning activities. These activities should be aligned to principle or a skill planned for students to learn and demonstrate proficiency at the end of the lesson to expose students to STEAM and giving them opportunities to explore STEAM-related concepts, they will develop a passion for it and, hopefully, pursue a job in a STEAM field.
Providing real life experiences and lessons, e.g., by involving students to actually solve a scientific, technological, engineering, or mathematical, or arts problem, would probably spark their interest in a STEAM career path. This is the theory behind STEAM education.

## STEAM-Based Assessment

STEAM-based assessment is closely linked to standards-based assessment where assessment is used to assess students' level of competency or proficiency of a specific knowledge, skill, value, or attitude taught using a set of performance standards (indicators or descriptors). The link also includes the main components such as the purpose, the assessment principles and assessment strategies and tools.
In STEAM-based assessment, assessments are designed for what students should know and be able to do. In STEAM learning, students are assessed in a variety of ways including portfolios, project/problem-based assessments, backwards design, authentic assessments, or other student-centered approaches.
When planning and designing the assessment, teachers should consider the authenticity of the assessment by designing an assessment that relates to a real world task or discipline specific attributes such as simulation, role play, placement assessment, live projects and debates. These tasks should make the activity meaningful to the student, and therefore be motivating as well as developing employability skills and discipline specific attributes.

## Effective STEAM-Based Assessment Strategies

The following are the six assessment tools and strategies to impact teaching and learning as well as help teachers foster 21$^{st}$ Century learning environment in their classrooms.

1. Rubrics
2. Performance-Based Assessments (PBAs)
3. Portfolios
4. Student self-assessment
5. Peer-assessment
6. Student Response Systems (SRS).

Although the list does not include all innovative assessment strategies, it includes what we think are the most common strategies, and ones that may be particularly relevant to the educational context of developing countries in this 21$^{st}$ Century. Many of the assessment strategies currently in use fit under one or more of the categories discussed. Furthermore, it is important to note that these strategies also connect in a variety of ways.

### *1. Rubrics*

Rubrics are both a tool to measure students' knowledge and ability as well as an assessment strategy. A rubric allows teachers to measure certain skills and abilities not measurable by standardized testing systems that assess discrete knowledge at a fixed moment in time. Rubrics are also frequently used as part of other assessment strategies including; portfolios, performances, projects, peer-review and self-assessment which are also elaborated in this section.

***2. Performance-Based Assessments***

Performance-Based Assessments (PBA), also known as project-based or authentic assessments, are generally used as a summative evaluation strategy to capture not only what students know about a topic, but if they have the skills to apply that knowledge in a "real-world" situation.
By asking them to create an end product. PBA pushes students to synthesize their knowledge and apply their skills to a potentially unfamiliar set of circumstances that is likely to occur beyond the confines of a controlled classroom setting.
The implementation of performance-based assessment strategies can also impact other instructional strategies in the classroom.

***3. Portfolio Assessment***

Portfolios are a collection of student work gathered over time that is primarily used as a summative evaluation method. The most salient characteristic of the portfolio assessment is that rather than being a snapshot of a student's knowledge at one point in time (like a single standardized test), it highlights student effort, development, and achievement over a period of time; portfolios measure a student's ability to apply knowledge rather than simply regurgitate. They are considered both student-centred and authentic assessments of learning.

***4. Self-assessment***

While the previous assessment tools and strategies listed in this report generally function as summative approaches, self-assessment is generally viewed as a formative strategy, rather than one used to determine a student's final grade. Its main purpose is for students to identify their own strengths and weakness and to work to make improvements to meet specific criteria.
Self-assessment occurs when students judge their own work to improve performance as they identify discrepancies between current and desired performance. In this way, self-assessment aligns well with standards- based education because it provides clear targets and specific criteria against which students or teachers can measure learning.

Self-assessment is used to promote self-regulation, to help students reflect on their progress and to inform revisions and improvements on a project or paper. In order for self-assessment to be truly effective four conditions must be in place: the self- assessment criteria is negotiated between teachers and students, students are taught how to apply the criteria, students receive feedback on their self-assessments and teachers help students use assessment data to develop an action plan.

***5. Peer assessment***

Peer assessment, much like self-assessment, is a formative assessment strategy that gives students a key role in evaluating learning. Peer assessment approaches can vary greatly but, essentially process develops both the assessor and assessee's skills and knowledge.

The primary goal for using peer assessment is to provide feedback to learners. This strategy may be particularly relevant in classrooms with many students per teacher since student time will always be more plentiful than teacher time. Although any single student's feedback may not be rich or in-depth as tea her's feedback, the research suggests that peer assessment can improve learning.

### *6. Student Response System*

Student response system (SRS), also known as classroom response (CRS), audience response system (ARS) is a general term that refers to a variety of technology-based formative assessment tools that can be used to gather student-level data instantly in the classroom. Through the combination of hardware, (voice recorders, PC, internet connection, projector and screen) and software.

Teachers can ask students a wide range of questions (both closed and open ended), where students can respond quickly and anonymously, and the teacher can display the data immediately and graphically. The use of technology also includes a use of video which examines how a range of strategies can be used to assess students' understanding.

The value of SRS comes from teachers analyzing information quickly and then devising real-time instructional solutions to maximize student learning. This includes a suggested approach to help teachers and trainers assess learning.

# Curriculum Integration

## What is Curriculum Integration?

Curriculum integration is making connections in learning across the curriculum. The ultimate aim of curriculum integration is to act as a bridge to increase students' achievement and engage in relevant curriculum.

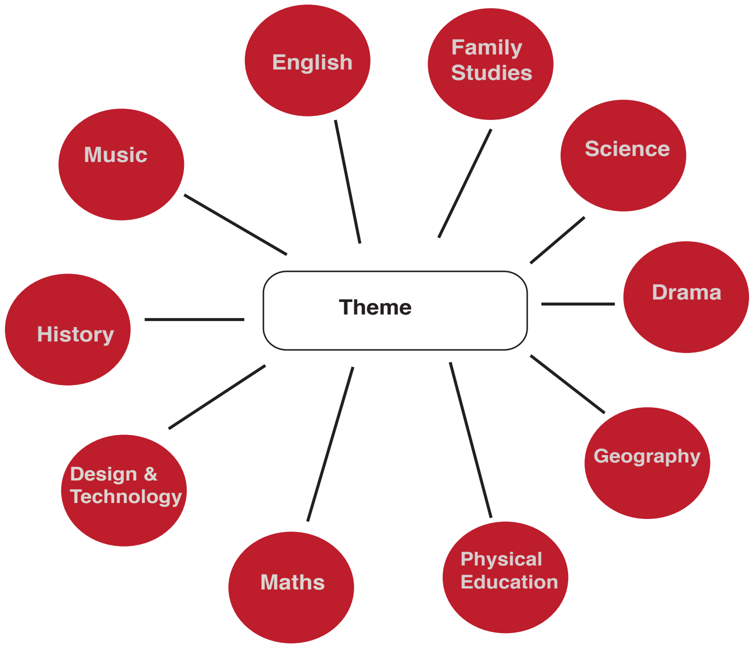
Teachers must develop intriguing curriculum by going beyond the traditional teaching of content based or fragmented teaching to one who is knowledge based and who should be perceived as a 21st Century innovative educator. Curriculum integration is a holistic approach to learning thus curriculum integration in PNG SBC will have to equip students with the essential knowledge, skills, values and attitudes that are deemed 21st Century.

There are three approaches that PNG SBC will engage to foster conducive learning for all its children whereby they all can demonstrate proficiency at any point of exit. Adapting these approaches will have an immense impact on the lives of these children thus they can be able to see themselves as catalyst of change for a competitive PNG. Not only that but they will be comparable to the world standards and as global citizens.

Engaging these three approaches in our curriculum will surely sharpen the knowledge and ability of each child who will foresee themselves as assets through their achievements thus contribute meaningfully to their country. They themselves are the agents of change. Integrated learning will bear forth a generation of knowledge based populace who can solve problems and make proper decisions based on evidence. Thus, PNG can achieve its goals like the Medium Term Development Goals (MTDG) and aims such as the Vision 2050 for a happy, healthy and wealthy society whereby, all its citizens should have access and fair distribution to income, shelter, health, education and general goods and services improving the general standard of living for PNG in the long run.

### 1. (i) Multidisciplinary Approach

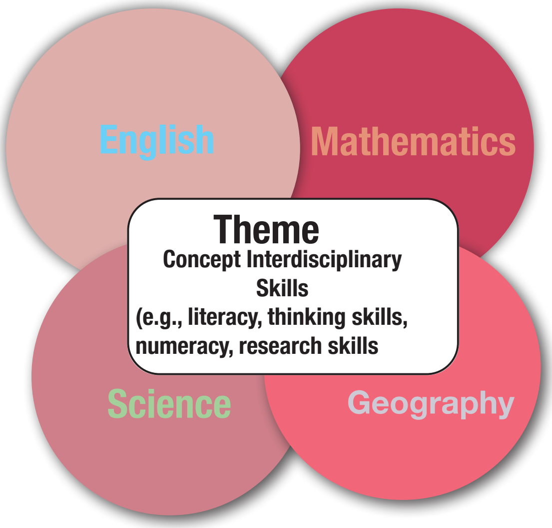
In this approach learning involves a theme or concept that will be taught right across all subject area of study by students. That is, content of a particular theme will be taught right across all subjects as shown in the diagram below. For instance, if the theme is global warming, subject areas create lessons or assessment as per their subjects around this theme. Social Science will address this issue, Science and all other subject likewise.




### 1. (ii) Interdisciplinary Approach

This approach addresses learning similarly to the multidisciplinary approach of integrated learning whereby learning takes place within the subject area. However, it is termed interdisciplinary in that the core curriculum of learning is interwoven into each subject under study by the students. For instance; in Social Science under the strand of geography students write essay on internal migration however, apart from addressing the issues of this topic, they are to apply the skill of writing text types in their essay such as argumentative essay, informative, explanatory, descriptive, expository and narrative essay while writing their essay. They must be able to capture the mechanics of English skills such as grammar, punctuation and so forth. Though these skills are studied under English they are considered as core skills that cut across all subjects under study. For example; if Science students were to write about human development in biology then the application of writing skills has to be captured by the students in their writing. It is not seen as an English skill but a standard essential skill all students must know and do regardless.

Therefore, essential knowledge, skills, values and attitudes comprising the core curriculum are interwoven and provide an essential and holistic framework for preparing all students for careers, higher education and citizenship in this learning.



### 2. Intradisciplinary Approach

This approach involves teachers integrate sub disciplines within a subject area. For instance, within the subject Social Science, the strands (disciplines) of geography, environment, history, political science and environment will all be captured studying a particular content for Social Science. For example, under global warming, students will study the geographical aspects of global warming, environmental aspect of global warming and likewise for history, political science and economics. Thus, children are well aware of the issues surrounding global warming and can address it confidently at each level of learning.

### 3. Trans disciplinary Approach

In this approach learning goes beyond the subject area of study. Learning is organized around students' questions and concerns. That is, where there is a need for change to improve lives, students develop their own curriculum to effect these need. The trans-disciplinary approach addresses real-life situations thus giving the opportunity to students to attain real life skills. This learning approach is more to do with Project–Based Learning also referred to as problem-based learning or place-based learning.

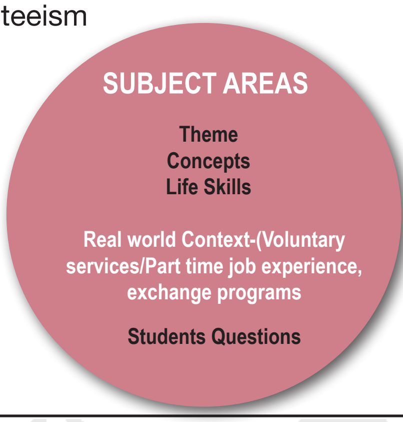
Below are the three steps to planning project based curriculum

1. Teachers and students select a topic of study based on student interests,
2. curriculum standards, and local resources.
3. The teacher finds out what the students already know and helps them generate questions to explore. The teacher also provides resources for students and
4. opportunities to work in the field
5. Students share their work with others in a culminating activity. Students display the results of their exploration and review and evaluate the project.

For instance; students may come up with slogans for school programs such as 'Our culture – clean city for a healthier PNG'. The main aim could be to curb betel nut chewing in public areas especially around bus stops and local markets. Here, students draw up their own instructions and criteria for assessment which is; they have to clean the nearest bus stop or local market once a week throughout the year. They also design and create posters to educate the general public as their program continues. They can also involve the town council and media to assist them especially to carry out awareness.

Studies have proven that Project based-programs have led to the following:

- Students go far beyond the minimum effort
- Make connections among different subject areas to answer open-ended questions
- Retain what they have learnt
- Apply learning to real-life problems
- Have fewer discipline problems
- Lower absenteeism




These integrated learning approaches will demand for teaches to be proactive in order to improve students learning and achievement. In order for PNG Standards-Based Curriculum to serve its purpose fully, these three approaches must be engaged for better learning for the children of PNG now and in the future

# Essential Knowledge, Skills, Values and Attitude & Mathematical Thinking

Students' level of proficiency and progression towards the attainment of content standards will depend on their mastery and application of essential knowledge, skills, values, and attitudes in real life or related situations. Provided here are examples of different types of knowledge, processes, skills, values, and attitudes that all students are expected to learn and master as they progress through the grades. These are expanded and deepen in scope and the level of difficulty and complexity are increased to enable students to study in-depth the subject content as they progress from one grade to the next.

These knowledge, skills, values and attitudes have been integrated into the content standards and benchmarks. They will also be integrated into the performance standards. Teachers are expected to plan and teach essential knowledge, skills, values and attitudes in their lessons, and assess students' performance and proficiency, and progression towards the attainment of content standards.

## Types of Knowledge

| There are different types of knowledge. These include; | |
|---|---|
| • Public and private (privileged) knowledge<br>• Specialised knowledge<br>• Good and bad knowledge<br>• Concepts, processes, ideas, skills, values, attitudes<br>• Theory and practice<br>• Fiction and non-fiction<br>• Traditional, modern, and postmodern knowledge | • Subject and discipline-based knowledge<br>• Lived experiences<br>• Evidence and assumptions<br>• Ethics and Morales<br>• Belief systems<br>• Facts and opinions<br>• Wisdom<br>• Research evidence and findings<br>• Solutions to problems |

## Types of Processes

| There are different types of processes. These include; | |
|---|---|
| • Problem-solving<br>• Logical reasoning<br>• Decision-making<br>• Reflection | • Cyclic processes<br>• Mapping (e.g. concept mapping)<br>• Modelling<br>• Simulating |
| Mathematics Inquiry processes include:<br><br>• Gathering information<br>• Analysing information<br>• Evaluating information<br>• Making judgements<br>• Taking actions | |

## Mathematical Thinking Processes

The five Mathematical process skills that can help the students improve their mathematical thinking.

**1. Mathematical Problem Solving**

- Understand the meaning of the problem and look for entry points to its solution
- Analyse information (givens, constrains, relationships, goals)
- Make conjectures and plan a solution pathway
- Monitor and evaluate the progress and change course as necessary
- Check answers to problems and ask, “Does this make sense?

**2. Mathematical Communication**

- Use definitions and previously established causes/effects (results) in constructing arguments
- Make conjectures and use counter examples to build a logical progression of statements to explore and support their ideas
- Communicate and defend mathematical reasoning using objects, drawings, diagrams, actions
- Listen to or read the arguments of others
- Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments.

**3. Mathematical Reasoning**

- Make sense of quantities and relationships in problem situations
- Represent abstract situations symbolically and understand the meaning of quantities
- Create a coherent representation of the problem at hand
- Consider the units involved
- Flexibly use properties of operations.

**4. Mathematical Connections**

- Look for patterns or structure, recognizing that quantities can be represented in different ways
- Recognize the significance in concepts and models and use the patterns or structure for solving related problems
- View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems
- Notice repeated calculations and look for general methods and short cuts
- Continually evaluate the reasonableness of intermediate results (comparing estimates) while attending to details and make generalizations based on finding.

**5. Mathematical Representation**

- Apply prior knowledge to solve real world problems
- Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
- Make assumptions and approximations to make a problem simpler
- Check to see if an answer makes sense within the context of a situation and change a model when necessary.

## Types of Skills

There are different types of skills. These include:

### 1. Cognitive (Thinking) Skills

Thinking skills can be categorized into **critical thinking** and **creative thinking** skills.

#### i. Critical Thinking Skills

| A person who thinks critically always evaluates an idea in a systematic manner before accepting or rejecting it. Critical thinking skills include; | |
|---|---|
| • Attributing<br>• Comparing and contrasting<br>• Grouping and classifying<br>• Sequencing<br>• Prioritising<br>• Analysing | • Detecting bias<br>• Evaluating<br>• Metacognition (Thinking about thinking)<br>• Making informed conclusions. |

#### ii. CreativeThinking Skills

| A person who thinks creatively has a high level of imagination, able to generate original and innovative ideas, and able to modify ideas and products. Creative thinking skills include; | |
|---|---|
| • Generating ideas<br>• Deconstruction and reconstruction<br>• Relating<br>• Making inferences<br>• Predicting<br>• Making generalisations<br>• Visualizing | • Synthesising<br>• Making hypothesis<br>• Making analogies<br>• Invention<br>• Transformation<br>• Modelling<br>• Simulating |

2. **Reasoning Skills** - Reason is a skill used in making a logical, just, and rational judgment.

3. **Decision-Making Skills** - Decision-making involves selection of the best solution from various alternatives based on specific criteria and evidence to achieve a specific aim.

4. **Problem Solving Skills** – These skills involve finding solutions to challenges or unfamiliar situations or unanticipated difficulties in a systematic manner.

### 5. Literacy Skills

A strong emphasis must be placed on various types of literacy, from financial to technological, from media to mathematical, from content to cultural. Literacy may be defined as the ability of an individual to use information to function in society, to achieve goals and to develop her or his knowledge and potential. Teachers emphasize certain aspects of literacy over others, depending on the nature of the content and skills they want students to learn.

| The following literacy skills are intended to be exemplary rather than defi nitive | |
|---|---|
| • Listens, read, write, and speak with comprehension and clarity<br>• Define and apply discipline-based conceptual vocabulary<br>• Describe people, places, and events, and the connections between and among them<br>• Arrange events in chronological sequence<br>• Differentiate fact from opinion<br>• Determine an author's purpose<br>• Determine and analyse similarities and differences<br>• Analyse cause and effect relationships<br>• Explore complex patterns, interactions and relationships<br>• Differentiate between and among various options | • Listens, read, write, and speak with comprehension and clarity<br>• Define and apply discipline-based conceptual vocabulary<br>• Describe people, places, and events, and the connections between and among them<br>• Arrange events in chronological sequence<br>• Differentiate fact from opinion<br>• Determine an author's purpose<br>• Determine and analyse similarities and differences<br>• Analyse cause and effect relationships<br>• Develop an ability to use and apply abstract principals<br>• Explore and/or observe, identify, and analyse how individuals and/or societies relate to one another |

6. **High Level Thinking Skills -** These skills include analysis, synthesis, and evaluation skills.

**i.** Analysis Skills – Analysis skills involve examining in detail and breaking information into parts by identifying motives or causes,underlying assumptions, hidden messages; making inferences and finding evidence to support generalisations, claims, and conclusions.

| Keywords | | | | |
|---|---|---|---|---|
| Analyse | Differences | Find | List | Similar to |
| Appraise | Discover | Focus | Motivate | Simplify |
| Arrange | Discriminate | Function | Omit | Take part in |
| Assumption | Discussion | Group | Order | Test for |
| Breakdown | Distinction | Highlight | Organize | Theme |
| Categorize | Distinguish | In-depth | Point out | |
| Cause & effect | Dissect | Inference | Research | |
| Choose | Divide | Inspect | See | |
| Classify | Establish | Isolate | Select | |
| Comparing | Examine | Investigate | Separate | |

ii. Synthesis Skills – Synthesis skills involve changing or creating something new, compiling information together in a different way by combining elements in a new pattern proposing alternative solutions.
iii. Evaluation Skills – Evaluation skills involve justifying and presenting and defending opinions by making judgments about information, validity of ideas or quality of work based on set criteria.

## Types of Values

Personal engagement and civic engagement strategies help young people to acquire and apply skills and dispositions that will prepare them to become competent and responsible citizens.

**1. Personal Values (importance, worth, usefulness, etc.)**

| Core values | Sustaining values |
|---|---|
| • Sanctity of life<br>• Truth<br>• Aesthetics<br>• Honesty<br>• Human<br>• Dignity<br>• Rationality<br>• Creativity<br>• Courage<br>• Liberty<br>• Affectivity<br>• Individuality | • Self-esteem<br>• Self-reflection<br>• Self-discipline<br>• Self-cultivation<br>• Principal morality<br>• Self-determination<br>• Openness<br>• Independence<br>• Simplicity<br>• Integrity<br>• Enterprise<br>• Sensitivity<br>• Modesty<br>• Perseverance |

**2. Social Values**

| Core values | Sustaining values |
|---|---|
| • Equality<br>• Kindness<br>• Benevolence<br>• Love<br>• Freedom<br>• Common good<br>• Mutuality<br>• Justice<br>• Trust<br>• Interdependence<br>• Sustainability<br>• Betterment of human kind<br>• Empowerment | • Plurality<br>• Due process of law<br>• Democracy<br>• Freedom and liberty<br>• Common will<br>• Patriotism<br>• Tolerance<br>• Gender equity and social inclusion<br>• Equal opportunities<br>• Culture and civilisation<br>• Heritage<br>• Human rights and responsibilities<br>• Rationality<br>• Sense of belonging<br>• Solidarity<br>• Peace and harmony<br>• Safe and peaceful communities |

## Types of Attitudes

| Attitudes - Ways of thinking and behaving, points of view | |
|---|---|
| • Optimistic<br>• Participatory<br>• Critical<br>• Creative<br>• Appreciative<br>• Empathetic<br>• Caring and concern<br>• Positive<br>• Confident<br>• Cooperative | • Responsible<br>• Adaptable to change<br>• Open-minded<br>• Diligent<br>• With a desire to learn<br>• With respect for self, life, equality and excellence, evidence, fair play, rule of law, different ways of life, beliefs and opinions, and the environment. |

# Teaching and Learning Strategies

Mathematics teaching emphasises and embraces the use of cognitive, reasoning, decision-making, problem solving and higher level thinking skills to teach to enhance students' understanding of inter-disciplinary concepts and issues in relation to environment, geography, history, politics and economic within PNG and globally. It aims to provide a meaningful pedagogical framework for teaching and learning essential and in demand knowledge, skills, values, and attitudes that are required for the preparation of students for careers, higher education and citizenship in the 21st Century.

Students must be prepared to gather and understand information, analyse issues critically, learn independently or collaboratively, organize and communicate information, draw and justify conclusions, create new knowledge, and act ethically. These teaching and learning strategies will help teachers to;

- familiarize themselves with different methods of teaching in the classroom.
- develop an understanding of the role of a teacher for application of various methods in the classroom.

Successful teachers always keep in view that teaching must "be dynamic, challenging and in accordance with the learner's comprehension. He/she does not depend on any single method for making his/her teaching interesting, inspirational and effective.

A detailed table of Teaching and Learning Strategies are outlined below:

| STRATEGY | TEACHER | STUDENTS |
|---|---|---|
| **CASE STUDY**<br>Used to extend students' understanding of real life issues | Provide students with case studies related to the topic of the lesson and allow them to analyse and evaluate. | Study the case study and identify the problem addressed. They analyse the problem and suggest solutions supported by conceptual justifications and make presentations. This enriches the students' existing knowledge of the topic. |
| **DEBATE**<br>A method used to increase students' interest, involvement and participation | Provide the topic or question of debate on current issues affecting a bigger population, clearly outlining the expectations of the debate. Explain the steps involved in debating and set a criteria/ standard to be achieved. | Conduct researches to gather supporting evidence about the selected topic and summarising the points. |
| **DISCUSSION**<br>The purpose of discussion is to educate students about the process of group thinking and collective decision. | The teacher opens a discussion on certain topic by asking essential questions. During the discussion, the teacher reinforces and emphasises on important points from students responses. | Students ponder over the question and answer by providing ideas, experiences and examples. |

| STRATEGY | TEACHER | STUDENTS |
|---|---|---|
| | Teacher guide the direction to motivate students to explore the topic in greater depth and the topic in more detail.<br>Use how and why follow-up questions to guide the discussion toward the objective of helping students understand the subject and summarise main ideas. | |
| **GAMES AND SIMULATIONS**<br>Encourages motivation and creates a spirit of competition and challenge to enhance learning | Being creative and select appropriate games for the topic of the lesson. Give clear instructions and guidelines. The game selected must be fun and build a competitive spirit to score more than their peers to win small prices. | Go into groups and organize.<br>Follow the instructions and play to win. |
| **OBSERVATION**<br>Method used to allow students to work independently to discover why and how things happen as the way they are. It builds curiosity. | Give instructions and monitor every activity students do. | Students possess instinct of curiosity and are curious to see the things for themselves and particularly those things which exist around them. A thing observed and a fact discovered by the child for himself becomes a part of mental life of the child. It is certainly more valuable to him than the same fact or facts learnt from the teacher or a book. Students<br>• Observe and ask essential questions<br>• Record<br>• Interpret |
| **PEER TEACHING & LEARNING**<br>(power point presentations, pair learning)<br>Students teach each other using different ways to learn from each other. It encourages; team work, develops confidence, feel free to ask questions, improves communication skills and most importantly develop the spirit of inquiry. | Distribute topics to groups to research and teach others in the classroom.<br>Go through the basics of how to present their peer teaching. | Go into their established working groups.<br>Develop a plan for the topic.<br>Each group member is allocated a task to work on.<br>Research and collect information about the topic allocated to the group. Outline the important points from the research and present their findings in class. |

| STRATEGY | TEACHER | STUDENTS |
|---|---|---|
| **PERFORMANCE-RELATED TASKS**<br>(dramatization, song/lyrics, wall magazines)<br>Encourages creativity and take on the overarching ideas of the topic and are able to recall them at a later date. | Students are given the opportunity to perform the using the main ideas of a topic.<br>Provide the guidelines, expectations and the set criteria. | Go into their established working groups.<br>Being creative and create dramas, songs/lyrics or wall magazines in line with the topic. |
| **PROJECT**<br>(individual/group)<br>Helps students complete tasks individually or collectively. | Teacher outline the steps and procedures of how to do and the criteria. | Students are involved in investigations and finding solutions to problems to real life experiences. They carry out researches to analyse the causes and effects of problems to provide achievable solutions. Students carefully utilise the problem-solving approach to complete projects. |
| **USE MEDIA & TECHNOLOGY** to teach and generate engagement depending on the age of the students. | Show a full movie, an animated one, a few episodes form documentaries, you tube movies and others depending on the lesson.<br>Provide questions for students to answer before viewing. | Viewing can provoke questions, debates, critical thinking, emotion and reaction.<br>After viewing, students engage in critical thinking and debate. |

# Strands, Units and Topics Suggested Lesson Titles

This section contains the overview of Mathematics content to be taught in Grade 11 Advanced Mathematics. The table below outlines strands, units, topics with suggested lesson titles. Teachers will use this to develop their own termly and yearly programs.

| Strand | Unit | Topic | Suggested Lesson Titles |
|---|---|---|---|
| **Number, Operation and Computation** | Numeracy | The real number system | Classification of Real Numbers |
| | | | Properties of Real Numbers |
| | | Surds | Properties of Surds |
| | | | Addition and Subtraction of Surds |
| | | | Multiplication of surds |
| | | | Division of surds by Rationalizing the denominator |
| | | Recurring and Non-Recurring Decimals | Expressing terminating and recurring Decimals |
| | | | Non terminating and non-recurring Decimals |
| | | Indices | Negative and Zero indices |
| | | | Fractional Indices |
| | | | Algebraic expressions with indices |
| | | | Solving Exponential Equation |
| | | Logarithm | Definition of Logarithm |
| | | | Proof of Logarithm Results |
| | | | Expressing Logarithmic Expressions as Single Logarithm |
| | | | Expanding Single Logarithmic Expressions |
| | | | Solving Logarithmic Equations |
| **Geometry, Measurement and Transformation** | Measurement | Conversion, Scales and dials | Conversion of metric and imperial units |
| | | | Reading Scale and dials |
| | | Length, Mass and Capacity | Application of length |
| | | | Application of Mass |
| | | | Application of capacity/volume |
| | Trigonometry | Application of Trigonometry | Trigonometry Ratios on Calculator |
| | | | Application of trigonometry ratios |
| | | | Application of Pythagoras Theorem |
| | | | Application of Angles of Elevation and Depression |
| | | Three-Dimensional application of Trigonometry | The angle between a line and a plane |
| | | | The angle between two planes |
| | | | Three-dimensional application |

| | | | |
|---|---|---|---|
| | Vectors | Definitions and Representation of vectors | Definition and representation of vectors |
| | | | Properties of Scalars and vectors |
| | | | Unit Vector Representation |
| | | | Position Vector Representation |
| | | Arithmetic operations on vectors | Addition and Subtraction of Vectors |
| | | | Scalar Multiplication |
| | | | Application of Parallel Vectors |
| | | Velocity vectors | Triangular velocity applied to aeroplane |
| | | | Triangular velocity applied to boat |
| | Geometry | Angles and Polygons | Transformation |
| | | | Tests for congruent triangles |
| | | | Constructing, investigating and proving for congruency |
| | | | Special and similar triangles |
| | | Geometric Proofs | Geometric construction |
| | | | Proof Geometric construction |
| | | Circle geometry | Angles at Centre and angles at Circumference |
| | | | Angles in the same segment |
| | | | Angles in opposite segment |
| | | | Application of chords and tangents in geometry |
| | | | Cyclic quadrilaterals and its properties |
| | | | Concyclic points |

| | | | |
|---|---|---|---|
| **Patterns and Algebra** | Linear Function | Gradient of straight line | The Gradient Intercept Form of a straight Line |
| | | | The Gradient of a Line through two points |
| | | Parallel and perpendicular Lines | Parallel and Perpendicular Lines |
| | | | Intersection of Two straight Lines |
| | | | Perpendicular bisector |
| | | Distance of a point from a line | The distance between two points |
| | | | The midpoint of an interval |
| | | | The perpendicular distance of a point from a line |
| | Functions and Graphs | Relations and Functions | The sketches of the functions and the vertical line test |
| | | | The applications of the functions |
| | | Domain and Range of a function | The main features of the functions including Domain and Range |
| | | | Calculate domain and range to functions |
| | | Absolute Value Functions | Absolute Value |
| | | | Graphing Absolute value function |
| | | | Absolute value equations |
| | | Linear, Quadratic and Exponential equations and Inequalities | Problems on Linear and Quadratic equations and Inequalities |
| | | | Problems on Exponential equations and Inequalities |
| | | Sketches of hyperbolic, exponential, Logarithmic functions and Asymptotes | Sketching Hyperbolic functions |
| | | | Sketching exponential functions |
| | | | Sketching Logarithmic functions |
| | | | Definition and application of asymptotes |
| | | Circles on Cartesian Plane | Radius and centre of a unit circle |
| | | | Deriving General form of circle equations from the graph |
| | | | Sketching the graphs of circles |
| | | Equation of Circles in Standard Form | Conversion of the equations of circles in Standard form: $(x-h)^2 + (y-k)^2 = r^2$ |
| | | | Sketch circles in standard form |

| | | | |
|---|---|---|---|
| **Statistics and Probability** | Data Analysis | Collecting and Organizing Data | Selecting sample data |
| | | | Frequency Tables for ungrouped Data |
| | | | Frequency Tables for grouped Data |
| | | | Frequency polygon |
| | | Measure of central tendency | Review of Measure of central tendency |
| | | Measure of dispersion | Review of Measure of dispersion |
| | | | Coefficient of variation |
| | Sets and Probability | Sets and Elements | Definition and Notations of Sets and Elements |
| | | | Field laws of Sets on Intersection and Union |
| | | Subsets and Venn Diagrams | Subsets and Equality of sets |
| | | | Representing sets Venn Diagrams |
| | | Solving problems using Venn diagrams | Solve problems using subsets and Venn diagrams |
| | | Theoretical and Experimental Probabilities | Theoretical Probability |
| | | | Experimental probability |
| | | Mutually and non-mutually exclusive events | Mutually Exclusive events |
| | | | Non-mutually Exclusive Events |
| | | Independent and dependent Events and Conditional Probability | Independent Events |
| | | | Dependent Events |
| | | | Conditional Probability |
| | | | Addition Rule for Probability ('OR') |
| | | | Multiplication Rule for Probability ('AND') |

# Grade 11
# Advanced Mathematics
## Teaching Content

## Strand 1: Number, Operations and Computation

**Content Standard:** Students will be able to represent numbers in various situations and forms, develop fluency in calculations through operations, use base ten as key for extending numbers and operations, and apply numbers in practical situations to develop number sense

| Unit | Benchmark | Topic | Lesson Title |
|---|---|---|---|
| Numeracy | **11.1.1.1** Apply arithmetic properties to operate on and simplify expressions that include radicals and other real numbers. | The real number system | Classification of Real Numbers |
| | | | Properties of Real Numbers |
| | | Surds | Properties of Surds |
| | | | Addition and Subtraction of Surds |
| | | | Multiplication of surds |
| | | | Division of surds by Rationalizing the denominator |
| | | Recurring and Non-Recurring Decimals | Expressing terminating and recurring Decimals |
| | | | Non terminating and non-recurring Decimals |
| | **11.1.1.2** Apply laws of indices and logarithms with different bases.<br><br>**11.1.1.3** Solve logarithmic equations in various problems. | Indices | Negative and Zero indices |
| | | | Fractional Indices |
| | | | Algebraic expressions with indices |
| | | | Solving Exponential Equation |
| | | Logarithm | Definition of Logarithm |
| | | | Proof of Logarithm Results |
| | | | Expressing Logarithmic Expressions as Single Logarithm |
| | | | Expanding Single Logarithmic Expressions |
| | | | Solving Logarithmic Equations |

## Strand 1: Number, Operations and Computation

**Unit : Numeracy** | **Topic: The real number system**

| | |
|---|---|
| **Benchmark** | **11.1.1.1** Apply arithmetic properties to operate on and simplify expressions that include radicals and other real numbers. |

**Learning Objective:** By the end of the topic, students will be able to define and classify the different types of real numbers in the real number system.

**Essential questions:**

- What are some types of numbers that make up the real number system?
- What are radical numbers?
- What are rational numbers?
- What are irrational numbers?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Appreciate learning about the classification of Real Numbers and their meanings. |
| **Skills** | Classify different types of Real Numbers. |
| **Knowledge** | The Real Number system. |
| **Mathematical Thinking** | Think about how real number systems are classified and their meaning. |

## Content Background

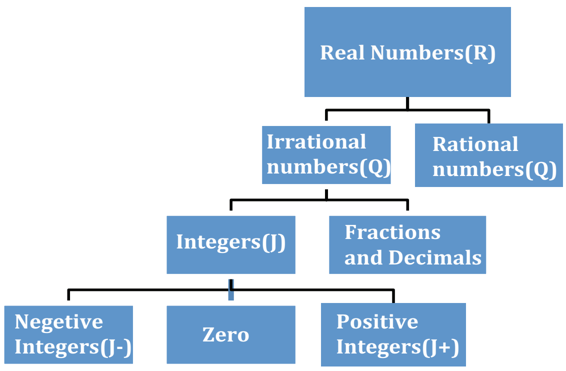
**Real Numbers**

All numbers on the number line are known as real numbers which include positive and negative whole numbers, zero, fractions, decimals, surds, π, etc. Real numbers are denoted by R.

There are different types of Real Numbers which are classified as:

- Rational and Irrational numbers
- Integers
- Fractions and decimals
- Natural or Counting numbers

The classification of real numbers is shown in the diagram below:



### Integers (J)

Integers are the set of negative numbers and whole numbers. An integer is a number that has no fractional part and no digits after the decimals point. An integer can be positive, negative or zero. It is denoted by (J). The set will be like {…-5,-4,-3,-2,-1,0,1,2,3,4,5…..}

### Fractions and decimals

The concept of "part of a whole" led to the development of fraction. A fraction is two quantities written one above the other that shows much of a whole thing we have. Later on, these fractions have their decimal representations.

### Rational Numbers

For many centuries whole numbers and fractions were thought to be the only types of numbers that we would ever have to deal with. They were collectively called the set of rational numbers.

Rational numbers:

- can be written as a ratio of two integers
- called rational numbers because the first five letters spell the word 'ratio'.
- can be expressed in the form $Q = \frac{a}{b}, b \neq 0;$ where, a and b are integers.
- can be a positive or negative.
- can have a zero as numerator, *e.g* $\frac{0}{6}$ since this number is equal to zero.
- cannot have a zero as denominator *e.g* $\frac{6}{0}$ since this number is undefined

| Unit: Numeracy | Topic: Surds |
|---|---|
| **Benchmark** | **11.1.1.1** Apply arithmetic properties to operate on and simplify expressions that include radicals and other real numbers. |

**Learning Objective:** By the end of the topic, students will be able to;

- define surds and apply properties of surds to simplify,
- perform arithmetic operations involving surds, and
- rationalise the denominators involving surds.

**Essential questions:**

- What are surds?
- Are surds part of the real number systems?
- What are surds classified as in the real number system?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Share ideas on simplifying and rationalizing surds. |
| **Skills** | Apply the properties of surds to reduce surds to their simplest forms. |
| **Knowledge** | Surds and the application of their properties. |
| **Mathematical Thinking** | Think about how to simplify and rationlize surds. |

## Content Background

### 1. Definition of surds and its properties

A root of a positive real quantity whose value cannot be exactly determined is called a surd. It is a number that cannot be simplified to remove the radical sign $\sqrt{\square}$ . Surds are Irrational numbers which have non –terminating decimals.

***Examples:*** $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{17}, \sqrt{19}$, etc

Properties of surds:

$$\text{(i)} \quad \sqrt{ab} = \sqrt{a} \text{ x} \sqrt{b} \qquad \text{(ii)} \quad (\sqrt{a})^2 = \sqrt{a} \text{ x} \sqrt{a} = a \qquad \text{(iii)} \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\text{(iv)} \quad \mathrm{a}\sqrt{\mathrm{c}} + \mathrm{b}\sqrt{\mathrm{c}} = (a+b)\sqrt{c} \qquad \text{(v)} \quad a\sqrt{c} - b\sqrt{c} = (a-b)\sqrt{c}$$

### 2. Application of the Properties

| Property | Application |
|---|---|
| $1. \sqrt{ab} = \sqrt{a} \text{ x} \sqrt{b}$ | $\sqrt{50} = \sqrt{25} \text{ x} \sqrt{2} = 5\sqrt{2}$ |
| $2. (\sqrt{a})^2 = \sqrt{a} \text{ x} \sqrt{a} = a$ | $(\sqrt{3})^2 = \sqrt{3} \text{ x} \sqrt{3} = \sqrt{9} = 3$ |
| $3. \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ | $\sqrt{\frac{75}{9}} = \frac{\sqrt{75}}{\sqrt{9}} = \frac{\sqrt{25 \text{ x} 3}}{3} = \frac{5\sqrt{3}}{3}$ |
| $4. a\sqrt{c} + b\sqrt{c} = (a+b)\sqrt{c}$ | $3\sqrt{7} + \sqrt{7} = (3 = +1)\sqrt{7} = 4\sqrt{7}$ |
| $5. a\sqrt{c} - b\sqrt{c} = (a-b)\sqrt{c}$ | $4\sqrt{11} - 2\sqrt{11} = (4-2)\sqrt{11} = 2\sqrt{11}$ |

## Content Background

### 3. Addition And Subtraction of Surds

Only Like surds can be added or subtracted. Before attempting to add or subtract surds, each surd should be reduced to its simplest form.

***Example*** simplify: $5\sqrt{27} - 3\sqrt{12}$

Solution:

$$\begin{aligned} 5\sqrt{27} - 3\sqrt{12} &= 5\sqrt{9x3} - 3\sqrt{4x3} \\ &= 5x3\sqrt{3} - 3x2\sqrt{3} \\ &= 15\sqrt{3} - 6\sqrt{3} \\ &= 9\sqrt{3} \end{aligned}$$

### 4. Fractions with surds in the denominator

To simplify a fraction such as $\frac{8}{\sqrt{2}}$ the denominator is changed into a rational number. This is called *rationalizing the denominator.*

***Example*** (*i*) $\frac{8}{\sqrt{2}} = \frac{8}{\sqrt{2}} x \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{\sqrt{4}} = 4\sqrt{2}$

(*ii*) $\frac{6}{5\sqrt{3}} = \frac{6}{5\sqrt{3}} x \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{5x3} = \frac{2\sqrt{3}}{5}$

### 5. Binomial Denominator

We have seen that the product of two conjugate surds is a rational number. This property is used to simplify fractions such as $\frac{3}{\sqrt{5}-2}$ by multiplying numerator and denominator by the conjugate of the denominator. (Conjugate means to change the operation sign)

***Example***

$$\frac{3}{\sqrt{5} - 2} = \frac{3}{\sqrt{5} - 2} x \frac{\sqrt{5} + 2}{\sqrt{5} + 2} = \frac{3(\sqrt{5} + 2)}{(\sqrt{5} - 2)(\sqrt{5} + 2)} = \frac{3\sqrt{5} + 6}{(5 - 4)} = 3\sqrt{5} + 6$$

| Unit: Numeracy | Topic: Recurring and Non-Recurring Decimals |
|---|---|
| **Benchmark** | **11.1.1.1** Apply arithmetic properties to operate on and simplify expressions that include radicals and other real numbers. |

**Learning Objective:** By the end of the topic, students will be able to;

- express non-recurring (terminating) and recurring decimals as vulgar fractions, and
- identify non-terminating decimals as irrational numbers.

**Essential questions:**

- What is another term that is used interchangeably for recurring decimals?
- What other term can be used to mean non-recurring decimals?
- Can decimals (recurring or non-recurring decimals) be expressed as fractions?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Appreciate learning about the recurring and non-recurring decimals. |
| **Skills** | Differentiate non-terminating decimals from terminating decimals. |
| **Knowledge** | Terminating and non-terminating decimals. |
| **Mathematical Thinking** | Think about how to differentiate non-terminating decimals from terminating decimals. |

## Content Background

### 1. Fractions and Decimals

We have seen that a calculator can be used to convert a fraction to a decimal. In fact all the fractions can be expressed as decimals which either terminate or recur.

Common fractions whose denominators involve factors of 2 and 5 only can be expressed as fractions with denominators that are powers of 10. This explains why such fractions when expressed as decimals always terminate.

***Example:***

Express as decimals

i $\frac{4}{5} = \frac{8}{10} = 0.8$ ii $\frac{9}{20} = \frac{45}{100} = 0.45$ iii $\frac{5}{8} = \frac{625}{1000} = 0.625$

### 2. Common Fractions as Recurring Decimals

When the denominator of fraction includes factors other than 2 or 5 the fraction always yields a recurring decimal.

***Example:*** i $\frac{1}{3} = 0.333333..., \textit{this is shortened to}\ 0.3$

ii $\frac{5}{11} = 0.454545...., \textit{this is shortened to}\ 0.45$

iii $\frac{2}{7} = 0.28571428714...., \textit{this is shortened to}\ 0.285714$

The converse of the above results states: Any terminating or recurring decimals can be expressed as a fraction.

**Recurring Decimals as Common Fractions**

To express recurring decimals as common fractions, follow these methods

***Example***

(i) Change $0.\dot{7}$ to a fraction

***Solution***

Let $x = 0.7$

*i.e.*

$x = 0.7777$

$10x = 7.77777\ldots$

By subtracting: $9x = 7$

$\therefore x = \frac{7}{9}$

*i.e.* $0.\dot{7} = \frac{7}{9}$

(ii) Change $0.\dot{4}\dot{3}$ to a fraction

***Solution***

Let $x = 0.\dot{4}\dot{3}$

*i.e.* $x = 0.434343\ldots$

$100x = 43.434343\ldots$

By subtracting: $99x = 43$

$\therefore x = \frac{43}{99}$

*i.e.* $0.\dot{4}\dot{3} = \frac{43}{99}$

(iii) Change $0.2\dot{6}$ to a fraction

***Solution***

Let $x = 0.2\dot{6}$

*i.e.* $10x = 2\dot{6}$

$100x = 26.\dot{6}\ldots$

By subtracting: $90x = 24$

$x = 24/90$

*i.e.* $0.2\dot{6} =$

| Unit: Numeracy | Topic: Indices |
|---|---|
| **Benchmark** | **11.1.1.2** Apply laws of indices and logarithms with different bases. |

**Learning Objective:** By the end of the topic, students will be able to;

- understand powers of numbers, become familiar with the laws of Indices and their applications, and
- identify and use laws of indices to solve questions related.

**Essential questions:**

- What are indices?
- What are the laws of indices?
- How can the laws of indices be related to some laws you know of?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Show confidence in solving application of laws of indices. |
| **Skills** | Simplify expressions involving indices. |
| **Knowledge** | Powers of numbers (indices). |
| **Mathematical Thinking** | Think about how to simplify expressions involving indices. |

## Content Background

### 1. The Laws of Indices

| LAW | APPLICATION |
|---|---|
| **1.** $a^m \times a^n = a^{m+n}$<br>When Multiplying powers of the same bases Add the indices | $3^2 \text{ x } 3^5 = 3^7$ |
| **2.** $a^m \div a^n = a^{m-n}$<br>To divide powers of the same base Subtract the indices | $48 \div 4^5 = 4^{8-5} = 4^3 = 64$ |
| **3.** $(a^m)^n = a^{m \times n}$<br>When raising the power of a quantity to anothe power, Multiply the indices | $(5^2)^3 = 5^{2 \times 3} = 5^6 = 15625$<br>$(2a^3)^2 = 2^2 a^6 = 4a^6$ |
| **4.** $a^0 = 1$ where $a \neq 0$<br>Any quantity raised to the power of 0 equal 1. | $4^8 \div 4^8 = 4^{8-8} = 4^0 = 1$<br>$(2a^3)^0 = 2^0 a^0 = 1\text{x}1 = 1$ |
| **5.** $a^{-m} = \frac{1}{a^m}$ where $a \neq 0$.<br>A Negative index indicates the reciprocal of the quantity | $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$ |
| **6.** $\sqrt[n]{a^m} = a\frac{m}{n}$<br>In general, $a^{\frac{1}{n}}$ means the $n^{th}$ root of $a$, *i.e.* $\sqrt[n]{a}$ and means the $n^{th}$ root of $a^m$,i.e. $\sqrt[n]{a^m}$ | $\sqrt[3]{8^2} = 8\frac{2}{3} = (\sqrt[3]{8})^2 = 2^2 = 4$ |

## Solutions of Equation Involving Indices

Equations with indices often simplify to equations of the form $x^n = C$. By raising the expression on each side of the equation to the reciprocal of the power of x. The solution can be found (the reciprocal of $n$ is $(\frac{1}{n})$.

***Example***

Solve $8x^4-3=2$, correct to 4 significant figures.

***Solution***

$$8x^4 - 3 = 2$$

$$\therefore 8x^4 = 5 \text{......................adding 5}$$

$$x^4 = \frac{5}{8} \text{..............................dviding by 8}$$

$$\therefore (x^4)^{\frac{1}{4}} = \left(\frac{5}{8}\right)^{\frac{1}{4}} \text{...........taking reciprocal power}$$

$$\therefore x = 0.8891\,(4sf)$$

Sometimes the unknown becomes as the part of the index. Equations of this nature are called exponential or indicial equations. When solving such equations, create same base on both sides of the equation then equate the indices to one another and solve for the unknown. $2^x=8$, $3^x=10$ and $\frac{1}{10^x} = 100$ are examples of exponential equations.

***Example***

Solve the following exponential equations:

**a.** $2^x = 4$ **b.** $9^x = \frac{1}{27}$ **c.** $2 \text{ x} \frac{1}{4^x} = 32$ **d.** $5^{x+2} = \frac{1}{\sqrt{5}}$

***Solution***

**a.**
$$2^x = 8$$
$$2^x = 2$$
$$\therefore x = 2$$

**b.**
$$9^x = \frac{1}{27}$$
$$3^{2(x)} = \frac{1}{3^3}$$
$$\therefore 3^{2x} = 3^{-3}$$
$$\therefore 2x = -3$$
$$\therefore x = -\frac{3}{2}$$

**c.**
$$2 \text{ x} \frac{1}{4^x} = 32$$
$$\frac{1}{4^x} = 16$$
$$\frac{1}{(2^2)^x} = 2^4$$
$$2^{-2x} = 2^4$$
$$\therefore -2x = 4$$
$$\therefore x = -2$$

**d.**
$$5^{x+2} = \frac{1}{\sqrt{5}}$$
$$5^{x+2} = \frac{1}{5^{\frac{1}{2}}}$$
$$\therefore x + 2 = -\frac{1}{2}$$
$$x = -2\frac{1}{2}$$

| Unit: Numeracy | Topic: Logarithm |
|---|---|
| **Benchmark** | **11.1.1.2** Apply laws of indices and logarithms with different bases.<br>**11.1.1.3** Solve logarithms equations in various problems. |

**Learning Objective:** By the end of the topic, students will be able to;

- define Logarithm, identify logarithm notation and express logarithmic expressions as single logarithm, and
- convert Logarithmic equations to exponential equations and solve logarithmic equations.

**Essential questions:**

- What is the definition of logarithm?
- How significant is logarithm?
- Why study logarithm?
- Is logarithm applicable in real life?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Show confidence in applying logarithmic laws to solve logarithmic equations. |
| **Skills** | Solve logarithmic equations and prove the results. |
| **Knowledge** | Laws of logarithm and solving logarithmic equations. |
| **Mathematical Thinking** | Think about how to apply logarithmic laws when solving and proving logarithmic equations. |

## Content Background

### 1. Logarithm

Definition of a logarithm: If $x>0$ and $b$ is a constant $(b\neq 1)$, then $y=\log_b x$ if and only if $b^y=x$.

In the equation $y=\log_b x, y$ , $y$ is referred to as the **logarithm**, $b$ is the **base**, and $x$ is the argument. The notation $\log_b x$ is read "the logarithm (or log) base $b$ of $x$." The definition of a logarithm indicates that a logarithm is an exponent.

$y=\log_b x, y$ is the logarithm form of $b^y=x$

$b^y=x$ is the exponential form of $y=\log_b x$

### 2. Properties of Logarithmic

If $b$, $a$ and $c$ are positive real numbers, $b\neq 1$, and $n$ is a real number then:

1. Product: $\log_b(a.b)=\log_b a+\log_b c$
2. Quotient: $\log_b\left(\frac{a}{c}\right)=\log_b a-\log_b a-\log_b c$
3. Power: $\log_b(a^n)=n.\log_b a$
4. $\log_b 1=0$
5. $\log_b b=1$
6. Inverse 1: $\log_b b^n=n$
7. Inverse 2: $b\log_b n=n, n>0$
8. One-to One: $\log_b a=\log_b c$ if and only if $a=c$
9. Change of Base: $\log_b a=\frac{\log_c a}{\log_c b}=\frac{\log_a}{\log_b}=\frac{\text{In } a}{\text{In } b}$

### 3. Evaluating a Logarithmic Equations

Evaluate: $\log_2 32=x$

***Solution*** $\log_2 32=x$ if and only if $2^x=32$

Since $32=2^5$, we have $2^5=2^5$

Thus by equality of exponents, $x=5$

**4. Rewriting Logarithmic Expressions Using Logarithmic Properties**

***Example:*** Use the properties of logarithms to rewrite each expression as a single logarithm:

**a.** $2\log_b x+\frac{1}{2}\log_b(x+4)$ **b.** $4\log_b(x+2)-3\log_b(x-5)$

*Solution*

**a.** $2\log_b x+\frac{1}{2}\log_b(x+4)$

$=\log_b x^2+\log_b(x+4)^{\frac{1}{2}}$.........power property

$=\log_b\left[x^2(x+4)^{\frac{1}{2}}\right]$.................product property

**b.** $4\log_b(x+2)-3\log_b(x-5)$

$=\log_b(x+4)^4-\log_b(x-5)^3$...power property

$=\log b\frac{(x+2)^4}{(x-5)^3}$.................Quotient property

**5. Changes between logarithmic and exponential forms**

***Example: 1.*** Write each equation in its exponential form.

**a.** $2=\log_7 x$ **b.** $3=\log_{10}(x+8)$ **c.** $\log_5 125=x$

*Solution*

Use the definition $y=\log_b x$ if and only if $b^y=x$

**a.** $2=\log_7 x$

$2=\log_7 x$ *if and only if*

$7^2=x$

**b.** $3=\log_{10}(x+8)$

$3=\log_{10}(x+8)$ if and only if

$10^3=(x+8)$

**c.** $\log_5 125=x$

$\log_5 125=x$ if and only if

$5^x=125$

**6. Write $x=25^{\frac{1}{2}}$ in its logaritm form**

*Solution:* $x=25^{\frac{1}{2}}$ if and only if $\frac{1}{2}=\log_{25}x$

***Equality of Exponents theorem:*** If $b$ is positive real number (b≠1)such that $b^x=b^y$, then $x=y$

## Strand 2: Geometry, Measurement and Transformation

**Content Standard:** Students will be able to comprehend the meaning and significant of geometry, measurements and spatial relationship including units and system of measurement and develop and use techniques, tools, and formulas for measuring the properties of objects and relationships among the properties and use transformations and symmetry to analyze mathematical situations.

<table>
<tr><th>Unit</th><th>Benchmark</th><th>Topic</th><th>Lesson Title</th></tr>
<tr><td rowspan="5">Measurement</td><td rowspan="5">11.2.2.1 Convert metric measurements to imperial or vice versa using length, mass and volume quantities.<br><br>11.2.2.2 Use analogue and digital devices to identify scale division.</td><td rowspan="2">Conversion, Scales and dials</td><td>Conversion of metric and imperial units</td></tr>
<tr><td>Reading Scale and dials</td></tr>
<tr><td rowspan="3">Length, Mass and Capacity</td><td>Application of length</td></tr>
<tr><td>Application of Mass</td></tr>
<tr><td>Application of capacity/volume</td></tr>
<tr><td rowspan="6">Trigonometry</td><td rowspan="3">11.2.2.3 Use right angle trigonometric ratios to determine an unknown length of a side or the measure of angle.<br><br>11.2.2.4 Apply trigonometric ratios to calculate unknown lengths and angles.<br><br>11.2.2.5 Apply the concepts of special right-angled triangles to real world situations.</td><td rowspan="3">Application of Trigonometry</td><td>Application of trigonometry ratios</td></tr>
<tr><td>Application of Pythagoras Theorem</td></tr>
<tr><td>Application of Angles of Elevation and Depression</td></tr>
<tr><td rowspan="3">11.2.2.6 Apply pythagoras theorem and trigonometry to solve 3-D problems invovling right angle triangles<br><br>11.2.2.7 Define plane and calculate angles between a line and plane.</td><td rowspan="3">Three-Dimensional application of Trigonometry</td><td>The angle between a line and a plane</td></tr>
<tr><td>The angle between two planes</td></tr>
<tr><td>Three - dimensional application</td></tr>
</table>

| | | | |
|---|---|---|---|
| **Vectors** | **11.2.2.8** Use vector notation to represent vectors and calculate position vector.<br><br>**11.2.2.9** Apply triangle and parallelogram laws and perform arithmetic operations on vectors.<br><br>**11.2.2.10** Use properties of vectors to sketch velocity vectors and apply to real life problems including triangular velocities. | Definitions and Representation of vectors | Definition and Notation of Vectors |
| | | | Properties of Scalars and vectors |
| | | | Unit Vector Representation |
| | | | Position Vector Representation |
| | | Arithmetic operations on vectors | Addition and Subraction of Vectors |
| | | | Scalar Multiplication |
| | | | Application of Parrallel Vectors |
| | | Velocity vectors | Triangular velocity applied to aeroplane |
| | | | Triangular velocity applied to boat |
| **Geometry** | **11.2.2.11** Draw similar triangles using scales.<br><br>**11.2.2.12** Use the concept of corresponding parts to prove that triangles and other polygons are congruent or similar.<br><br>**11.2.2.13** Use protractor and compass to construct geometric angles and shapes.<br><br>**11.2.2.14** Use circle properties and angles in cyclic quadrilaterals or polygons. | Angles and Polygons | Transformation |
| | | | Tests for congruent triangles |
| | | | Constructing, investigating and proving for congruency |
| | | | Special and similar triangles |
| | | Geometric Proofs | Geometric construction |
| | | | Proof Geometric construction |
| | | Circle geometry | Angles at Centre and angles at Circumference |
| | | | Angles in the same segment |
| | | | Angles in opposite segment |
| | | | Application of chords and tangents in geometry |
| | | | Cyclic quadrilaterals and its properties |
| | | | Concyclic points |

**Unit: Measurement** | **Topic: Conversion, Scales and dials**

**Benchmark**

- **11.2.2.1** Convert metric measurements to imperial or vice versa using length, mass and volume quantities.
- **11.2.2.2** Use analogue and digital devices to identify scale division.

**Learning Objective:** By the end of the topic, students will be able to;

- create measurements using appropriate equipment, scales and units, and
- convert between imperial and International standard units.

**Essential questions:**

- What is the difference between imperial and international standard units?
- What equipment are used for measurement?

**Key Concepts(ASK-MT)**

| | |
|---|---|
| **Attitudes/Values** | Confidently convert measurements and read scales. |
| **Skills** | Convert between units and create measurements using appropriate equipment, scales and units. |
| **Knowledge** | Conversion of units ,and scales and dials. |
| **Mathematical Thinking** | Think about how to convert between units and read with reasonable accuracy. |

## Content Background

### Conversion Tables

Many countries have officially converted to metric (SI) units however have had trouble convincing their population to work on this units. This is particularly evident in Britain and the United States of America where there are often references to the Imperial System of units. Below are tables that can be used for conversion from imperial to metric or vice versa

### Length

| Imperial | | Metric |
|---|---|---|
| 1 inch (in) | | 2.54cm |
| 1 yard | 12 in | 0.3048m |
| 1 yard | 3 ft. | 0.9144 m |
| 1mile | 1760 yard | 1.6093km |

### Area

The units for length are just squared to get the units for area which also means that if the units are squared then the equivalent length values must be squared as well to get its corresponding equivalent value in area.

| Metric | | Imperial |
|---|---|---|
| 1 $cm^2$ | 100 $mm^2$ | 0.1550$in^2$ |
| 1 $m^2$ | 10 000 $cm^2$ | 1.1960 $yd^2$ |
| 1ha | 10 000 $m^2$ | 2.4711 acres |
| 1$m^2$ | 10 000 $cm^2$ | 0.3861 $mile^2$ |

## Mass

Here are some useful conversion chart for metric and imperial followed by the formula converting between the two

| METRIC | | |
|---|---|---|
| **Unit** | **Metric Equivalent** | **Imperial equivalent** |
| 1 mg | | 0.0154 grain |
| 1 g | 1000 mg | 0.0353 ounces (oz) |
| 1 kg | 1000 g | 2.2046 pounds (lb) |
| 1 t | 1000 kg | 0.9842 ton |
| **IMPERIAL** | | |
| Unit | Imperial Equivalent | Metric Equivalent |
| 1 ounce (oz) | 437.5 grains | 28.350 g |
| 1 lb | 16 oz | 0.4536 kg |
| 1 stone (st) | 14 lb | 6.3503 kg |
| 1 hundredweight (cwt) | 112 lb | 50.802 kg |
| 1 tonne | 20cwt/22450 | |

### Mass Conversion

Converting Metric units to Imperial units

| To convert | multiply by |
|---|---|
| Gram (g) to Ounce (oz) | 0.0353 |
| Gram (g) to pound (lb) | 0.0022 |
| Kilogram (kg) to pound (lb) | 2.2046 |
| Kilogram (Kg) to tonnes | 0.000 98 |
| tonnes (t) to tonnes | 0.9842 |

### Converting Imperial units to Metric units

| To convert | ***multiply by |
|---|---|
| Ounce (oz) to gram (g) | 28.35 |
| pound (lb) to gram (g) | 453.592 |
| pound (lb) to Kilogram (kg) | 0.4536 |
| tonnes to Kilogram (Kg) | 1016.05 |

### Capacity/Volume

Imperial Units for volume are cubic inches ($in^3$), cubic feet ($ft^3$) and cubic yard ($yd^3$). For liquids, the units are fluid ounces (fl oz), the pint (pt), where 1 pint equals 20 fl oz and the gallon (gal), where 1 gallon equals 8 pints

| METRIC | | |
|---|---|---|
| **Unit** | **Metric Equivalent** | **Imperial equivalent** |
| 1 $cm^3$ | | 0.0610 in3 |
| 1 $dm^3$ | 1 000 $cm^3$ | 0.0353 ft3 |
| 1 $m^3$ | 1 000 $dm^3$ | 1.3080 yd3 |
| 1 L | 1 $dm^3$ | 1.76 pt |
| **IMPERIAL** | | |
| Unit | Imperial Equivalent | Metric Equivalent |
| 1 $in^3$ | | 16.387 $cm^3$ |
| 1 $ft^3$ | 1 728 $in^3$ | 0.0283 $m^3$ |
| 1 fl oz | | 28.413 mL |
| 1 pt | 20 fl oz | o.5683 L |
| 1gal | 8 pt | 4.5461 L |

**Scales & Dials**

Measurements are usually made by using various tools and instruments such as measuring tapes, digital scale. Stop watches or metres. On many such instruments taking the measurement means reading a scale or dial

A scale is shown by marks at set intervals, such as those on a ruler or protractor, a bathroom scale or speedometer of a car. Small divisions between markings on a scale indicate intermediate values that can be read with reasonable accuracy.

| Unit: Measurement | Topic: Length, Mass and Capacity |
|---|---|
| **Benchmark** | • **11.2.2.1** Convert metric measurements to imperial or vice versa using length, mass and volume quantities.<br>• **11.2.2.2** Use analogue and digital devices to identify scale division. |

**Learning Objective:** By the end of the topic, students will be able to;

- perform conversion of metric measurements using length, mass and volume quantities,
- identify scale divisions using analogue and digital devices.

**Essential questions:**

- What units of measurements are converted in length, Mass and volume?
- How are the conversions of the measurements done?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Recognising metric measurements and enjoy converting measurements in lengths, mass and volume. |
| **Skills** | Coverting measurements in lengths, mass and volume. |
| **Knowledge** | Conversions of length, mass and volume. |
| **Mathematical Thinking** | Converting metric measurements to imperials and using of analogue and digitial devices to assist in the conversion. |

## Content Background

Papua New Guinea declared the ***International System of Units (SI)*** to be the legal units of measurement in 1980.

The International System of Units (SI) define seven units of measurement as a basic set from which all other SI units are derived. These SI basic units and their physical quantities are;

- Metre for length
- Kilogram for mass
- Second for time
- Ampere for electric current

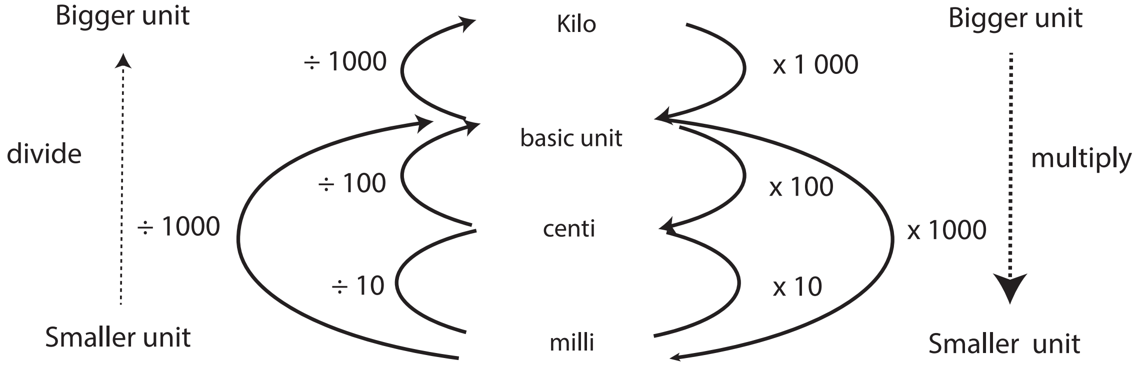
If the basic unit of length is the metre than all the units of length, area and volume are defined in terms of the basic unit.

In the metric system, each quantity measured has a basic unit (metre, litre), of which other units are multiples. These other units are identified by the prefix such as ‘kilo’, ‘centi’ and ‘milli’. By using the prefixes measurements can be expressed with suitably sized numbers. *Eg:* not 23 000 m but 23 km.

The table below shows the commonly used prefixes with their respective value

| Prefix | kilo (k) | centi (c) | milli (m) |
|---|---|---|---|
| means | $10^3$<br>1 000<br>One thousand | $10^{-2}$<br>$\frac{1}{100}$<br>One hundredth | $10^{-3}$<br>$\frac{1}{1000}$<br>One thousandth |

To change from one unit to another, either multipliers or divisors are used. This conversion method is show in the diagram below for prefixes kilo, centi and milli.




To convert from one unit to another follow the arrows, eg to change from centi to base unit divide by 100 and to change from kilo to base unit multiply by 1000.

- To change from a smaller unit to a bigger unit, use division, since there will be fewer of the bigger unit
- To change from a bigger unit to a smaller unit, use division, since there will be more of the smaller unit

## Length

The base unit for length in the metric system is the metre (symbol m). Other commonly used units for length are kilometre (km), centimetre (cm) and millimetre (mm)

1 000 mm = 1 m    100 cm = 1 m    1 km = 1 000 m

## Mass

Earlier the base unit for mass was **gram** (symbol **g**) but since it was too small, the base unit for mass now, agreed upon as SI unit is **kilogram** (symbol **kg)**. Other units commonly used for mass are

1 000 mg = 1 g    1 000 g = 1 kg    1 t = 1 000 kg

**milligram (mg)**, and **tonne (t)**

**Note:** Mass is commonly (but incorrectly) referred to as weight. ***State the difference between the two measures

## Capacity

The capacity of a container is the amount of liquid or other material it will hold. The **Litre** (symbol **L**) and millilitre (symbol **mL**) are the commonly used units of capacity in the metric system. Other units of capacity include **kilolitre (kL)**

1 kL = 1 000 L    1 000 mL = 1 L

| Unit: Trigonometry | Topic: Application of Trigonometry |
|---|---|
| **Benchmark** | • **11.2.2.3** Use right angle trigonometric ratios to determine an unknown length of a side or the measure of an angle.<br>• **11.2.2.4** Apply trigonometric ratios to calculate unknown lengths and angles.<br>• **11.2.2.5** Apply the concepts of special right triangles to real-world situations. |

**Learning Objective:** By the end of the topic, students will be able to;

- calculate the length of a side or the angle in a triangle, and
- solve authentic problems by incorporating the concepts of special right angle triangle.

**Essential questions:**

- Where in life is trigonometry applied?
- What do you think of when you hear the word trigonometry?
- What are some things or concepts that will be captured in trigonometry?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Enjoy solving real life problems using the application of trigonometry. |
| **Skills** | Calculate unknown length or angle of a triangle and use the trigonometric ratio to solve everyday situations. Skilful use of the calculator practicing the trigonometric function keys. |
| **Knowledge** | Application of Trigonometry. |
| **Mathematical Thinking** | Thinking about how to solve real life problems using the application of trigonometry. |

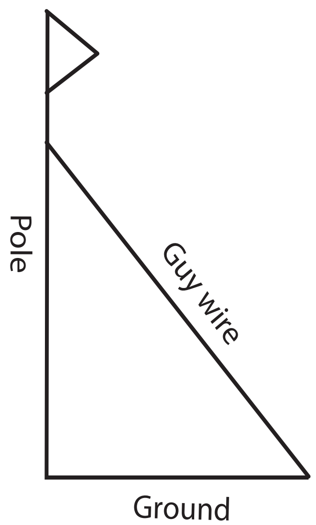
## Content Background

**Pythagoras Theorem** $(c = a^2 + b^2)$

Pythagoras' Theorem is used to find the length of a missing side in a right angled triangle.

**Example**

(i) A guy (support) wire is attached 3.2 m up a pole and at a point 2.1 m from the pole. The ground and the pole are perpendicular (at right angles). What is the length of the guy wire?

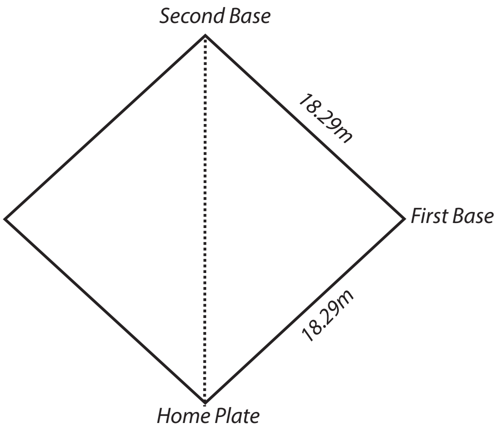



Let the length of the guy wire be l.

$$h^2 = a^2 + b^2$$
$$l^2 = 3.2^2 + 2.1^2$$
$$l^2 = 14.65$$
$$l = \sqrt{14.65}$$
$$l = 3.83$$

The length of the guy wire is 3.83m

(ii) On a softball diamond, the distance between bases is 60 feet or 18.29 m. How far must the catcher (at home base) throw the ball to the player on second base?




The angle at first base is a right angle. Let the distance from Home plate to second base $d$ ?

$h^2 = a^2 + b^2$

$d^2 = 18.29^2 + 18.29^2$

$d^2 = 669.05$

$d = \sqrt{669.05}$

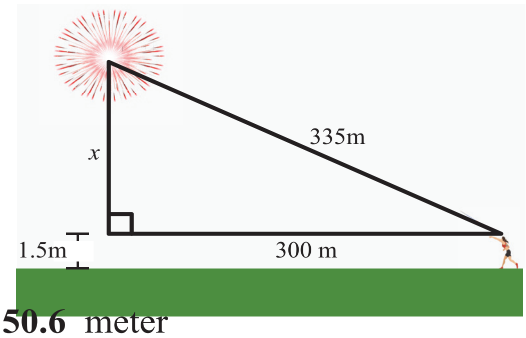
$d = 25.87$

The distance from home plate to second base is 25.87m

**Pythagorean Theorem-Real Life Application**

***Example:***

Find the height of the firework. Round your answer to the nearest tenth.




$a^2 + b^2 = c^2$

$x^2 + 300^2 = 335^2$

$x^2 + 90,000 = 112,225$

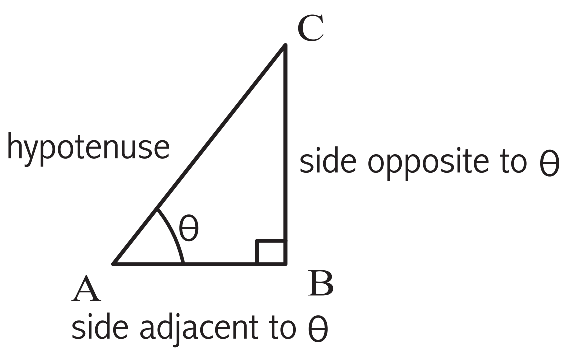
$x^2 = 22,225$

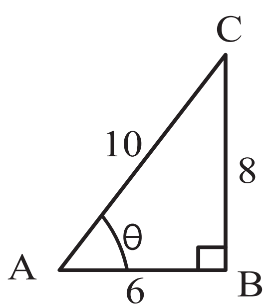
$\sqrt{x^2} = \sqrt{22,225}$

$\therefore$ The height of the firework is about $149.1 + 1.5 = \mathbf{150.6}$ meter

**The Trigonometrical Ratios-Sine,cosine and tangent Ratios**

Study the right-angled triangle ABC shown on the right and the one with known lengths on the left




The side opposite the right-angle is called the hypotenuse. The side opposite to θ is BC. The remaining side, AB, is said to be adjacent to θ.

We can then divide the length of one side by the length of one of the other sides.

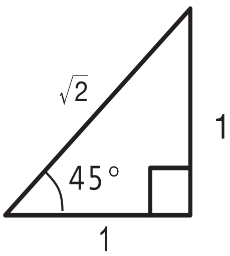
According to the right angled triangle on the left:

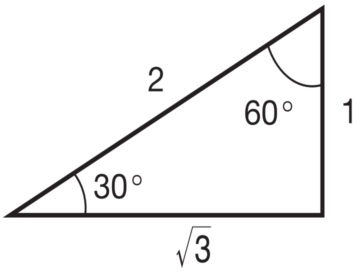
The ratio $\frac{BC}{AC}$ is known as the $\sin$ of angle $\theta$. This is abbreviated to $\sin\theta$. In the triangle shown we see that $\sin\theta = \frac{8}{10} = 0.8$

The ratio $\frac{AB}{AC}$ is known as the *cosine* of angle $\theta$. This is abbreviated to $\cos\theta$. In the triangle shown we see that $\cos\theta = \frac{6}{10} = 0.6$
The ratio $\frac{BC}{AB}$ is known as the tangent of angle θ. This is abbreviated to tanθ. In the triangle shown we see that $\tan\theta = \frac{8}{6} = 1.3333$
In any right-angled triangle we define the trigonometrical ratios as follows:

$$\sin\theta = \frac{Opposite}{Hypothenuse} = \frac{BC}{AC} \qquad \cos\theta = \frac{Adjacent}{Hypothenuse} = \frac{AB}{AC} \qquad \tan\theta = \frac{Opposite}{Adjacent} = \frac{BC}{AB}$$

**Some Common or Standard Triangles for Exact Values**




$$\sin 45^0 = \frac{1}{\sqrt{2}}, \cos 45^0 = \frac{1}{2}, \tan 45^0 = 1$$

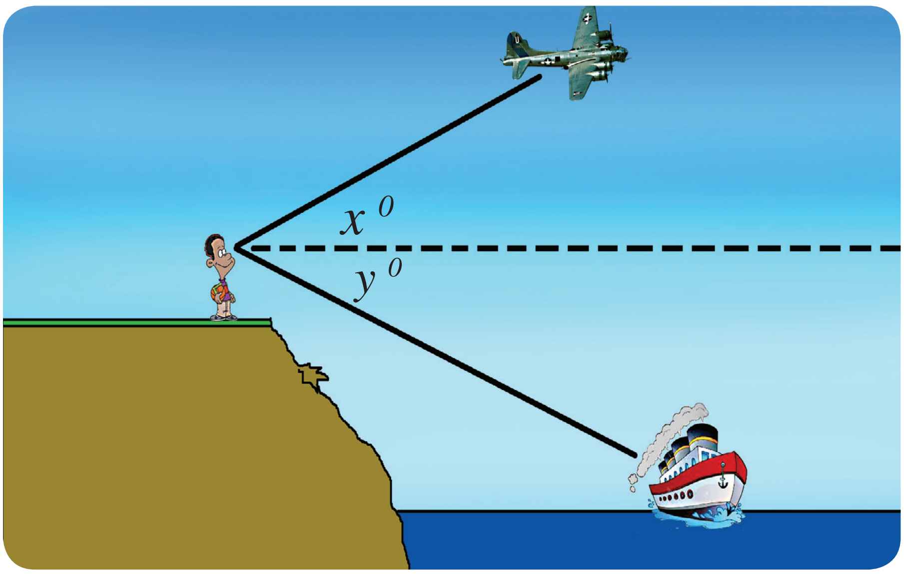
$$\sin 30^0 \frac{1}{2}, \cos 30^0 = \frac{\sqrt{3}}{2}, \tan 30^0 = \frac{1}{\sqrt{3}}$$

$$\sin 60^0 = \frac{\sqrt{3}}{2}, \cos 60^0 = \frac{1}{2}, \tan 60^0 = \sqrt{3}$$

**Application of trigonometry**

Now let us see some real life applications of Trigonometry. Trigonometry is interesting and has many useful applications in the field of astronomy, geography etc. One of the useful applications of trigonometry is to calculate the height of a tower or a peak or the distance of a ship sailing in the sea. We do not need measuring scales. All we need to know is the angle of elevation or angle of depression.

The angle of elevation and the angle of depression can be demonstrated by the diagram as shown below.




Line of sight is a straight line from our eye to the object.

$x^0$ is the angle of elevation and $y^0$ is the angle of depression

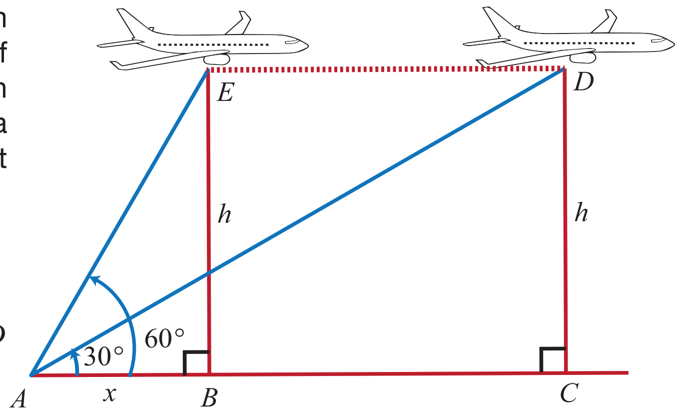
***Example***

1. Two ships are sailing in the sea on the two sides of a lighthouse. The angle of elevation of the top of the lighthouse is observed from the ships are $30^0$ and $45^0$ respectively. If the lighthouse is 100 m high. What is the distance between the two ships?

*Solution*
Let $AB$ be the lighthouse and $C$ and $D$ be the positions of the ships. Here, we need to find $CD$

$$AB = 100m, \angle ACB = 30^0 \ and \angle ADB = 45^0$$
$$\frac{AB}{AC} = \tan 30^0 = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \text{ x } \sqrt{3} = 100\sqrt{3m}$$
$$\frac{AB}{AC} = \tan 45^0 = 1 \Rightarrow AB = AD = 100m$$
Now
$$CD = (AC + AD)$$
$$= (100\sqrt{3} + 100)\, m$$
$$= 100(\sqrt{3} + 1) = (100 \text{ x } 2.73)$$
$$= 273\, m$$

2. The angle of elevation of an aeroplane from a point A on the ground is $60^0$. After a light of 15 seconds horizontally, the angle of elevation changes to $30^0$. If the aeroplane is flying at a speed of 200m/s, then find the constant height at which the aeroplane is flying.




*Solution:* Let $A$ be the point of observation.
Let $E$ and $D$ be positions of the aeroplane initially and after 15 seconds respectively. Let $BE$ and $CD$ denote the constant height at which the aeroplane is flying. Given that $\angle DAC$ =$30^0$ and $\angle EAB$ = $60^0$.

Let $BE = CD = h$ metres
Let $AB = x$ metres

The distance covered in 15 seconds, $ED$=200 x 15 = 3000 $m$
Thus, $BC = ED$ =3000 $m$

In right –angled triangle $ADC$, $\tan 30^0 = \frac{CD}{AC} \Longrightarrow CD = ACx \tan 30^0$

$$\text{Thus, } h = (x + 3000)\, x\frac{1}{\sqrt{3}} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$
$$\text{In right} - \text{angled triangle } AEB, \tan 60^0 = \frac{BE}{AB} \Longrightarrow BE = AB \text{ x } \tan 60^0$$
$$\text{Thus, } h = \sqrt{3} \text{ x } x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$
From (1) and (2), we have
$$\sqrt{3} \text{ x } x = (x + 3000)\, x\frac{1}{\sqrt{3}}$$
$$\Longrightarrow 3x = x + 3000$$
$$\Longrightarrow x = 1500m$$
$$\text{Thus from (2), } h = 1500\sqrt{3}\, m$$

| Unit: Geometry | Topic: Three-Dimentional application of Trigonometry |
|---|---|
| **Benchmark** | • **11.2.2.6** Apply pythagoras theorem and trigonometry to solve 3-D problems invovling right angle triangles.<br>• **11.2.2.7** Define plane and calculate angles between a line and plane. |

**Learning Objective:** By the end of the topic, students will be able to;

- define and calculate angles that are formed between a line and a plane,
- define and calculate angles formed between two planes,
- solve 3-D problems using Pythagoras Theorem.

**Essential questions:**

- What concepts or ideas are captured in geometry?
- How can you calculate angles formed between lines?
- How is Pythagoras theorem used in sovling 3-D problems?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Creatively define angles using artistic diagrams in 2-D or 3-D. |
| **Skills** | Provide reasons to justify the sizes of angles form between lines. |
| **Knowledge** | Knowing special and similar triangles, the angles between a line and a plane or between two planes and three dimensional application. |
| **Mathematical Thinking** | Think about how to calculation of angles between a line and a plane, two planes and 3-D applications. |

## Content Background

### 1. Using Pythagoras Theorem and Trigonometry in Three Dimensions

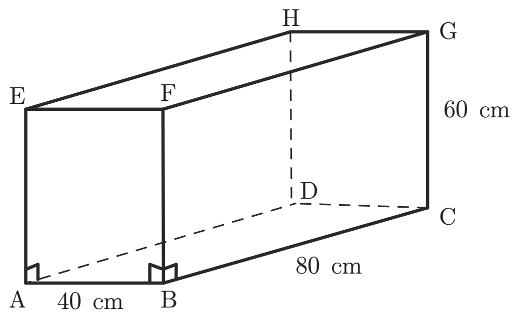
Pythagoras theorem and the trigonometry units can be applied in three dimensional problems. The main technique is to reduce the problem to a two dimensional situation by identifying suitable triangles to work with. The standard results can then be applied to these triangles.

***Example 1:*** A box has the dimensions shown in the diagram. A game is to be packed into this box. Part of the game is a rod that will just fit into the box. Find the length of this rod.

***Solution***

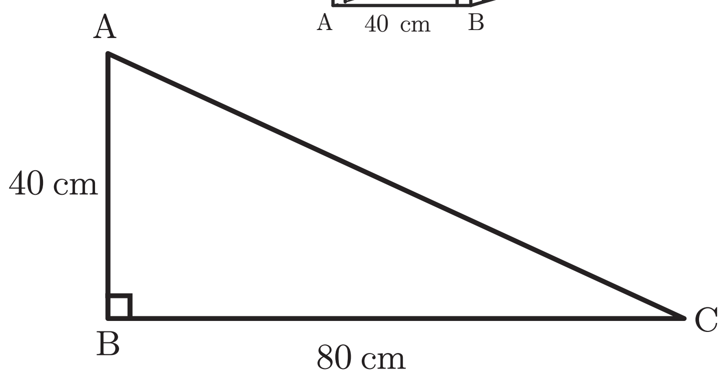
The longest rod that can fit into this box will have one end at $A$ and the end at $G$ or lie along a similar diagonal, the problem is to find the length $AG$.

The first stage is to find the length of $AC$, the diagonal in the base directly below $AG$.



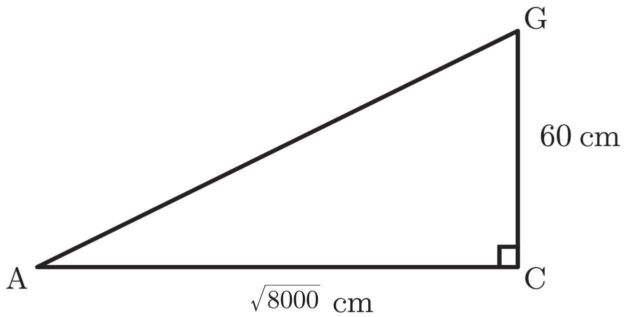

From the Triangle $ABC$ as shown,

$$AC^2 = AB^2 + BC^2$$
$$= 40^2 + 80^2$$
$$= 8000$$
$$AC = \sqrt{8000}\,cm$$

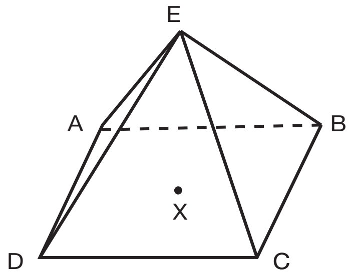


Now AG can be found considering the triangle $ACG$.

$$\begin{aligned} AG^2 &= AC^2 + CG^2 \\ &= 800 + 60^2 \\ &= 11600 \\ AG &= \sqrt{11600} \approx 107.7cm \end{aligned}$$




***Example 2:*** A square pyramid has four equilateral triangles with sides of length 10 cm. Find the height of the pyramid.

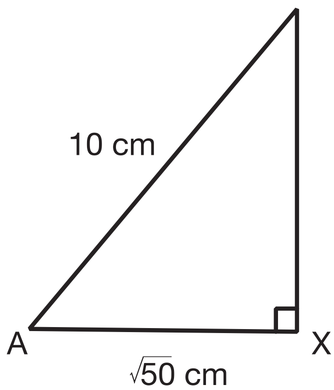



***Solution***

The point marked $X$ on the diagram is the centre of the base directly above $E$. The distance $XE$ is the required height.

Consider first the base of the pyramid.

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 10^2 + 10^2 \\ &= 200 \\ AC &= \sqrt{200} \\ AX &= \frac{1}{2}AC = \frac{\sqrt{2002}}{2} = \sqrt{50} \end{aligned}$$

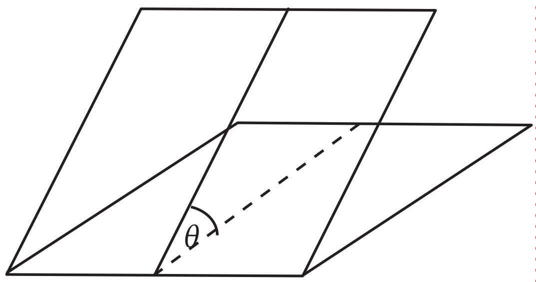



Next work in the triangle $AXE$

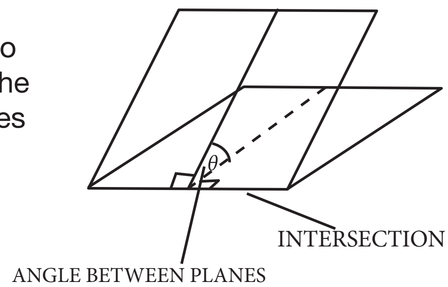
$$\begin{aligned} AE^2 &= AX^2 + XE^2 \\ 10^2 &= 50 + XE^2 \\ \therefore XE &= \sqrt{50} \approx 7.07cm \end{aligned}$$

**Angles and Planes**

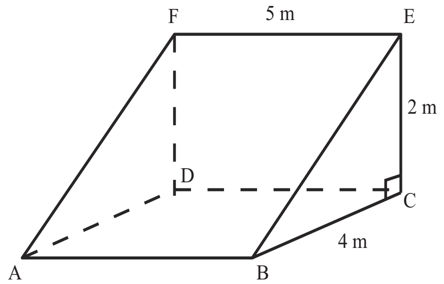
When calculating the angle between a line and plane you should always find the smallest angle between the line and the plane. Consider a skier travelling in a straight line down a slope. The angle between the path of the skier and the horizontal plane is the angle between the path and line directly below the path in the horizontal plane.




When finding the angle between the two planes it is important to consider where the planes intersect and the line that it forms. The angle between the two planes is equal to the angle between lines in each plane that are perpendicular to the line formed by the intersection.



**Example:** The diagram shows are wedge.

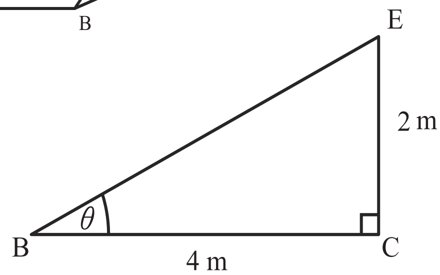



Find the angle between:

**a.** The line BE at the plane ABCD
**b.** The line BF and the plane ABCD
**c.** Plane ABCD and plane ABEF
**d.** The lines BD and BE

*Solution:*

**a.** Consider the triangle BCE. The angle between the line and the plane has been labelled $\theta$ on the diagram.



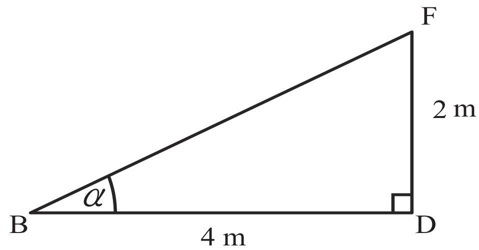

$$\tan\theta = \frac{2}{4} \Rightarrow \theta \approx 26.6^0$$

**b.** The required angle can be found from the triangle BDF because the line BD is directly below BF. This is labelled $\alpha$ on the diagram. The length BD can be found by considering the base of the wedge.

$$BD^2 = 4^2 + 5^2 = 41$$
$$BD = \sqrt{41}$$

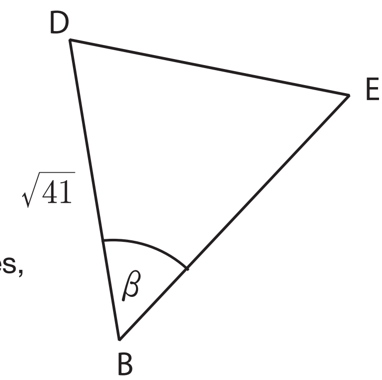
The next angle $\alpha$ can be found

$$\tan\alpha = \frac{2}{\sqrt{41}} \Rightarrow \alpha = 17.3^0$$




**c.** The angle between the planes ABCD and ABEF is the same as $C\hat{B}E$ and $D\hat{A}F$. In part (a) $C\hat{B}E$ was found as 26.6⁰

**d.** Consider the triangle BDE. The BD is known as $\sqrt{41}$. The two sides must be calculated.




$$BE^2 = 4^2 + 2 = 20 \Longrightarrow BE = \sqrt{20}$$

and

$$DE^2 = 5^2 + 2^2 = 29 \Longrightarrow DE = \sqrt{29}$$

Using the cosine rule, because BDE is not a right angled triangle, gives,

$$Cos\beta = \frac{BD^2 + BE^2 - DE^2}{2 \text{ x } BD \text{ x } BE}$$
$$= \frac{41 + 20 - 29}{2 \text{ x} \sqrt{41} \text{ x} \sqrt{20}}$$
$$= \frac{32}{2\sqrt{820}}$$
$$\beta \approx 56.0^0$$

| Unti: Vectors | Topic: Definition and Representation of Vectors |
|---|---|
| **Benchmark** | **11.2.2.8** Use vector notation to represent vectors and calculate position vector. |

**Learning Objective:** By the end of the topic, students will be able to;

- define vectors and scalars,
- define unit vector and position vectors, and
- add and subtract position vectors.

**Essential questions:**

- What are vectors and scalars?
- What is a unit vector?
- What is a position vector?
- How are unit and position vectors represented?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Confidently describe physical quantities as scalar or vectors and define unit and position vectors . |
| **Skills** | Describe physical quantities as vector and define unit and position vectors. |
| **Knowledge** | Vectors, scalars, Vector addition, representation of vectors. |
| **Mathematical Thinking** | Think about how to represent unit and position vectors and perform arithmetic operations. |

## Content Background

Physicists were responsible for first conceiving the idea of a vector, but the mathematical concept of vectors has become important in its own right and have extremely wide application, not only in the sciences but in mathematics as well.

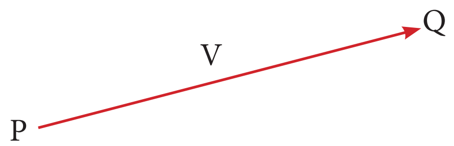
### 1. Scalars and Vectors

A quantity which is completely specified by a certain number associated with a suitable unit without any mention of direction in space is known as scalar. Examples of scalar are time, mass, length, volume, density, temperature, energy, distance, speed etc. The number describing the quantity of a particular scalar is known as its magnitude. The scalars are added, subtracted, multiplied and divided by the usual arithmetical laws.

A quantity which is completely described only when both their magnitude and direction are specified is known as **vector.** Examples of vector are force, velocity, acceleration, displacement, torque, momentum, gravitational force, electric and magnetic intensities etc. A vector is represented by a Roman letter in bold face and its magnitude, by the same letter in italics. Thus **V** means vector and $|V|$ is magnitude.

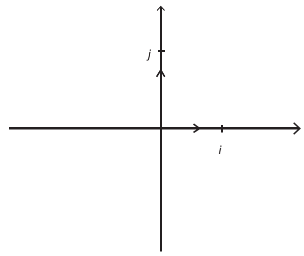
### 2. Vector Representations

A vector quantity is represented by a straight line segment, say $\overrightarrow{\mathbf{PQ}}$ . The arrow head indicates the direction from **P** to **Q**. The length of the Vector represents its magnitude. Sometimes the vectors are represented by single letter such as **V** or $\vec{\mathrm{V}}$ . The magnitude of a vector is denoted by |**V**| or by just $V$, where$[\vec{\mathrm{V}}]$ means modulus of $\vec{\mathrm{V}}$ which is a positive value.



### 3. Unit and Position Vectors (i,j)

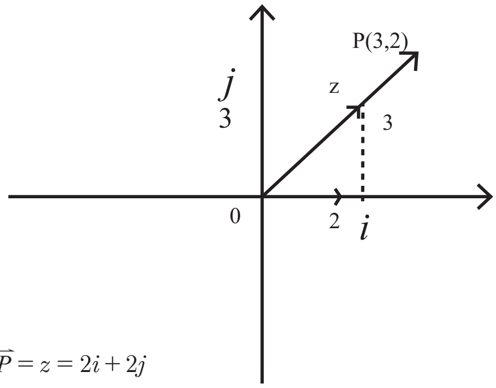
Consider the position vectors i and j in the figure below.



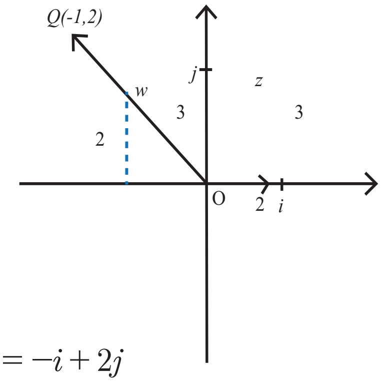

$i$ and $j$ are called unit vectors (magnitude) and are perpendicular to each other. Vectors can be given in terms of these unit vectors. For instance;

**a.** $z = 2i + 3j$ **b.** $w = -i + 2j, etc$

These two variables can be represented in the above diagram as;




$\overrightarrow{OP} = z = 2i + 2j$




$\overrightarrow{OQ} = w = -i + 2j$

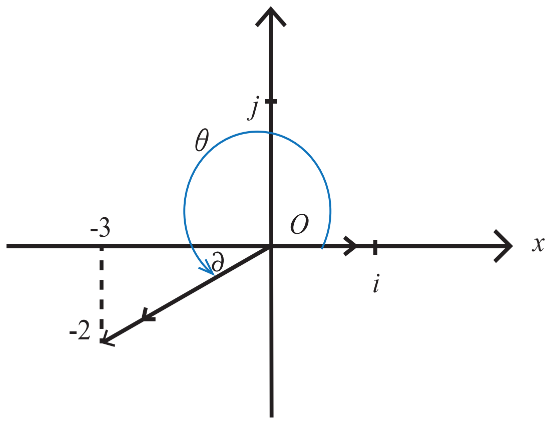
The origin ($O$) is the initial point and $P$ and $Q$ are the terminal points of these two vectors respectively. The i and j components determine which quadrant the terminal point lies.

If $= xi + yi;$ then the magnitude (modulus) of $v$ is given by $|v| = \sqrt{x^2 + y^2}$ .

***Example:***

$$v = -3i - 2j, its \bmod ulus\ is$$
$$|v| = \sqrt{(-3)^2 + (-2)^2}$$
$$= \sqrt{9 + 4}$$
$$= \sqrt{13}\ units$$

Diagram:



The modulus is the hypotenuse of the right triangle formed by the two components. Direction of the given vector is determined by $Q$.

$$\tan\phi = \frac{2}{3} \qquad \phi = 33.7^0$$

### Addition and Subtraction of Position Vectors

Add or subtract component wise. For instance,

$$\begin{aligned} v_1 &= 2i - 3j \quad \text{and}\, v_2 = -i + 2j \\ v_1 + v_2 &= (2 + (-1))\,i + (-3 + 2)\,i \\ &= 1i - j = i - j \\ v_1 - v_2 &= (\,2 - (-1)\,)i + (-3 - 2)j \\ &= 3i - 5j \end{aligned}$$

### Multiplication of Position Vectors

Remember: $i \cdot j = 0$

$i \cdot i = 1\, and\; j \cdot j = 1$

**Example:** $v_i = 4i + 5j, v_2 = -3i + 2j$

$$\begin{aligned} v_v \cdot v_2 &= (4i + 5j)\,(-3i + 2j) \\ &= (-12)\,(1) + 8\,(0) + (-15)\,(0) + 10\,(1) \\ &= -12 + 0 + 0 + 10 \\ &= -12 + 10 \\ &= -2 \\ v_1 \cdot v_2 &= -2 \end{aligned}$$

***Note:*** *Multiplication of two vectors is known as scalar product or dot product. The final product is a scalar (not a vector).*

| Unit: Vectors | Topic : Arithmetic Operations on Vectors |
|---|---|
| **Benchmark** | **11.2.2.9** Apply triangle and parallelogram laws and perform arithmetic operations on vectors. |

**Learning Objective:** By the end of the topic, students will be able to;

- add and subtract vectors using triangle and parallelogram laws, and
- multiply vector by scalar.

**Essential questions:**
- How can vectors be added, subtracted or multiplied?
- What is the result of multiplying a vector by a scalar?

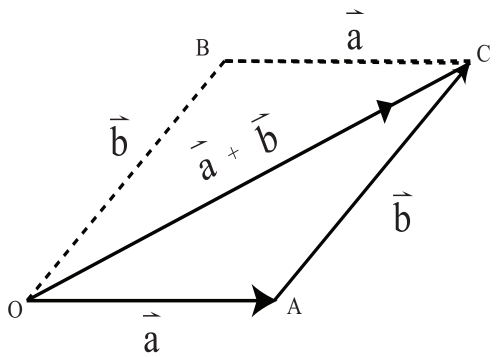
| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Appreciate triangular and parallelogram laws to perform vector arithmetic. |
| **Skills** | Apply of the triangle and parallelogram laws to add and subtract vectors. |
| **Knowledge** | Add, subtract, and multiply vectors and scalar multiplication. |
| **Mathematical Thinking** | Think about how to add, subtract, multiply vectors and apply scalar multiplication. |

## Content Background

### Addition and subtraction of vectors

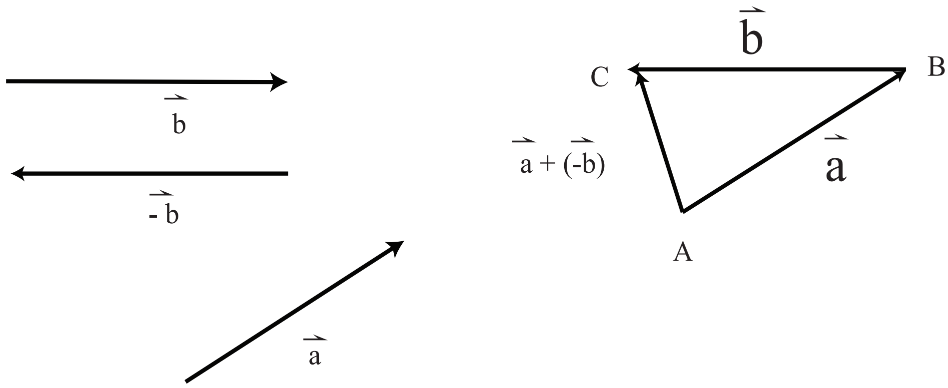
#### 1. Addition of Vectors

Suppose $\vec{a}$ and $\vec{b}$ are any two vectors. Choose point $A$ so that $\vec{a} = \overline{OA}$ and choose point $C$ so that $\vec{b} = \overline{AC}$ . The sum, $\vec{a} + \vec{b}$ of $\vec{b}$ and $\vec{b}$ is the vector $\overline{OC}$ . Thus the sum of two vectors $\vec{a}$ *and* $\vec{b}$ is performed by the ***Triangle Law of Addition.***

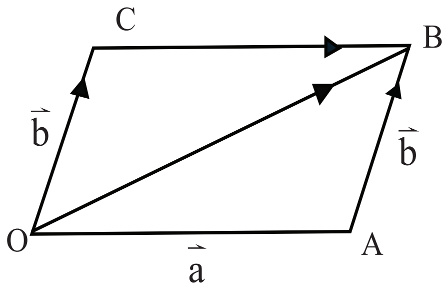



#### 2. Subtraction of Vectors

If a vector $\vec{b}$ is to be subtracted from a vector $\vec{a}$ , the difference vector $\vec{a}$ - $\vec{b}$ can be obtained by adding vectors $\vec{a}$ and $\vec{b}$ . The vector $\vec{b}$ is a vector which is equal and parallel to that of vector but its arrow-head points in opposite direction. Now the vectors $\vec{a}$ and $\vec{b}$ can be added by the head-to-tail rule. Thus the line $\overline{AC}$ represents, in magnitude and direction, the vector $\vec{a} - \vec{b}$ .



### 3. Properties of Vector Addition

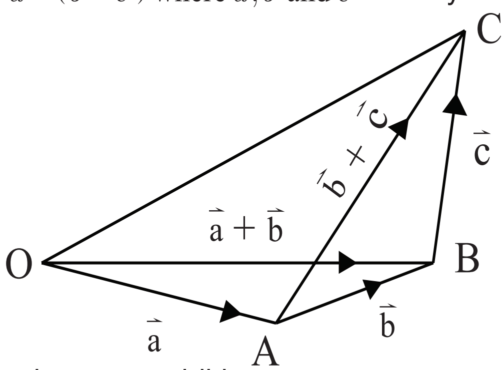



**(i)** Vector addition is commutative

*i.e.*, $\vec{a}+\vec{b}=\vec{a}+(\vec{b}+\vec{c})$ where $\vec{a}$ and $\vec{b}$

**(ii)** Vector Addition is Associative

*i.e.*, $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ where $\vec{a},\vec{b}$ and $\vec{c}$ are any three vectors.




**(iii)** $\vec{0}$ is the identity in vector addition

For every vector $\vec{a}, \vec{a}+\vec{0}=\vec{a}$ where $\vec{0}$ is the zero vector.

***Note:*** Non-parallel vectors are not added or subtracted by the ordinary algebraic Laws because their resultant depends upon their directions as well.

### 4. Multiplication of a vector by a Scalar

If $\vec{a}$ is any vector and K is a scalar, then K $\vec{a}$ = $\vec{a}$ K is a vector with magnitude $|K|.|a|$i.e.,$|K|$ times the magnitude of $\vec{a}$ and whose direction is that of vector $\vec{a}$ or opposite to vector $\vec{a}$ accordingly as K is positive or negative respectively. In particular $\vec{a}$ and - $\vec{a}$ are opposite vectors.

### 5. Properties of Multiplication of Vector by Scalars

(i) The scalar multiplication of a vector satisfies

$$m(n\vec{a}) = (mn)\vec{a} = n(m\vec{a})$$

(ii) The scalar multiplication of a vector satisfies the distributive laws

*i.e.*, $(m+n)\vec{a} = m\vec{a} + n\vec{a}$

and $m(\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$ where m and n are scalars and $\vec{a}$ and $\vec{b}$ are vectors.

| Unit: Vectors | Topic : Velocity Vectors |
|---|---|
| **Benchmark** | **11.2.2.10** Use properties of vectors to sketch velocity vectors and apply to real life problems including triangular velocities. |

**Learning Objective:** By the end of the topic, students will be able to;

- sketch velocity vector diagram, and
- calculate vector velocities applied to aeroplane and boats.

**Essential questions:**
- What are vector velocities?
- What is the difference between triangle of velocities applied to aeroplane and boats?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Show confidence solving velocity vectors problems. |
| **Skills** | Describing, sketching and calculating velocity vectors. |
| **Knowledge** | Velocity vectors. |
| **Mathematical Thinking** | Think about how to sketch velocity vector diagram and calculate. |

## Content Background

### Velocity vectors

Velocity is a speed in a given direction. Hence velocity is a vector quantity. Common examples of velocity vectors are the flight of aeroplanes and the motion of a boat through water.

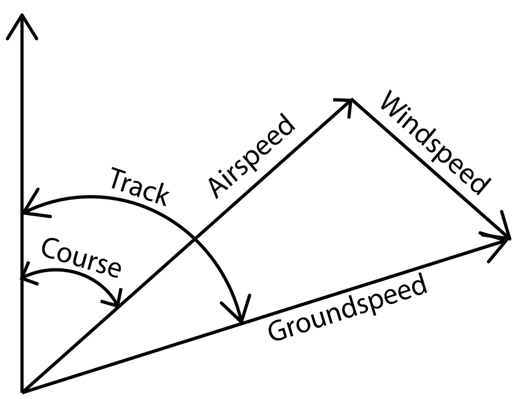
**1. The triangle of Velocities applied to aeroplane**

When an aeroplane is flying it has two velocities as follows:

**a.** Its velocity through air due to the engine,
**b.** The velocity due to the wind.

These two velocities combine to give a resultant velocity which is the groundspeed.

In order to determine this resultant velocity we draw a triangle of velocities similar to one shown below. In drawing this triangle note carefully that the arrows on the airspeed and windspeed vectors follow nose to tail.

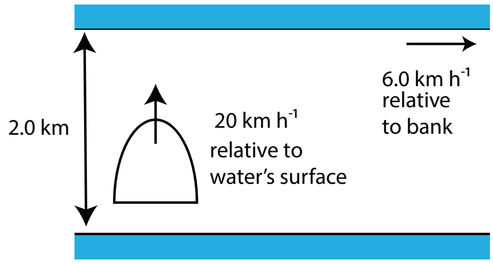


### 2. The triangle of Velocities applied to boats

The motion of a boat through water is very similar to the flight of an aeroplane through air. The corresponding terms are: for wind velocity read current velocity; for airspeed read the speed of the boat through water (i.e., waterspeed); for groundspeed the track read resultant speed of the direction of the motion of the boat.

***Example***

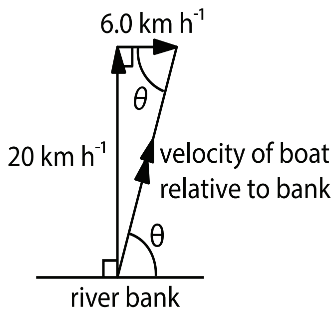
A boat starts to cross a river which is 2.0 km wide. The boat travels at 20 km $h^{-1}$ relative to the surface of the water and points directly across the river. The river is moving at 6.0 km $h^{-1}$ downstream relative to the river bank.




**a.** What is the boat's velocity relative to the river bank?
**b.** How long does the boat take to cross the river?
**c.** At what point does the boat land on the other side?

**Solution:**

The current is relative to the river bank and the river bank (and any observer standing on the bank) is at rest.




**a.** The boat's velocity relative to the river bank is combined result of its velocity relative to the river and the velocity of the river relative to the river bank:

$$V_{\sim \text{boat rel river}} + V_{\sim \text{river rel bank}} + V_{\sim \text{boat rel bank}}$$

Using Pythagoras and trigonometry, the boat moves with a speed of:
$21 km\, h^{-1} (\sqrt{20^2 + 6.0^2})$ at an angle downstream of $73^0$ $\left(\tan\theta = \frac{20}{6.0}\right)$ to the river bank.

The boat continuous to travel across the river at 20 km $h^{-1}$, despite the fact that it is also being carried downstream. The river's motion does not affect the boat's movement across the river because the motion of the river is at right angles to the direction in which the boat is pointed.

Time taken to cross the river is $t = \frac{d}{v}$ [rearranging $d = vt$]

$$= \frac{2.0}{20}$$

$$= 0.10\, h\, (6 \text{ minutes})$$

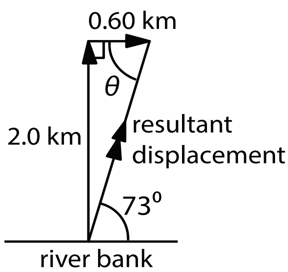
As well as travelling across the river, the boat is carried at 6.0 km $h^{-1}$ downstream for 0.01 hour (the time taken to cross the river). The distance travelled downstream is:

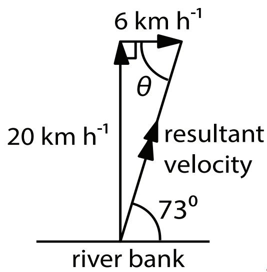
$$d = vt$$

$$= 6.0 \text{ x } 0.01$$

$$= 0.60\, km$$

The boat touches the opposite river bank 0.06 km downstream from a point directly across the river. The displacement vector diagram forms a triangle which is similar to the velocity diagram:



| Unit: Geometry | Topic: Angles and Polygons |
|---|---|
| **Benchmark** | • **11.2.2.11** Draw similar triangles using scales.<br>• **11.2.2.12** Use the concept of corresponding parts to prove that triangles and other polygons are congruent or similar. |

**Learning Objective:** By the end of the topic, students will be able to;

- transform polygons especially triangles using scale factors,
- test for congruency in triangles using the properties of congruency, and
- construct polygons and investigate for similarity or congruency.

**Essential questions:**

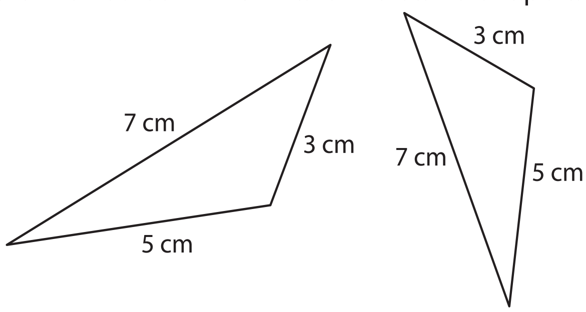
- How are angles formed?
- What are polygons?
- What are congruent polygons?
- How are similar polygons identified?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Confidently transform polygons using scale factors and also testing for congruency and similarity. |
| **Skills** | Transform polygons using scale factors and test for congruency. |
| **Knowledge** | Transformation of polygons and test for congruency. |
| **Mathematical Thinking** | Think about how to transform and construct polygons and prove for congruency. |

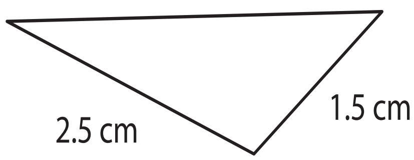
## Content Background

### 1. Congruence and Similarity Congruence

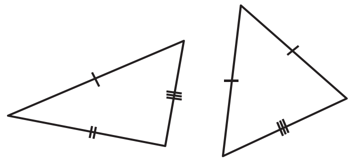
Two shapes are said to be congruent if they are the same shape and size: that is, the corresponding sides of both shapes are the same length and corresponding angles are the same.
The two triangles shown here are congruent.
Shapes which are of different sizes but which have the same shape are said to be similar.




The triangle below is similar to the triangles above and not congruent because the size is different to the triangles above.

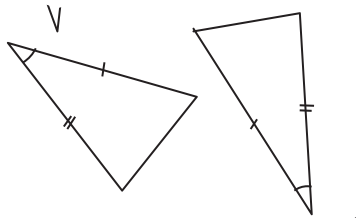


There are four tests for congruence which are outlined below.



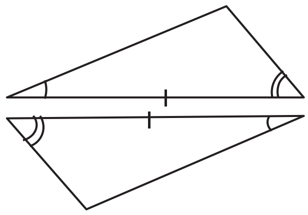
**Test 1: (Side, Side, Side)**
If all three sides of one triangle are the same as the lengths of the sides of the second triangle, then the two triangles are congruent.

This test is referred to as SSS.



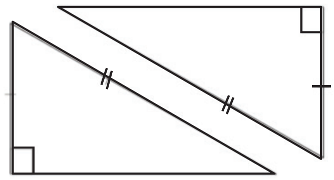
**TEST 2 (Side, Angle, Side)**
If two sides of one triangle are the same length as two sides of the other triangle and the angle between these two sides is the same in both triangles, then the triangles are congruent.

This test is referred to as SAS.



**TEST 3 (Angle, Angle, Side)**
If two angles and the length of one corresponding side are the same in both triangles, then they are congruent.

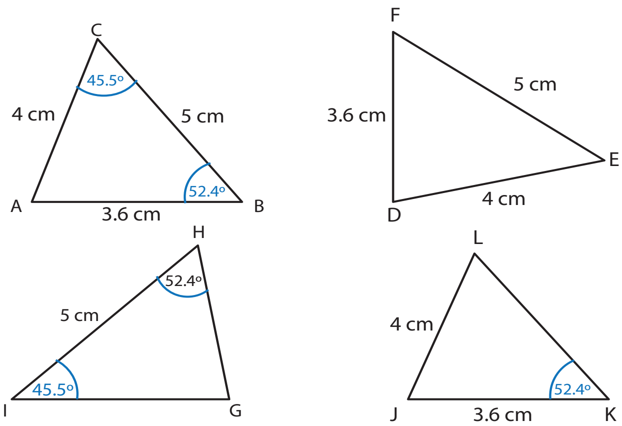
This test is referred to as AAS.



**TEST 4 (Right angle, Hypotenuse, Side)**
If both triangles contain a right angle, have hypotenuses of the same length and one other side of the same length, then they are congruent.

This test is referred to as RHS.

***Example***
Which of the triangles below are congruent to the triangle ABC, and why?




***Solution***
Consider first the triangle DEF:
AB = DF
BC = EF
AC = DE

As the sides lengths are the same in both triangles the triangles are congruent. (SSS)

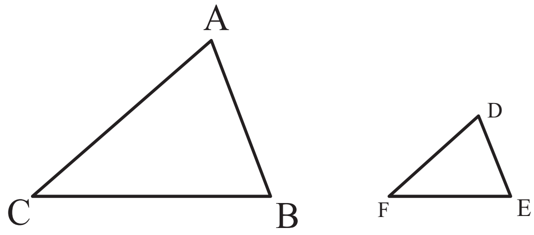
Consider the triangle GHI:
BC = HI
$A\hat{B}C = G\hat{H}I$
$A\hat{C}B = G\hat{I}H$

As the triangles have one side and two angles the same, they are congruent. (AAS)

Consider the triangle JKL:
Two sides are known but the angle between them is unknown, so there is insufficient information to show that the triangles are congruent.

**2. Similar Triangles**

Two triangles are similar if corresponding angles are equal in measure.
For similar triangles corresponding sides are in proportion.

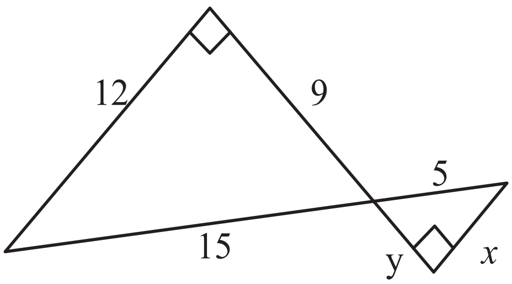



If triangle ABC and triangle DEF are similar $(\square ABC \sim \square DEF)$ then

$$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F \text{ and } \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

If a pair of triangles has two pairs of congruent corresponding angles then they are similar.

***Example:***
In the figure below, find x and y.




Similarly since the sides of the small triangle are one-third the length of their corresponding sides in the larger triangle, $y = 3$

*Solution* $\frac{x}{12} = \frac{5}{15}$

$15x = 12 \times 5$

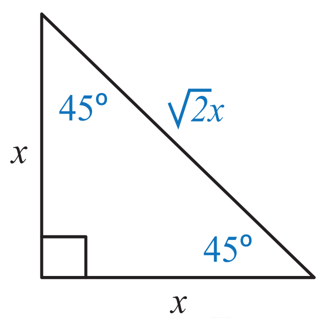
$15x = 60$

$x = 4$

**3. Special Triangles**

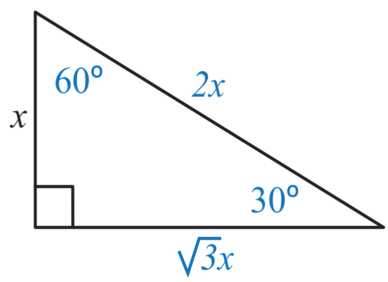
Right triangles whose angle measures are 45°,45°,90° or 30°,60°,90° are called special right triangles.

In a 45°,45°,90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each length.




Hypotenuse: = $\sqrt{2} \times$ length

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter length, and the longer leg is $\sqrt{3}$ times as long as the shorter length.



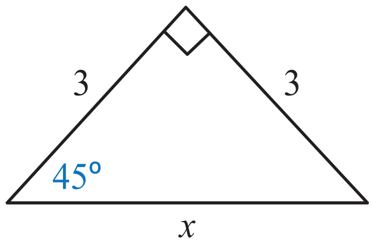

Hypotenuse: = 2 × shorter length
Longer length: = $\sqrt{3}$ × shorter length

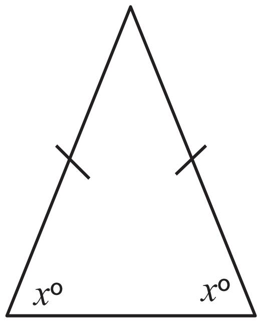
***Example:*** Find the value of $x$.

*Solution*

By the Triangle Sum Theorem, the measure of the third angle is 45°. The triangle is a 45°- 45°- 90° right triangle, so the length $x$ of the hypotenuse is times the length 3.

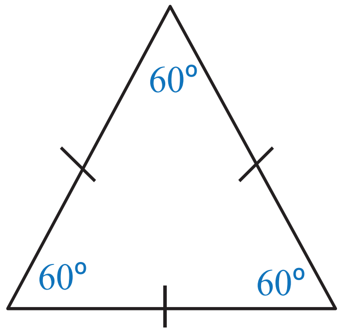
Hypotenuse: $\sqrt{2}$ length $=\sqrt{2}\text{x}3 \Longrightarrow \therefore x = 3\sqrt{2}$



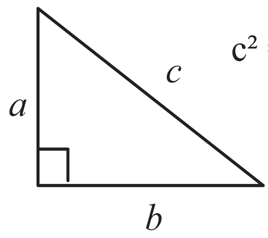

Isosceles

A triangle with two equal sides is called an ***isosceles triangle.*** The angles opposite the two equal sides are equal in measure.




Equilateral

A triangle with three equal sides is called an ***equilateral triangle.*** All three angles of an equilateral triangle are 60°.

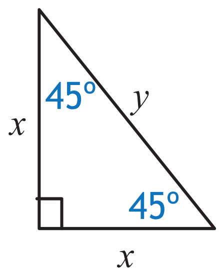



$c^2 = a^2 + b^2$

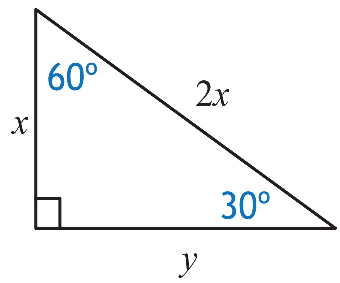
***Pythagorean Theorem:*** In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the two lengths.

***Example:*** Find y for each of the triangles below.

**a.**




**b.**




*Solution*

$$y^2 = x^2 + x^2$$
$$y^2 = 2x^2$$
$$y = x\sqrt{2}$$

$$x^2 + y^2 = (2x)^2$$
$$y^2 = 3x^2$$
$$y = x\sqrt{3}$$

**Unit: Geometry** | **Topic: Geometric Proofs**

| | |
|---|---|
| **Benchmark** | **11.2.2.13** Use protractor and compass to construct geometric angles and shapes. |

**Learning Objective:** By the end of the topic, students will be able to;

- construct Geometric angles and shapes using compass and protractors, and
- proof geometric constructions using the geometric properties.

**Essential questions:**

- How can the compass and protractors be used in constructing angles?
- What are the geometric properties?

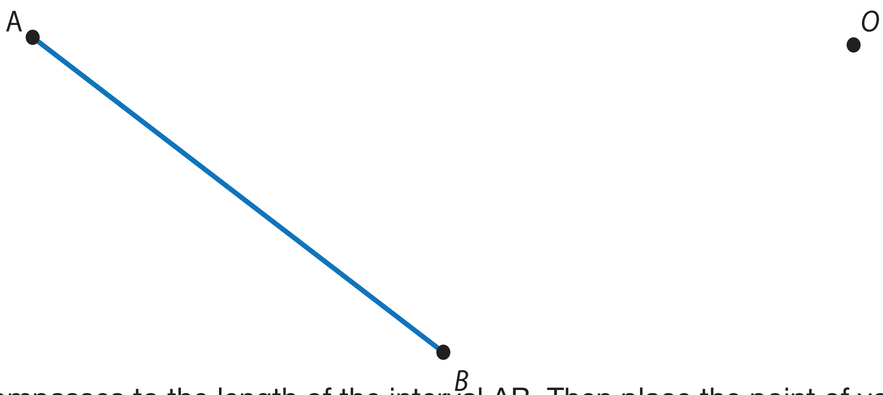
| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Enjoy and creatively design geometric shapes and angles using compass and protractors. |
| **Skills** | Design and construct angles and justify using the geometric properties. |
| **Knowledge** | Proving geometric constructions, angles and shapes. |
| **Mathematical Thinking** | Think about how to construct and prove angles and shapes. |

## Content Background

Geometrical constructions should be drawn with a sharp, hard (2H) pencil. A pencil eraser, a ruler marked in centimetres (often used as a straight edge), a pair of compasses, two set squares (90°,60°,30° and 90°,45°,45°) and a protractor with a radius of at least 5 cm make up the basic set of geometrical instruments. Students should be encouraged to draw accurate and large diagrams.

### Using compasses to draw a circle

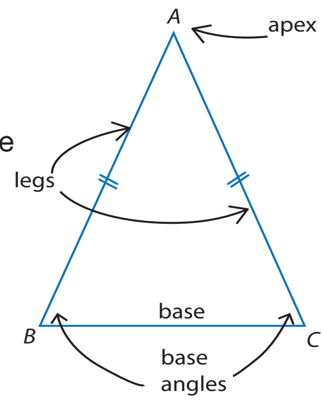
Suppose we are given an interval AB and a point O. In geometry we always include the end points in an interval.




Open your compasses to the length of the interval AB. Then place the point of your compasses firmly into the point O. Holding the compasses only by the very top, draw a circle.
This is called drawing a circle with centre O and radius AB. Notice that every point on the circle is the same distance from the centre O, because the distance between the point O and the pencil lead never changes.

## 1. Isosceles and equilateral triangles

Triangles with two or three sides equal have some interesting properties. Using simple constructions we can investigate or demonstrate these properties.

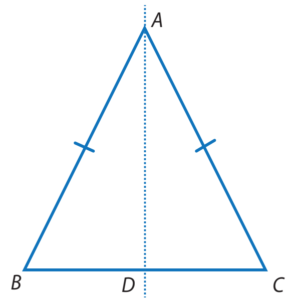



An isosceles triangle is a triangle with two (or more) sides equal.

- The equal sides *AB* and *AC* of the isosceles triangle *ABC* to the right are called the legs. They have been marked with double dashes to indicate that they are equal in length.
- The vertex *A* where the legs meet is called the apex and the third side *BC* is called the base.
- The angles $\angle B$ and $\angle C$ at the base are called base angles.

The word isosceles is a Greek word meaning "equal legs". The prefix iso means "equal", and-sceles is related to the word for leg.

## 2. Constructing an isosceles triangle and demonstrating the base angles are equal



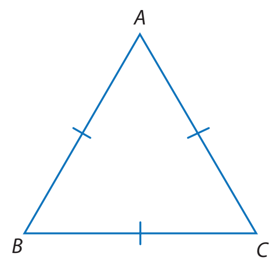

Draw a large circle (or an arc) with centre *A*. Draw two radii *AB* and *AC* (*A,B,C* not collinear). Triangle *ABC* is isosceles. Check with a protractor that $\angle B = \angle C$.

Note that the isosceles triangle ABC has an axis of symmetry AD with D the mid-point of BC.

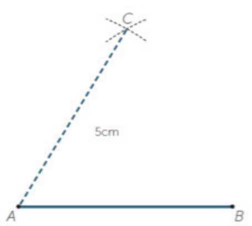
This means that reflecting $\Delta ACD$ in the line *AD* produces $\Delta ABD$. Alternatively, if we fold along the line *AD*, the points *B* and *C* coincide.

If a triangle has two equal angles, then the two sides opposite those angles are equal and the triangle is isosceles.

## 3. Constructing an equilateral triangle using two circles




An equilateral triangle is a triangle in which all three sides have equal length. The diagram to the right shows an equilateral triangle *ABC*. Notice that it is an isosceles triangle in three different ways, because the base could be taken as *AB, BC,* or *CA.*




The construction is straightforward. Draw an interval *AB* of length, say, 5 cm. Using compasses draw arcs of two circles centre *A* and *B,* both of radius 5 cm. The point *C* is the point where the arcs meet.

Since an equilateral triangle is isosceles with base *AB*, $\angle A = \angle B$. Similarly $\angle A = \angle C$. But $\angle A + \angle B + \angle C = 180°$, so all angles are 60°.

## 4. Basic constructions with straight edge and compass

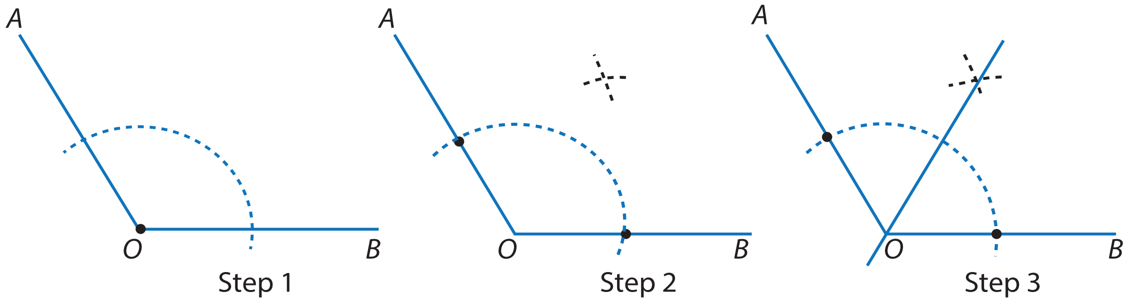
Careful constructions with compasses and straight edge have always been an essential part of geometry. These constructions are based on a fundamental fact about circles:

All radii of a circle are equal.

Whenever you draw a circle using compasses, as the pencil lead moves, it always remains exactly the same distance from the fixed point.

**(i) Construction – Bisecting a given angle**

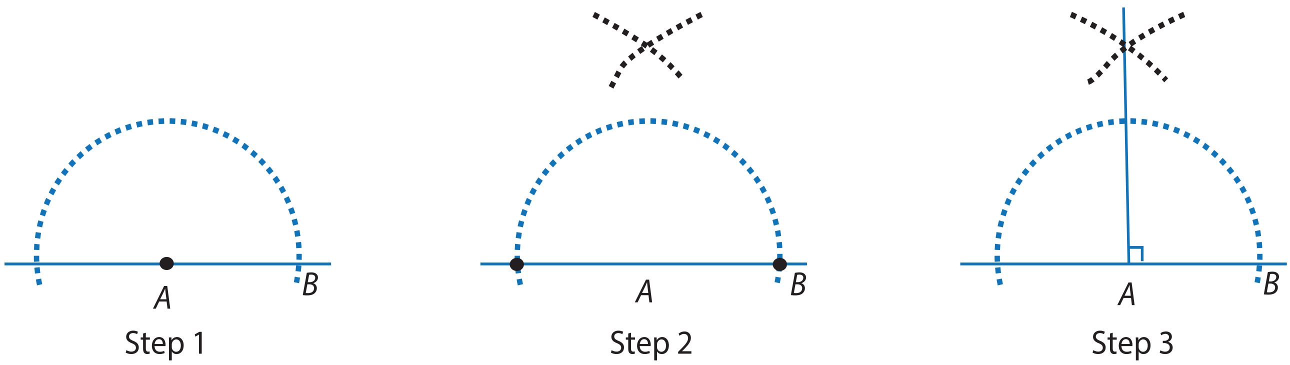
The diagram below shows the steps to follow to bisect a given angle $\angle AOB$.




The two arcs in step 2 can have a different radius from the arc in step 1.
Folding the paper along the constructed line provides an informal proof or demonstration that the construction works. The arms OA and OB then fall exactly on top of each other, so $\angle$AOB has been cut into two equal pieces. The formal proof uses SSS congruence.
The line you have constructed also bisects the reflex angle $\angle AOB$. (Can you prove this?)

**(ii) Construction – A right angle at the endpoint of an interval**

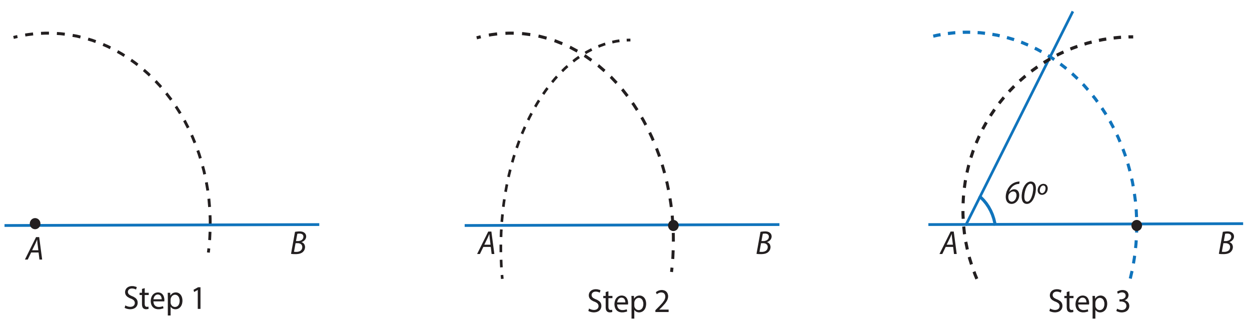
A right angle is half a straight angle. Thus bisecting a straight angle using the previous construction will give a right angle. We begin by producing (extending) the interval *BA.*




The two arcs in step 2 will need to have a larger radius than the arc in step 1.

**(iii) Construction – An angle of 60° at the endpoint of an interval**

The angles of an equilateral triangle are all 60°. Thus constructing an equilateral triangle will give an angle of 60°.




This time the arcs in steps 1 and 2 must have the same radius.

| Unit: Geometry | Topic: Circle geometry |
|---|---|
| **Benchmark** | **11.2.2.14** Use circle properties and angles in cyclic quadrilaterals or polygons. |

**Learning Objective:** By the end of the topic, students will be able to;

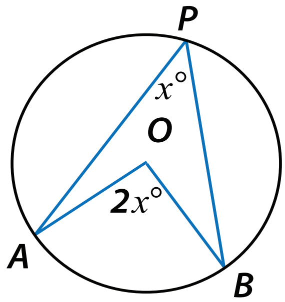
- identify and explain angles formed to the centre and circumference of a circle,
- discuss and identify angles in the same and opposite segment,
- solve the application of chords and tangents, and
- explain concyclic points and verify angles formed in a cyclic quadrilaterals using the properties.

**Essential questions:**
- Are the features to a circle?
- What angles are formed in a circles?
- How are the angles in a circle formed?
- What are the angle properties of a circle?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Confidently explain angles formed within a circle and describe the properties. |
| **Skills** | Provide logical reasons to justify angles form within a circle. |
| **Knowledge** | The properties of angles formed within a circle. |
| **Mathematical Thinking** | Think about the application of circle properties to verify or justify values of angles formed within circles. |

## Content Background

### Angle properties of the circle



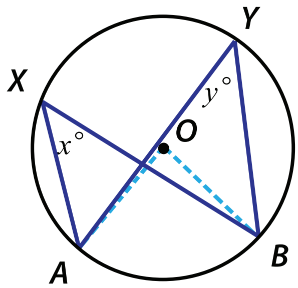

**Theorem 1**: The angle at the centre of a circle is twice the angle at the circumference subtended by the same arc *AB*

**Theorem 2:** The angles in the same segment of a circle are equal.

*Proof*

Let $\angle AXB = x^0$ and $\angle AYB = y^0$ , Then by Theorem 1 $\angle AOB = 2x^0 = 2y^0$

$\therefore x = y$




**Theorem 3:** The angle subtended by a diameter at the circumference is equal to a right angle ($90^0$).

*Proof*

The angle subtended at the centre is $180^0$. Theorem 1 gives the result.

A quadrilateral inscribed in a circle is called a ***cyclic quadrilateral.***

**Theorem 4:** The opposite angles of a quadrilateral inscribed in a circle sum to two right angles ($180^0$) . (The opposite angles of a cyclic quadrilateral are supplementary). The converse of this result also holds.

*Proof*

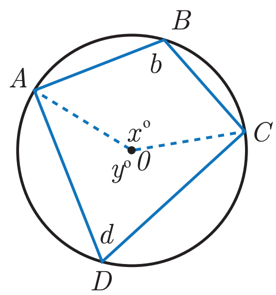
*O* is the centre of the circle

By Theorem $1y = 2b$ and $x = 2d$

Also $x + y = 360$

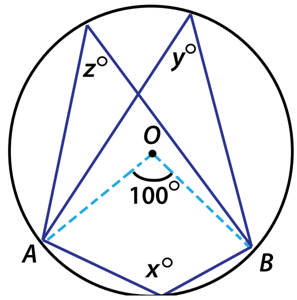
Therefore $2b + 2d = 360$

*i.e.* $b + d = 180$




The converse states: if a quadrilateral has opposite angles supplementary then the quadrilateral is inscribed in a circle.

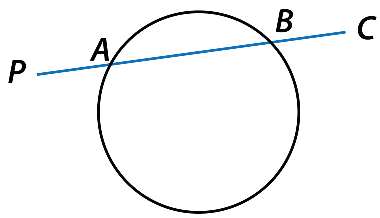
***Example:*** Find the value of each of the pronumerals in the diagram. *O* is the centre of the circle and $\angle AOB = 100^0$

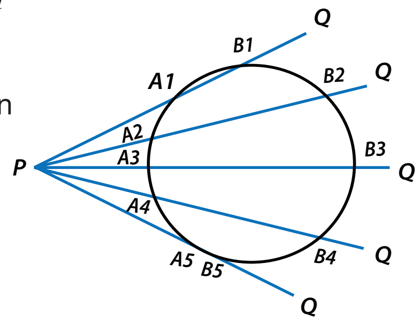



*Solution*
Theorem 1 gives that $z = y = 50$
The value of $x$ can be found by observing either of the following.
Reflex angle AOB is $260^0$. Therefore $x$=130 (theorem 1) or $x+y$ =180 (theorem 4)
Therefore $x$ =180-50=130

**Tangents**

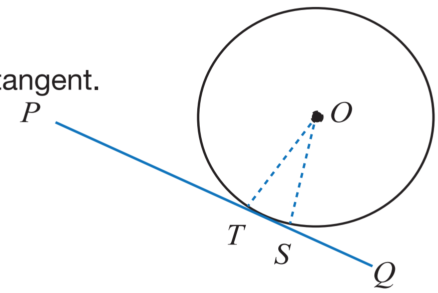
Line $PC$ is called a secant and line segment $AB$ a chord. If the secant is rotated with $P$ as the pivot point a sequence of pairs of points on the circle is defined. As $PQ$ moves towards the edge of the circle the points of the pairs become closer until they eventually coincide. When $PQ$ is in this final position (*i.e.*, where the intersection points $A$ and $B$ collide) it is called a **tangent** to the circle. $PQ$ touches the circle. The point at which the **tangent** touches the circle is called the **point** of tangency. The length of a tangent from a point $P$ outside the tangent is the distance between $P$ and the point o tangency.



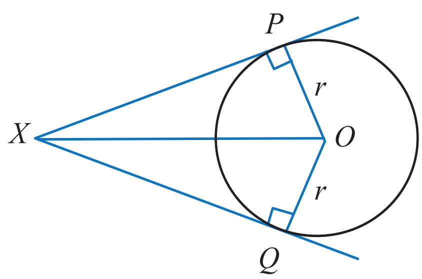

**Theorem 5:** A tangent to a circle is perpendicular to the radius drawn to the point of tangency.

*Proof*
Let $T$ be the point of tangency to the tangent $PQ$.
Let $S$ be the point on $PQ$, not $T$, such that $OSP$ is a right angle.
Triangle $OST$ has a right angle at $S$.
Therefore $OT > OS$ as $OT$ is the hypotenuse of triangle $OTS$.
∴ $S$ is inside the circle as $OT$ is a radius.
∴ The line through $T$ and $S$ must cut the circle again. But $PQ$ is a tangent.
A contradiction.
Therefore $T = S$ and angle $OTP$ is a right angle.



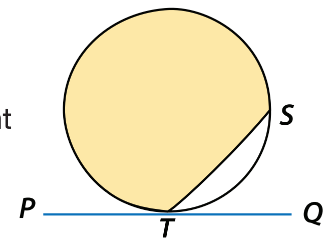

**Theorem 6:** The two tangents drawn from an external point to a circle are of the same length.
*Proof*
Triangle $XPO$ is congruent to triangle $XQO$ as $XO$ is a common side.

$\angle XPO = \angle XQO = \angle 90^0$
$OP = OQ$ (radii)
Therefore $XP = XQ$




**Alternate segment theorem:** The shaded segment is called the alternate segment in relation to $\angle STQ$.
The unshaded segment is alternate to $\angle PTS$

**Theorem 7:** The angle between a tangent and a chord drawn from the point of contact is equal to any angle in the alternate segment.



*Proof*

Let $\angle STQ = x^0$, $\angle RTS = y^0$ and $\angle TRS = z^0$ where *RT* is a diameter

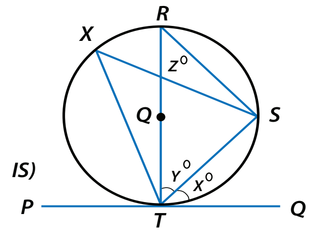
Then $\angle$RST=$90^0$ (Thorem 3, angle subtended by a diameter)

Also $\angle RTO = 90^0$ (theorem 5, tangent is perpendicular to radius)

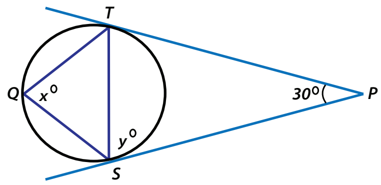
Hence $x+y = 90$ and $y+z = 90$

Therefore $x = z$

But $\angle TXS$ is the same segment as $\angle TRS$ and therefore $\angle TXS = x^0$




***Example:* (i)** Find the magnitude of the angles $x$ and $y$ in the diagram.

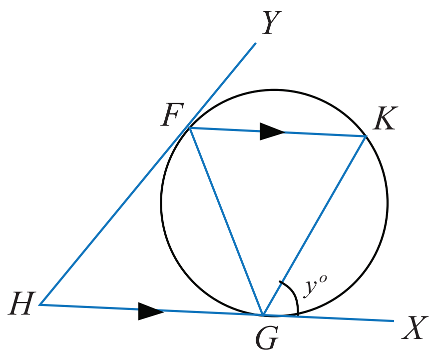



*Solution*

Triangle *PTS* is isosceles (Theorem 6, two tangents from the same point) and therefore $\angle$PTS = $\angle$PST

Hence $y = 75$. The alternate segment theorem gives that $x = y = 75$

**(ii)** The tangents to a circle at *F* and *G* meet at *H*. If a chord *FK* is drawn parallel to *HG*, prove that triangle *FGK* is isosceles.




*Solution*

Let $\angle XGK = y^0$

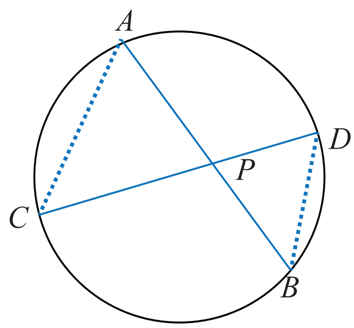
Then $\angle$GFK = $y^0$ (alternate segment theorem)

and $\angle$GKF = $y^0$ (alternate angles)

Therefore triangle *FGK* is isosceles with *FG* = *KG*

## Chords in Circles

**Theorem 8:** If *AB* and *CD* are two cords which cuts a point *P* (which may be inside or outside the circle) then $PA \cdot PB = PC \cdot PD$




*Proof*

Case 1: (The intersection point is inside the circle)

Consider triangles *APC* and *BPD.*

$\angle$APC=$\angle$BPD (vertically opposite)

$\angle$CDB=$\angle$CAB (angles in the same segment)

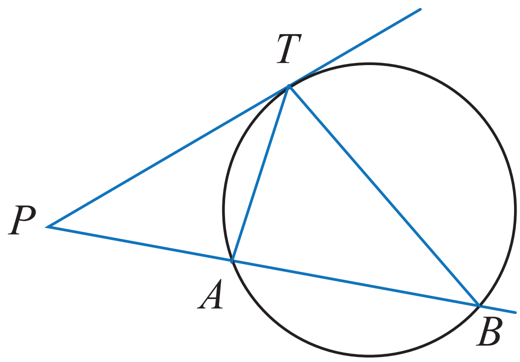
$\angle$ACD=$\angle$DBA (angles in the same segment)

Therefore triangle CAP is similar to triangle BDP

Therefore $\frac{AP}{PD} = \frac{CP}{PD}$

and $AP \cdot BP = CP \cdot PD$, which can be written as $AP \cdot PB = PC \cdot PD$

Case 2: (The intersection point is outside the circle.) Show triangle APD is similar to triangle CPB




*Proof*

$\angle$PTA=$\angle$TBA (alternate segment theorem)

$\angle$PTB=$\angle$TAP (angle sum of a triangle)

Therefore triangle *PTB* is similar to triangle *PAT*

**Theorem 9:** If *P* is a point outside a circle and *T,A,B* are points on the circle such that *PT* is a tangent and *PAB* is a secant then $\angle PT^2 = PA \angle PB$

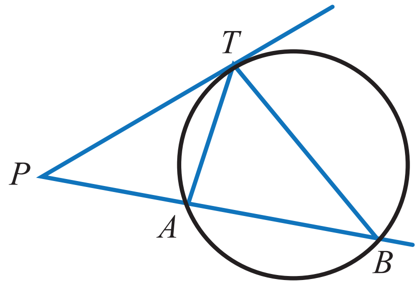
*Proof*

$\angle PTA = \angle TBA$ (alternate segment theorem)

$\angle PTB = \angle TAP$ (angle sum of a triangle)

Therefore triangle *PTB* is similar to triangle *PAT*

$\frac{PT}{PA} = \frac{PB}{PT}$ which implies $PT^2$= PA·PB

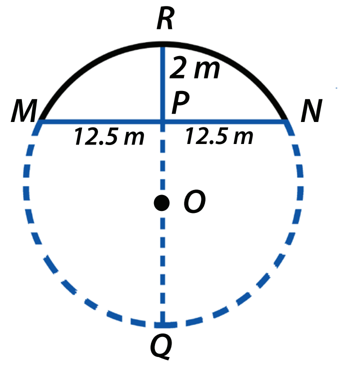



**Examples**

**(i)** The arch of a bridge is to be in the form of an arc of a circle. The span of the bridge is to be 25 m and the height in the middle 2 m. Find the radius of the circle.

*Solution*

**By theorem 8:** *RP·PQ = MP·PN*




Therefore $2PQ = (12.5)^2 \quad \therefore PQ = \frac{(12.5)^2}{2}$

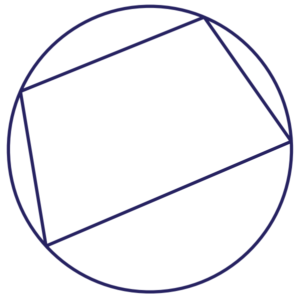
Also $PQ = 2r - 2$

where $r$ is the radius of the circle

Hence $2r - 2 = \frac{(12.5)^2}{2}$

and $r = \frac{1}{2}\left(\frac{(12.5)^2}{2} + 2\right) = \frac{641}{16} m$

### Cyclic quadrilateral and its properties



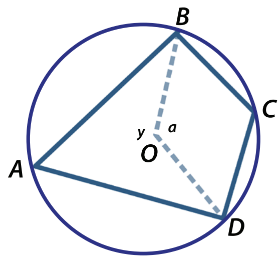
A cyclic quadrilateral is a quadrilateral whose vertices all lie on a circle.

**The opposite angles of a cyclic quadrilateral**

The distinctive property of a cyclic quadrilateral is that it's opposite angles are supplementary. The following proof uses the theorem that an angle at the circumference is half the angle at the centre standing on the same arc.

**Theorem:** The opposite angles of a cyclic quadrilateral are supplementary.

Proof: Let *ABCD* be a cyclic quadrilateral with *O* the centre of the circle. Join the radii *OB* and *OD*. Let α and γ be the angles at the centre, as shown on the diagram.




Then $\alpha + \gamma = 360^0$ (angles in a revolution at *O*)

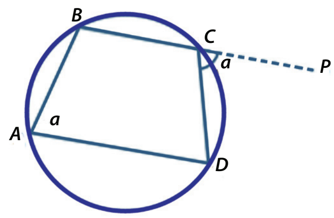
Also $\angle A = \frac{1}{2}\alpha$ (angles on the same arc *BCD*)

and $\angle C = \frac{1}{2}\gamma$ (angles on the same arc *BAD*)

so $\angle A + \angle C = \frac{1}{2}\alpha + \frac{1}{2}\gamma = 180^0$

Hence also $\angle ABC + \angle ADC = 180^0$ (angle sum of quadrilateral *ABCD*)

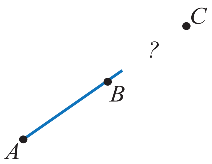
### Exterior angles of a cyclic quadrilateral




An exterior angle of a cyclic quadrilateral is supplementary to the adjacent interior angle, so is equal to the opposite interior angle. This gives us the corollary to the cyclic quadrilateral theorem:
Theorem: An exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

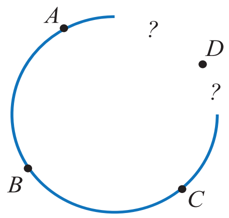
*Proof*
In the diagram to the right, *BC* is produced to *P* to form the exterior angle ∠*PCD*. This exterior angle and ∠A are both supplementary to ∠*BCD*, so they are equal **Concyclic Points**

One of the basic axioms of geometry is that a line can be drawn through any two distinct points A and B. When there are three distinct points A, B and C, we can ask whether the three points are collinear, or form a triangle.



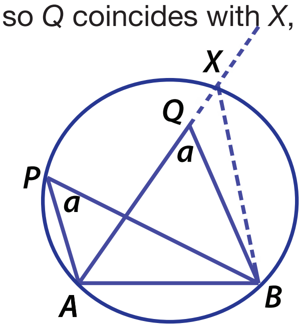

Similarly, any three non-collinear points *A, B* and *C* are Concyclic. When there are four points *A, B, C* and *D*, no three collinear, we can ask whether these four points are Concyclic, that is, do they lie on a circle.

**Theorem:** If an interval subtends equal angles at two points on the same side of the interval, then the two points and the endpoints of the interval are Concyclic.




*Proof*
Let the interval *AB* subtend equal angles at points *P* and *Q* on the same side of ***BC?. Construct the circle through *A, B* and *P,* and suppose, by way of contradiction, that the circle does not pass through *Q.*
Let *AQ*, produced if necessary; meet the circle again at *X.*
Then ∠*AXB* = α
So *BX*||*BQ*

Hence *BQ* and *BX* are the same line, so *Q* coincides with *X*, which is a contradiction.



## Strand 3: Patterns and Algebra

**Content Standard:** Students will be able to interpret various types of patterns and functional relationships, use symbolic forms to represent, model, and analyze mathematical situations and collect, organise, and represent data to answer questions.

| Unit | Benchmark | Topic | Lesson Title |
|---|---|---|---|
| Factorization | **11.3.3.1** Factorize quadratic and cubic expressions.<br><br>**11.3.3.2** Simplify algebraic fractions.<br><br>**11.3.3.3** Factorize, solve and sketch quadratic expressions and equations.<br><br>**11.3.3.4** Simplify algebraic fractions to solve simultaneous equations using elimination, substitution and graphical methods. | Factorization of Algebraic Expressions | Simple algebraic expressions |
| | | | Factorizing by difference of squares |
| | | | Difference of two squares |
| | | | The product of two binomial expressions |
| | | | Factors of quadratic expressions |
| | | Algebraic Fractions | Addition and Subtraction of algebraic fractions |
| | | | Multiplying and Division of algebraic fractions |
| | | | Simplify rational expression by factorizing |
| Equations and Inequalities | **11.3.3.5** Solve inequalities and plot on number line or plane. | Quadratic Equations with (a) not equal 1 | Solving Quadratic Equations by Factorization |
| | | | Solving quadratic equations by completing the square |
| | | | Using the formula to solve quadratic equations |
| | | | Discriminant of quadratic equations |
| | | | Graphing quadratic functions which can be factorized |
| | | | Problems involving quadratic equations/ functions. |
| | | Solutions To Simultaneous Equations | Substitution method |
| | | | Elimination method |
| | | | Graphical method |
| | | | Solutions of Non-Linear Simultaneous Equations |
| | | Inequalities | Combined inequalities |
| | | | Inequality involving absolute value |
| | | | Graphs of linear inequalities |

| | | | |
|---|---|---|---|
| Linear Function | **11.3.3.6** Calculate the gradient of a straight line using intercept form and line through two points. | Gradient of straight line | The Gradient Intercept Form of a straight Line |
| | | | The Gradient of a Line through two points |
| | **11.3.3.7** Apply the concepts of parallel and perpendicular lines to find gradient and equation. | Parallel and Perpendicular lines | Parallel and Perpendicular Lines |
| | | | Intersection of Two straight Lines |
| | | | Perpendicular bisector |
| | **11.3.3.8** Calculate the distance of a point from a line, distance between two points and intersection of two straight lines. | Distance of a point from a line | The distance between two points |
| | | | The midpoint of an interval |
| | | | The perpendicular distance of a point from a line |
| Functions and Graphs | **11.3.3.9** Define and describe a function using the vertical line test. | Relations and Functions | The sketches of the functions and the vertical line test |
| | | | The applications of the functions |
| | **11.3.3.10** Recognize and explain different functions and their features and calculate their range and domain. | Domain and Range of a function | The main features of the functions including Domain and Range |
| | | | Calculate domain and range to functions |
| | **11.3.3.11** Define absolute value of a number and graph absolute value functions. | Absolute Value Functions | Absolute Value |
| | | | Graphing Absolute value function |
| | | | Absolute value equations |
| | **11.3.3.12** Solve problems involving linear, quadratic and exponential and inequality equations involving absolute values individually or simultaneously using algebraic or graphs. | Linear, Quadratic and Exponential equations and Inequalities | Problems on Linear and Quadratic equations and Inequalities<br>Problems on Exponential equations and Inequalities |
| | | Absolute Value Function | Introductions of absolute value functions |
| | | | Graphs of absolute value functions |
| | **11.3.3.13** Derive and sketch graphs of hyperbolic, exponential, logarithm functions and discuss their asymptotes and applications. | Sketches of hyperbolic, exponential, Logarithmic functions and Asymptotes | Sketching Hyperbolic functions |
| | | | Sketching exponential functions |
| | | | Sketching Logarithmic functions |
| | | | Definition and application of asymptotes |
| | **11.3.3.14** Derive and sketch graphs of circles. | Circles on Cartesian Plane | Radius and centre of a unit circle |
| | | | Deriving General form of circle equations from the graph |
| | | | Sketching the graphs of circles |
| | **11.3.3.15** Convert and write equations of circles in standard form $(x-h)^2 + (y-k)^2 = r^2$ | Equation of Circles in Standard Form | Conversion of the equations of circles in Standard form: $(x-h)^2 + (y-k)^2 = r^2$ |
| | | | Sketch circles in standard form |

| Unit: Factorization | Topic: Factorization of Algebrain Expressions |
|---|---|
| **Benchmark** | **11.3.3.1.** Factorize quadratic and cubic expressions. |

**Learning Objective:** By the end of the topic, students will be able to;

- factorize and simplify simple quadratic and cubic expressions,
- factorize by difference of two squares, perfect squares and grouping methods, and
- factorize the general quadratic and cubic expressions.

**Essential questions:**

- What is factorization?
- What are the rules in factorising quadratic and cubic expressions?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Confidently discuss their ideas of factorization. |
| **Skills** | Apply rules of factorisation. |
| **Knowledge** | Factorisation of quadratic and cubic expressions. |
| **Mathematical Thinking** | Think about how to factorise and quadratic cubic expressions. |

## Content Background

### 1. Simple Quadratic and Cubic expressions

A simple quadratic expression is an expression of the form $x^2 + bx + c$, where $b$ and $c$ are given numbers. Generally, when we expand $(x + p)(x + q)$, we get $x^2 + px + qx + pq = x^2 + (p+q)x + pq$
The coefficient of $x$ is the sum of $p$ and $q$, and the constant term is the product of $p$ and $q$
A quadratic expression can sometimes be factorized into two brackets in the form of $(x + p)$ and $(x + q)$,where $p$ and $q$ can be found by using a product and sum method
Expanding the brackets $(x + 2)(x + 3)$ gives $x^2 + 3x + 2x + 6$ which simplifies to $x^2 + 5x + 6$.
Factorizing is the reverse process of expanding brackets, so factorizing $x^2 + 5x + 6$ gives $(x+2)(x+3)$

***Example:*** Factorize $x^2+7x+10$.

*Solution*
To factorize this expression, find two numbers that have a product +10 and the sum of +7.
There are a couple of ways of making +10 by multiplying two numbers. These are 1 × 10 and 2 × 5.
Only the combination of 2 and 5 will also give a sum of +7, so the two numbers are 2 and 5.
$x^2 + 7x + 10 = (x + 2)(x + 5)$

To check this answer is correct, expand out the brackets.

This is the original expression, so $(x + 2)(x + 3)$ is the correct factorization.

### 2. Cubic Expressions

A Cubic expression has the highest power of the variable 3. A cubic expression can be in the following forms
$ax^3$, binomial $ax^3 + bx^2$, $ax^3 + cx$, $ax^3 + d$, trinomial $ax^3 + bx^2 + cx$, $bx^3 + bx^2 + d$, or a polynomial $ax^3 + bx^2 + cx + d$.

The general strategy for factorizing a cubic expression is to reduce it to quadratic expression and then simplify a quadratic by the usual means, either by factorization or grouping. This is also applicable to cubic equations.

***Example***

Suppose we wish to factorize $x^3$-$5x^2$- $2x + 24$ given that $x = -2$ is a solution.

There is a theorem called the Factor Theorem which we do not prove here. It states that if $x = -2$ is a solution (if this was a cubic equation), then $x+2$ is a factor of this whole expression. This means that $x^3 - 5x^2 - 2x + 24$ can be written in the form $(x + 2)(x^2 + px + q)$ where p and q are numbers. Our task now is to find p and q, and we do this by adapting process called ***synthetic division. This involves looking at the coefficients of the original cubic expression; $x^3$-$5x^2$-2x+24, which are 1,-5,-2 and 24. These are written down in the first row of a table, the starting layout for which is

| 1 | - 5 | -2 | 24 | $x = -2$ |
|---|---|---|---|---|
| | | | | |

Notice that to the right of the vertical line we write down the known root $x$ =-2. We have left a blank line which will be filled in shortly. In the first position on the bottom row we have brought down the number 1 from the first row.

| 1 | - 5 | -2 | 24 | $x = -2$ |
|---|---|---|---|---|
| 1 | | | | |

The next step is to multiply the number 1, just brought down by the known root, -2, and write the result, -2, in the blank row in the position shown

| 1 | - 5 | -2 | 24 | $x = -2$ |
|---|---|---|---|---|
| 1 | - 2 | | | |
| 1 | | | | |

The numbers in the second column are then added,-5 +-2 =-7, and the result written in the bottom row as shown

| 1 | - 5 | -2 | 24 | $x = -2$ |
|---|---|---|---|---|
| 1 | - 2 | | | |
| 1 | - 7 | | | |

Then, the number just written down,-7, is multiplied by the known root,-2, and we write the result, 14, in the blank row in the position shown. Then the numbers in this column are added:

| 1 | - 5 | -2 | 24 | $x = -2$ |
|---|---|---|---|---|
| 1 | - 2 | 14 | | |
| 1 | - 7 | | 12 | |

The process continues

Note that the final number in the bottom row (obtained by adding 24 and -24) is zero. This is confirmation that $x$=-2 is a root of the original cubic. If this value turns out to be non-zero then we do not have a root.

At this stage the coefficients in the quadratic that we are looking for are the first three numbers in the bottom row. So the quadratic is $x^2$-$7x+12$ So we have reduced our cubic to $(x+2)(x^2 -7x + 12)$

The quadratic term can be factorized to give $(x + 2)(x^2$-$7x+12) = (x +2)(x$-$3)(x$-$4)$

### 3. The difference of two squares

The difference of two numbers is found by subtracting. The difference of two squares means one squared term subtract another squared term. For example, $(x^2–9)$ would be the difference of two squares as $x^2$ is a squared term ($x$ has been multiplied by itself) and 9 is a square number (3×3). The difference of two squares can be factorized into brackets using the method above for factorizing quadratics.

***Example***

Factorize $(x^2 - 4)$

*Solution*

Quadratics can be factorized into the form $(x + a)(x - b)$ therefore $(x^2 - 4)$ can be written as $x^2+0x-4$. To factorize this expression, find two numbers that have a product -4 and the sum of 0. The factor pairs that make – 4 are either $-1\times4$, $1x-4$, or $-2\times2$. The factor pair that has a product of –4 but the sum of 0 is –2 × 2 because $-2 \times 2 = -4$ and $-2 + 2 = 0$. Therefore the two numbers to go into the brackets are –2 and 2. This gives $(x^2 - 4) = (x-2)(x+2)$.

To confirm this answer, expand the brackets to see if the answer gives the original expression. If it does, then this is a correct factorization.
$(x - 2)(x + 2) = (x \times x) + (x \times 2) + (-2 \times x) + (-2 \times 2) = x^2 + 2x - 2x - 4$

Collecting the like terms gives the original expression of $x^2 - 4$
If the quadratic is in the form $x^2 - a^2$ then its factorized form is $x - a(x + a)$

**4. Perfect squares**

We have already discussed perfect squares in our previous years.

What we need to know is to "Remember" these patterns so that we can be at the look-out for them when factoring

$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$

***Example***

Factorize $(a + b)^2 + 12(a + b) + 36$

Does this fit the pattern of a perfect square trinomial?
Yes. Both $(a + b)^2$ and 36 are perfect squares, and $12(a + b)+$ is twice the product of $(a + b)$ and 6. Since the middle term is positive, the pattern is $(a + b)^2 = a^2 + 2ab + b^2$

Let $a = (a + b)$ and $b = 6$.

Answer: $((a + b) + 6)^2$ or $(a + b + 6)^2$

**5. Factorizing quadratics when the coefficient of x squared ≠1**

Quadratic expressions can be written in the form $ax^2 + bx + c$, where $a, b,$ and $c$ are numbers. $a$ is called the coefficient of $x^2$ and $b$ is the coefficient of $x$ . $c$ is a constant term (a number that is not multiplied by the variable $x$).

***Example***
For the quadratic expression, $6x^2 - 7x - 3$...$a = 6$, $b = -7$ and $c = -3$

*Solution*
To factorize this quadratic, first multiply the coefficient of $x^2$ by the constant term (-3). Find two numbers which have a product of -18 x 2 and a sum of -7x. (2 × -9 = -18 and 2 + -9 = -7 so the numbers are 2 and -9)
Rewrite the center term as these two numbers, so $-7x$ becomes $2x - 9x$

Factorize each pair of terms. Check that the bracket created is the same. The brackets have now been found. The first bracket is the common factor of $(3x + 1)$ and the second bracket is the factorized terms outside of each bracket $(2x - 3)$

| Unit: Factorization | Topic: Algebraic Fractions |
|---|---|
| **Benchmark** | **11.3.3.2** Simplify algebraic fractions. |

**Learning Objective:** By the end of the topic, students will be able to evaluate algebraic fractions and simplify rational expressions by factorizing, adding, subtracting, multiplying and dividing.

**Essential questions:**

- What are algebraic fractions?
- How are algebraic fractions factorised and simplified?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Appreciating the steps involved in simplifying algebraic fractions. |
| **Skills** | Add, subtract, multiply, divide and factorize algebraic fractions. |
| **Knowledge** | Algebraic fractions, restrictions to variable denominators, simple rational expressions. |
| **Mathematical Thinking** | Think about how to add, subtract, multiply, divide and factorize algebraic fractions. |

## Content Background

### Algebraic fractions - Review of Algebraic fractions

Algebraic fractions are fractions using a variable in the numerator or denominator, such as $\frac{x}{3} or \frac{2}{y}$ Because division by zero (0) is impossible, variables in the denominator have certain restrictions; the denominator can never be 0. Therefore in the fractions be aware of the following restrictions of conditions

$\frac{2}{x}$ x cannot equal zero $(x \neq 0)$

$\frac{5}{x-3}$ x cannot equal 3 $(x \neq 3)$

$\frac{9}{a-b}$ $(a-b)$ cannot equal zero $(a-b) \neq 0$), $a$ cannot equal $b$ $(a \neq b)$

$\frac{4}{ab}$ neither a nor b can equal 0 $(a \neq 0, b \neq 0)$

Algebraic expressions in fraction form are rational. Methods of adding, subtracting, multiplying and dividing fractions plus expanding and factorizing can be used to simplify rational expressions.

### Adding and subtracting rational expressions

Adding and subtracting algebraic fractions is a similar process to adding and subtracting normal fractions.
Fractions can only be added or subtracted when there is a common denominator and therefore, adding and subtracting of algebraic fractions uses the same methods.

***Example***

**1.** Write $\frac{2}{y} + \frac{1}{y}$ as a single fraction.

*Solution*
In this example, the denominators of the two fractions are the same, so the numerators can simply be added.

This gives $\frac{2}{y}+\frac{1}{y}=\frac{2+1}{y}=\frac{3}{y}$.This fraction cannot be simplified so $\frac{3}{y}$ is the final answer.

**2.** Write $\frac{5}{3t}-\frac{2}{7t}$ as a single fraction.

*Solution*

These fractions do not have a common denominator. There are also no common factors between the denominators, so the only way to create a common denominator is to multiply the two expressions together.

$$\frac{5}{3t}-\frac{2}{7t}$$

$$=\frac{7t(5)-3t(2)}{21t^2} \qquad \textit{Multiply denominator} \quad (3t)(7t)$$

$$=\frac{35t-6t}{21t^2}$$

$$=\frac{29^t}{21t^2}$$

$$=\frac{29}{21t}$$

Simplify

$$\frac{2}{y+4}+\frac{3}{y-2} \qquad \textit{Multiply denominators} \quad (y+4)(y-2)$$

$$\frac{2}{y+4}+\frac{3}{y-2}$$

$$=\frac{2(y-2)+3(y+4)}{(y+4)(y-2)} \qquad \textit{Divide the numerators into the denominator}$$

$$=\frac{5y+8}{(y+4)(y-2)} \qquad \textit{Expand and simply the numerator}$$

Expanding brackets means everything inside the bracket has to be multiplied by the term outside the bracket.
This fraction cannot be simplified, so this is the final answer.

## Multiplying and dividing rational expressions

Multiplying and dividing algebraic fractions works using the same methods in multiplying and dividing rational expressions.

### Multiplying rational expressions

To multiply two rational expressions, multiply the numerators together and the denominators together. (cancel if there is common factors)

The method to multiply fractions is to multiply the numerators together, multiply the denominators together and then cancel down if necessary.

***Example***

Simplify

$$\frac{5m^2}{6m}\text{x}\frac{3}{4m}$$

*Solution*

$$\frac{5m^2}{6m} \text{x} \frac{3}{4m} = \frac{5m^2 \text{ x } 3}{6m \text{ x } 4m}$$
$$= \frac{15m^2}{24m^2}$$
$$= \frac{5}{8}$$

The fraction now has no further common factors, so this is the final answer. This fraction could also be simplified by removing any common factors before multiplying together. To simplify the fraction in this way, first check all numerators and denominators and cancel out any common factors. There is a common factor of 3 and m between the numerators and denominators.

$$\frac{5m}{26m}x\frac{13}{4} = \frac{5}{2}x\frac{1}{4} = \frac{5x1}{2x4} = \frac{5}{8}$$

Note that the answer is the same regardless of whether common factors are divided first or last.

**Dividing rational expressions**

The method to divide fractions is to keep the first fraction the same, turn the divide sign into a multiply and turn the second fraction upside down. This is known as multiplying by the reciprocal. The expression then becomes multiplying two fractions, which is done using the method above.

***Example***

Simplify $\frac{7p}{2} \div \frac{2}{3p}$

*Solution* $\frac{7p}{2} \div \frac{2}{3p} = \frac{7p}{2} \text{x} \frac{3p}{2}$

$$= \frac{7p \text{ x } 3p}{2 \text{ x } 2}$$
$$= \frac{21p^2}{4}$$

There are no common factors so this is the final answer.

**Simplifying rational expressions**

Simplifying rational expressions or algebraic fractions works in the same way as simplifying normal fractions. A common factor must be found and divided throughout.

***Example***

Simplify $\frac{6m^2}{3m}$

*Solution*
To simplify this, look for the highest common factor of $6m^2$ and 3m. This is 3m
Take this common factor out of each part of the fraction.

This gives $\frac{6m^2 \div 3m}{3m \div 3m} = \frac{2m}{1} = 2m$. (This fraction cannot be simplified any further so this is the final answer.)

**Simplifying rational expressions with factorizing**

Some rational expressions do not have obvious common factors. In these cases, it is necessary to factorize either the numerator or the denominator, or both, to find common factors.

***Example***

**1.** Simplify $\frac{3t+6}{3t}$

*Solution*

The numerator of this fraction can be factorized as there is a common factor of 3. This gives $\frac{3(t+2)}{3t}$, Now, there is clearly a common factor of 3 between the numerator and denominator. Cancelling this through the fraction gives $\frac{t+2}{t}$. There are no more common factors in this expression. Note $t$ cannot be cancelled as there is no t term in the +2 in the numerator.

**2.** Simplify. $\frac{x^2+5x+4}{4x+16}$

In this example, both the numerator and the denominator can be factorized. The numerator is a quadratic with no common factors which will therefore factorize into two factors. The denominator does contain a common factor of 4 so will factorize into one bracket. Thus, $\frac{x^2+5x+4}{4x+16}=\frac{(x+4)(x+1)}{4(x+4)}=\frac{x+1}{4}$

**Unit: Equations and Inequalities** | **Topic: Quadratic Equations with not equal 1**

| | |
|---|---|
| **Benchmark** | **11.3.3.3** Factorize, solve and sketch quadratic expressions and equations. |

**Learning Objective:** By the end of the topic, students will be able to;

- solve Quadratic Equations by Factorization, completing the square and using the quadratic formula,
- identify the discriminant of quadratic equations and graph the functions, and
- graph quadratic functions that can be factorized and solve Problems involving quadratic equations/functions.

**Essential questions:**

- How can you identify a quadratic equation?
- How are quadratic equations with a>1 or a<1 factorised?

| **Key Concepts(ASK-MT)** | |
|---|---|
| **Attitudes/Values** | Explore various methods of solving quadratic equations where a ≠ 1. |
| **Skills** | Solve and graph quadratic equations. |
| **Knowledge** | Solving Quadratic equations by factorization, completing the squares and quadratic formula, identifying discriminant of quadratic equation, graph of a quadratic function. |
| **Mathematical Thinking** | Think about how to solve quadratic equations by factorization, completing the squares and using the quadratic formula, graph quadratic functions and solve quadratic functions related problems. |

## Content Background

**Quadratic Equations with the coefficient of $x^2$ not 1 ($ax^2 + bx + c = 0, a \neq 1$)**

A quadratic equation contains terms up to $x^2$. There are different methods of solving quadratics. All quadratic equations can be written in the form $ax^2 + bx + c = 0$ where $a, b$ and $c$ are numbers ($a$ cannot be equal to $0$ or 1, but $b$ and $c$ can be any number).

**1. Solving Quadratic Equations By Factorizations and grouping**

To solve quadratic equations of the form $ax^2 + bx + c = 0$ , where $a \neq 0$ or $a \neq 1$, the following steps could be used;

- If the coefficients have a numerical common factor, divide both side of the equation by that factor
- To factorize the quadratic expression such as $ax^2 + bx + c = 0$, find two numbers α and β whose product is ac and whose sum is b. write the middle term as $\alpha x + \beta x$, and factorize by grouping.

***Example:*** Solve $12x^2 = 23x - 5$

*Solution*

$12x^2 = 23x - 5$

$12x^2 - 23x + 5 = 0$ $12x^2 - 20x$ x $- 3x + 5 = 0$

(Find two numbers whose product is 60, *ie* (5×12) and whose sum is (-23) ie, (-20)+(-3)

Split the middle term

$(12x^2 - 20x) - (3x + 5) = 0$

$4x(3x - 5) - 1(3x - 5) = 0$

$(3x - 5)(4x - 1) = 0$

$3x - 5 = 0$ $\quad$ $4x - 1 = 0$

$3x = 5$ $\quad$ $4x = 1$

$x_1 = \frac{5}{3}$ $\quad$ $x_2 = \frac{1}{4}$

## 2. Completing the Squares

Another method of solving quadratic equations that works with both real and imaginary roots is called completing the square.

1. Put the equation into the form $ax^2 + bx = -c$ .
2. Make sure that a = 1 (if a ≠ 1, multiply through the equation by $\frac{1}{a}$ before proceeding).
3. Using the value of $b$ from this new equation, add $(\frac{b}{2})^2$ to both sides of the equation to form a perfect square on the left side of the equation.
4. Find the square root of both sides of the equation.
5. Solve the resulting equation.

Simplify

$$4x^2 - 2x - 5 = 0$$

$$4x^2 - 2x = 5 \qquad \textit{put the equation in the form } ax^2 - bx = -c$$

$$\frac{1}{4}(4x^2 - 2x) = \frac{1}{4}(5) \qquad \textit{making } a = 1, \textit{ if } a \neq 1$$

$$x^2 - \frac{1}{2}x = \frac{5}{4}$$

$$x^2 - \frac{1}{2}x + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 + \frac{5}{4} \qquad \textit{adding } \left(\frac{b}{2}\right)^2 \textit{ to both sides}$$

*write out the squared numbers in binomial form*

$$\left(x - \frac{1}{4}\right)^2 = \frac{21}{16}$$

$$\left(x - \frac{1}{4}\right) = \pm\sqrt{\frac{21}{16}}$$

$$x - \frac{1}{4} = \pm\frac{\sqrt{21}}{16}$$

$$x = \pm\frac{\sqrt{21}}{4} + \frac{1}{4}$$

$$\therefore x = \frac{1 \pm \sqrt{21}}{4}$$

**Quadratic Formula**

Many quadratic equations cannot be solved by factoring. This is generally true when the roots, or answers, are not rational numbers. A second method of solving quadratic equations involves the use of the following formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a$,$b$,and $c$ are taken from the quadratic equation written in its general form of $ax^2+bx+ c = 0$ where $a$ is the coefficient of $x^2$, $b$ is the coefficient of $x$, and $c$ is the numeral with no variable next to it (a.k.a., "the constant").

When using the quadratic formula, you should be aware of three possibilities. These three possibilities are distinguished by a part of the formula called the discriminant. The discriminant is the value under the radical sign, $b^2 - 4ac$. A quadratic equation with real numbers as coefficients can have the following:

1. Two different real roots if the discriminant $b^2 - 4ac$ is a positive number.
2. One real root if the discriminant $b^2 - 4ac$ is equal to 0.
3. No real root if the discriminant $b^2 - 4ac$ is a negative number.

## Graphing Quadratic Functions

Discriminant

The general form of a quadratic is " $y = ax^2 + bx + c$ ". For graphing, the leading coefficient "a" indicates how "fat" or how "skinny" the parabola will be. For | a | > 1 (such as $a$ = 3 or $a$ = –4), the parabola will be "skinny", because it grows more quickly (three times as fast or four times as fast, respectively, in the case of our sample values of a).

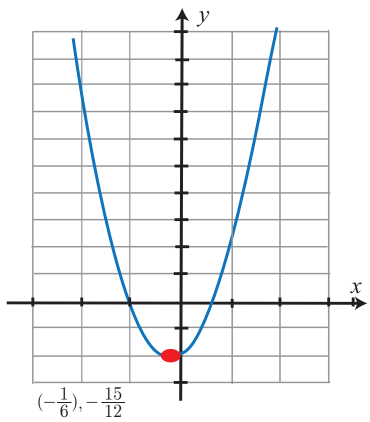
If the quadratic is written in the form $y = (x - h)^2 + k$, then the vertex is the point ($h,k$). This makes sense, if you think about it. The squared part is always positive (for a right-side-up parabola), unless it's zero. So you'll always have that fixed value $k$, and then you'll always be adding something to it to make y bigger, unless of course the squared part is zero. So the smallest y can possibly be is $y = k$, and this smallest value will happen when the squared part, $x – h$, equals zero. And the squared part is zero when $x – h = 0$, or when $x = h$. The same reasoning works, with k being the largest value and the squared part always subtracting from it, for upside-down parabolas.

(Note: The "$a$" in the vertex form "$y = a(x-h)^2+k$" of the quadratic is the same as the "$a$" in the common form of the quadratic equation, " $y = ax^2 + bx + c$ ".

Since the vertex is a useful point, and since you can "read off" the coordinates for the vertex from the vertex form of the quadratic, you can see where the vertex form of the quadratic can be helpful, especially if the vertex isn't one of your T-chart values. However, quadratics are not usually written in vertex form. You can complete the square to convert $ax^2 + bx + c$ to vertex form, but, for finding the vertex, it's simpler to just use a formula.

***Example*** $y = 3x^2 + x - 2$

| | | | | | |
|---|---|---|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 |
| y | 8 | 0 | 2 | 2 | 12 |




Write down the exact coordinates: "( –1/6, –25/12 )". But for graphing purposes, the decimal approximation of "(–0.2, –2.1)" may be more helpful, since it's easier to locate on the axes.

The only other consideration regarding the vertex is the "axis of symmetry". If you look at a parabola, you'll notice that you could draw a vertical line right up through the middle which would split the parabola into two mirrored halves. This vertical line, right through the vertex, is called the axis of symmetry. If you're asked for the axis, write down the line "$x = h$", where h is just the $x$-coordinate of the vertex. So in the example above, then the axis would be the vertical line $x = h = –1/6$.

Helpful note: If your quadratic's $x$ -intercepts happen to be nice neat numbers, a shortcut for finding the axis of symmetry is to note that this vertical line is always exactly between the two $x$-intercepts. So you can just average the two intercepts to get the location of the axis of symmetry and the $x$-coordinate of the vertex. However, if you have messy $x$-intercepts or if the quadratic doesn't actually cross the $x$-axis, then you'll need to use the formula to find the vertex.

| Unit: Equations and Inequalities | Topic: Solutions to Simultaneous Equations |
|---|---|
| **Benchmark** | **11.3.3.4** Simplify algebraic fractions to solve simultaneous equations using elimination, substitution and graphical methods. |

**Learning Objective:** By the end of the topic, students will be able to;

- solve equations using the substitution, elimination and the graphical methods, and
- calculate solutions of non-linear simultaneous equations.

**Essential questions:**

- What are simultaneous equations?
- What are the methods involved in solving simultaneous equations?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Appreciate solving simultaneous equation using the substitution, elimination and graphing method. |
| **Skills** | Solve simultaneous equations. |
| **Knowledge** | Substitution, elimination and graphing method to solve both linear and non-linear simultaneous equations. |
| **Mathematical Thinking** | Think about how to solve simultaneous equations using substitution, elimination or graphing methods. |

## Content Background

### Solutions to Simultaneous Equations

Simultaneous equations require algebraic skills to find the values of letters within two or more equations. They are called simultaneous equations because the equations are solved at the same time. Equations that have more than one unknown can have an infinite number of solutions.
*For example*, $2x + y = 10$ could be solved by:

$x = 1$ and $y = 8$
$x = 2$ and $y = 6$
$x = 3$ and $y = 4$

To be able to solve an equation like this, another equation needs to be used alongside it. In that way it is possible to find the only pair of values that will solve both equations at the same time. These are known as ***simultaneous equations.***

An example of this is:
$3x + y = 11$ and $2x + y = 8$

The unknowns of x and y have the same value in both equations. This fact can be used to help solve the two simultaneous equations at the same time and find the values of $x$ and $y$

### Solving simultaneous equations by elimination

The most common method for solving simultaneous equations is the elimination method which means one of the unknowns will be eliminated from each equation. The remaining unknown can then be calculated. This can be done if the **coefficient** of one of the variables is the same, regardless of sign.

***Example***

Solve the following simultaneous equations:

$3x + y = 11$

$2x + y = 8$

First, identify which unknown has the same coefficient. In this example this is the letter $y$, which has a coefficient of 1 in each equation.

Either add or subtract the two equations from each other to eliminate the letter. In this example the equations will need to be subtracted from each other as $-y = 0$ .

If the equations were added together, then $y + y = 2$, and so the letter $y$ would not be eliminated.

$$3x + y = 11$$

$$2x + y = 8$$

$$-\begin{cases} 3x + y = 11 \\ 2x + y = 8 \end{cases}$$

$$x + 0 = 3$$

$$\therefore x = 3$$

The value of $x$ can now be **substituted** into either of the equations to find the value of $y$.

Substitute $x = 3$

$$3x + y = 11$$

$$2(3) + y = 11$$

$$9 + y = 11$$

$$y = 11 - 9$$

$$y = 2$$

Check the answers by substituting both values into the other original equation. If the equation balances, then the answers are correct.

**Solving simultaneous equations with no common coefficients**

Some pairs of simultaneous equations may not have any common **coefficients.**

For example, the simultaneous equations $3a + 2b = 17$ and $4a - b = 30$ have no common coefficient as the coefficients of $a$ are 3 and 4, and the coefficients of bare 2 and -1.

Remember that a common coefficient is needed, regardless of sign. This means that -1 and 1 would be seen as a common coefficient.

In examples like this, one or both equations must be multiplied by a factor to create a common coefficient.

$3a + 2b = 17$

$4a - b = 30$

Create a common coefficient for either $a$ or $b$. In this case, making a common coefficient for $b$ will be easier, as $-b$ can be doubled to create $-2b$ , which will be a common coefficient throughout the equations.

Multiply the bottom equation to create a common coefficient of $2b$ .

$3a + 2b = 17$

$8a - 2b = 60$

These equations can now be used to find the values of $a$ and $b$.
Add or subtract the two sets of equations together to solve them.

**DASS stands for Different Add Same Subtract. Look at the signs in front of the common coefficient. If the signs are different, add the equations together. If the signs are the same, subtract them.**

The signs in front of the common coefficients are different, so the equations should be added together:

### Simultaneous equations with one linear and one quadratic

A **linear equation** does not contain any powers higher than 1. A **quadratic equation** contains a variable that has the highest power of 2.

For example: $y = x + 3$ is a linear equation and $y = x^2 + 3x$ is a quadratic equation.
Solving simultaneous equations with one linear and one quadratic
Algebraic skills of **substitution** and **factorizing** are required to solve these equations.
When solving simultaneous equations with a linear and quadratic equation, there will usually be two pairs of answers.

### Solving simultaneous equations graphically

**Simultaneous equations** can be solved **algebraically** or **graphically**. Knowledge of plotting **linear** and **quadratic graphs** is needed to solve equations graphically.

To find solutions from graphs, look for the point where the two graphs cross one another. This is the solution point

***Example***
Solve the simultaneous equations

$x + y = 5$ and $y = x + 1$ using graphs.

To solve this question, first construct a set of axes, making sure there is enough room to plot the two graphs.

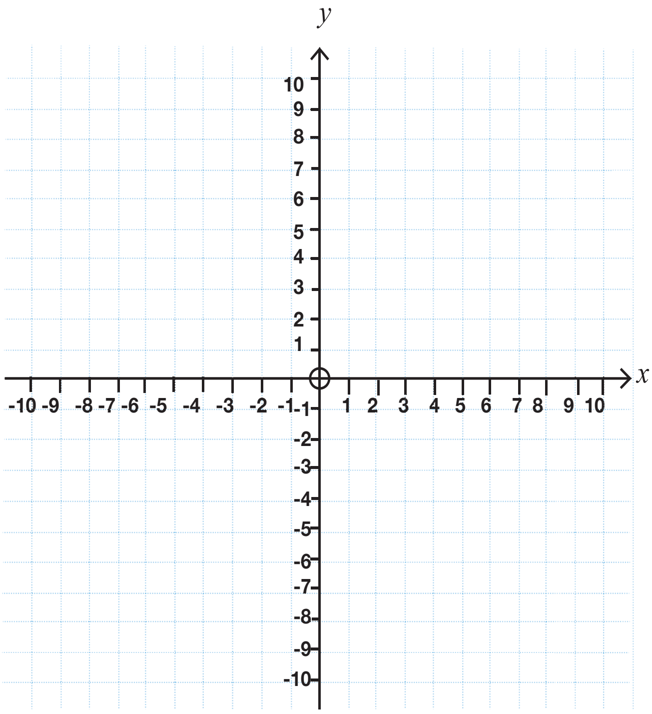
Now draw the graphs for $x + y = 5$ and . $y = x+1$
To draw these graphs, use a *table of values*:

$y = x + 1$

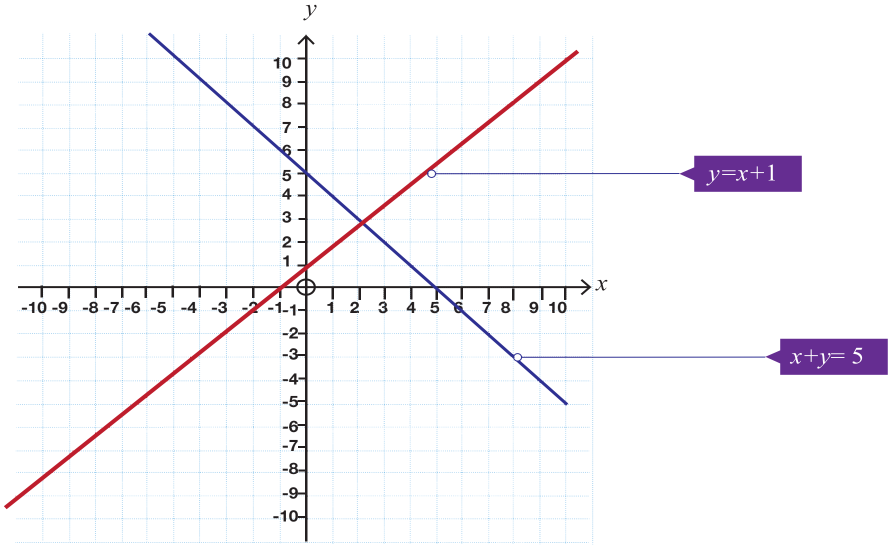
| x | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|---|
| y | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 5 |

$y = 5 - x$

| x | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|---|
| y | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 5 |



Plot these graphs onto the axes and label each graph.




**Solving linear and quadratic equations graphically**

Simultaneous equations that contain a quadratic and linear equation can also be solved graphically. As with solving algebraically, there will usually be two pairs of solutions.

***Example***

Solve the simultaneous equations $y = x^2$ and $y = x + 2$

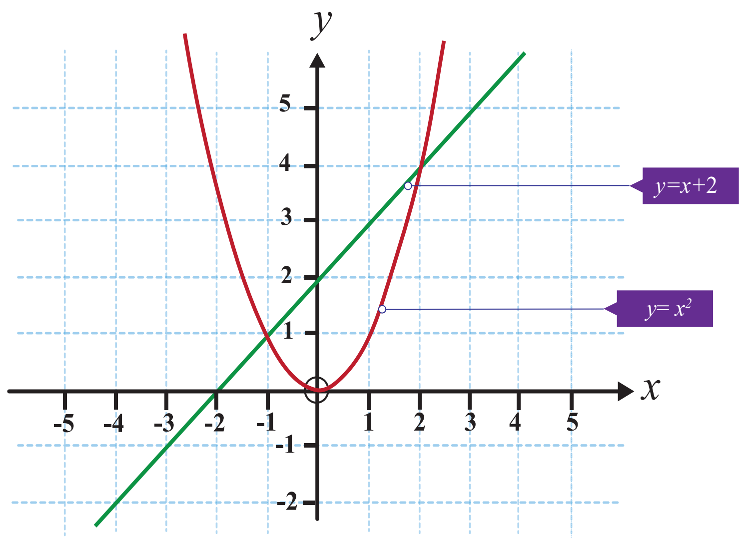
$y = x^2$

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 16 | 9 | 4 | 1 | 0 | 1 | 4 | 9 | 16 |

$y = x + 2$

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

Plot the graphs on the axes and look for the points of intersection.




The two points of intersection are at (2, 4) and (-1, 1) so $x = 2$ and $= 4$ , and $x =-1$ and $y = 1$.

| Unit: Equations and Inequalities | Topic: Inequalities |
|---|---|
| **Benchmark** | **11.3.3.5** Solve inequalities and plot on number line or plane. |

**Learning Objective:** By the end of the topic, students will be able to;

- solve inequalities,
- solve Inequalities involving absolute value, and
- graph of linear inequalities.

**Essential questions:**

- What are parallel lines?
- What are perpendicular lines?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Share their ideas on combining, solving and graphing inequalities. |
| **Skills** | Combine, solve and graph inequalities. |
| **Knowledge** | Combining, solving and graphing inequalities on a number line or number plane. |
| **Mathematical Thinking** | Think about how to solve inequalities and represent inequalities on the number line or number plane. |

## Content Background

### Inqualities

The process to solve inequalities is the same as the process to solve equations, which uses inverse operations to keep the equation or inequality balanced. But instead of using an equal sign, however, the inequality symbols are used throughout.

***Example***

*Solve* $3m + 2 > -4$.

The inequality will be solved when *m* is isolated on one side of the inequality. This can be done by using inverse operations at each stage of the process.

$$3m + 2 > -4$$
$$3m + 2 - 2 > -4 - 2$$
$$\frac{3m}{3} > \frac{-6}{3}$$
$$m > -2$$

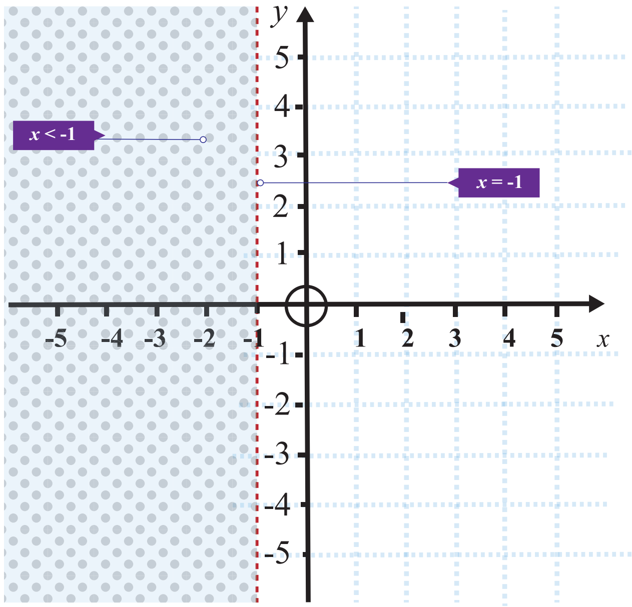
The final answer is > -2 ., which means *m* can be any value that is bigger than -2, not including -2 itself. If this answer was to be placed on a number line, an open circle would be needed at -2 with a line indicating the numbers that are greater than -2.


### Graphing Inequality

An inequality can be represented graphically as a region on one side of a line.

Inequalities that use < or > symbols are plotted with a dashed line to show that the line is not included in the region. Inequalities that use ≤ or ≥ symbols are plotted with a solid line to show that the line is included in the region.

For example, this graph shows the inequality < -1 . This can be seen as there is a dashed line at $x = -1$, and the region where the $x$ coordinates are less than -1 is shaded.




***Example***

Show the region satisfied by the inequality $-2 < x \leq 3$

Identify the two regions shown by the inequalities. These are $-2<x$ (or $x > -2$) and $x \leq 3$

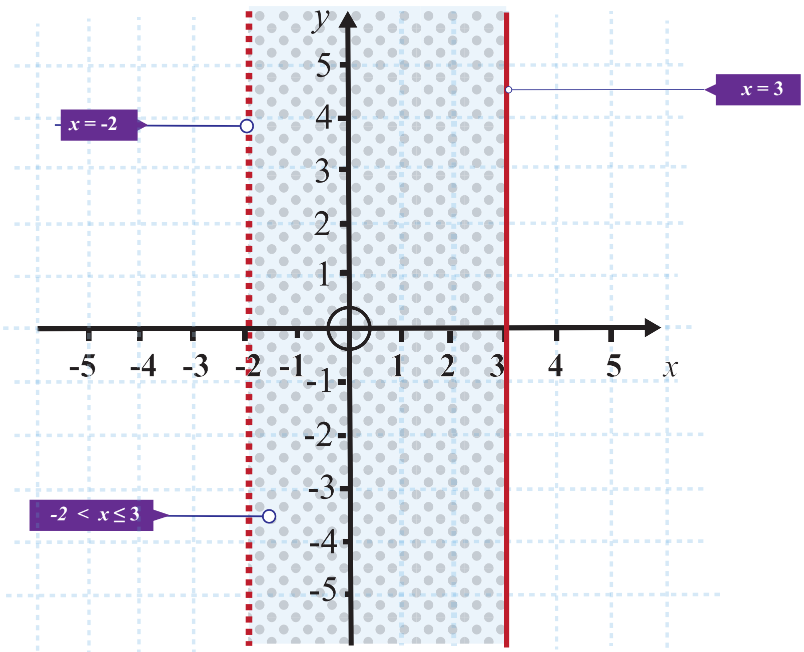
$x > -2$: draw a dotted line at $x = -2$

$x = -2$ is the graph made by coordinates points where $x$ is equal to -2, for example (-2, 5),(-2, 4), (-2, 3),(-2, 2) and so on.

$x \leq 3$ draw a solid line at $x = 3$

$x = 3$ is the graph made by coordinate points where $x$ is equal to 3, for example (3, -4),(3, -3),(3, -2), (3, -1) and so on.

$x$ is the values in between these two inequalities, so shade this region.



| Unit: Linear Function | Topic: Gradient of Straight Line |
|---|---|
| **Benchmark** | **11.3.3.6** Calculate the gradient of a straight line using intercept form and line through two points. |

**Learning Objective:** By the end of the topic, students will be able to;

- identify and explain what a gradient to a straight line is, and
- calculate gradients to straight lines.

**Essential questions:**

- How can a gradient be defined?
- Why is it important to calculate gradient to straight lines?
- How is gradient calculated?

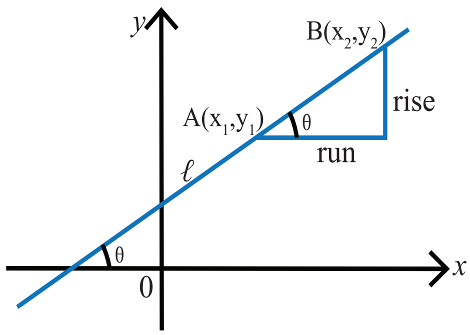
| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Confidently draw a straight line, show and discuss what the gradient is. |
| **Skills** | Calculate the gradients to straight lines and examine the equation of the line. |
| **Knowledge** | Gradients and equations of straight lines. |
| **Mathematical Thinking** | Think about how to evaluate the equations and gradients to any given straight lines. |

## Content Background

In coordinate geometry, points are ordered pairs $(x, y)$, lines are given by equations $ax + by + c = 0$. The most useful and most often met application of coordinate geometry is to solve geometrical problems.

### 1. Gradients and the angle of inclination

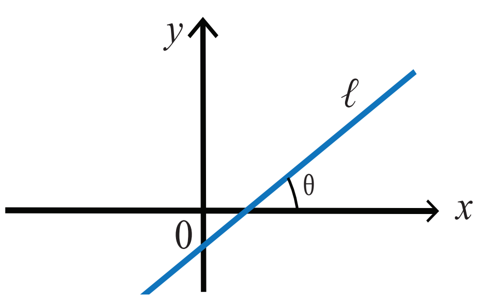
Suppose l is a line in the number plane not parallel to the $y$-axis or the $x$-axis.
Let $\theta$ be the angle between l and the positive $x$-axis, where $0^0 < \theta < 90^0$ or $90^0 < \theta < 180^0$



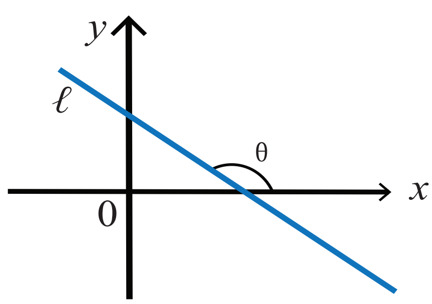

Suppose $A(x_1,y_1)$ and $B(x_2,y_2)$ are two points on $l$. Then, by definition, the gradient of the interval AB is

$$m = \frac{Rise}{Run} = \frac{y_2 - y_2}{x_2 - x_1}$$

From the diagram above, this is equal to tan$\theta$. So tan$\theta$ is the gradient of the interval $AB$.
Thus the gradient of any interval on the line is constant. Thus we may sensibly define the gradient of $l$ to be tan$\theta$.

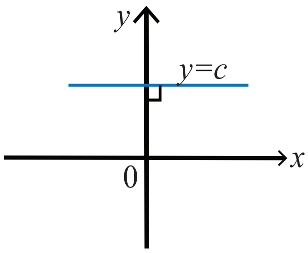



Positive Gradient
$0^0 < \theta < 90^0$

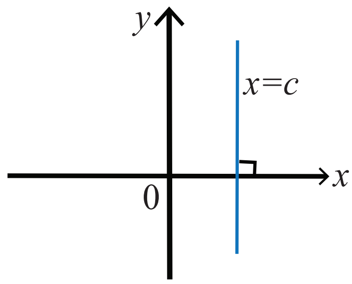



Negative Gradient
$90^0 < \theta < 180^0$

In the case where the line is parallel to the $x$ -axis, we say that the gradient is 0. In the case where the line is parallel to the $y$ -axis, we say that $\theta = 90^0$.

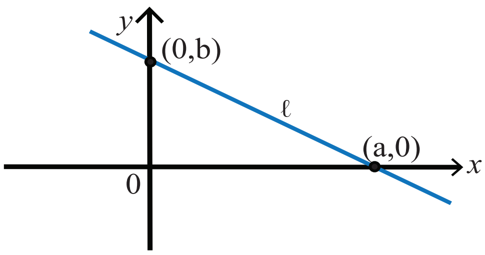



Zero Gradient
$\theta$ not defined




No Gradient
$\theta = 90^0$

**1. Intercepts and Equation of Lines**




All lines, except those parallel to the $x$-axis or the $y$-axis, meet both coordinate axes. Suppose that a line $l$ passes through ($a$, 0) and (0, $b$). Then $a$ is the $x$-intercept and b is the $y$-intercept of $l$ . The intercepts a and b can be positive, negative or zero. All lines through the origin have $a$ =0 and $b$ = 0.

**2. Equation of Lines**

One of the axioms of Euclidean geometry is that two points determine a line. In other words, there is a unique line through any two fixed points. This idea translates to coordinate geometry and, as we shall see, all points on the line through two points satisfy an equation of the form $ax + by + c = 0$, with $a$ and $b$ not both 0. Conversely, any 'linear equation' $ax + by + c = 0$ is the equation of a (straight) line. This is called the general form of the equation of a line.

**3. Point-Gradient Form**

Consider the line $l$ which passes through the point $(x_l, y_l)$ and has gradient $m$.
Let P$(x,y)$ be any point on $l$, except for $(x_1,y_1)$ Then,

$$m = \frac{y - y_1}{x - x_1}$$

and so $y - y_1 = m(x - x_1)$

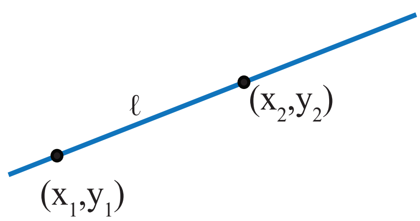
This equation is called the **point–gradient form** of the equation of the line $l$ .

Suppose that $(x_1, y_l) = (0, c)$. Then the equation is $y - c = mx$ or, equivalently, $y = mx + c$. This is often called the **gradient–intercept form** of the equation of the line.

**4. Line through two points**

To find the equation of the line through two given points $(x_1,y_1)$ and $(x_2,y_2)$ first find the gradient

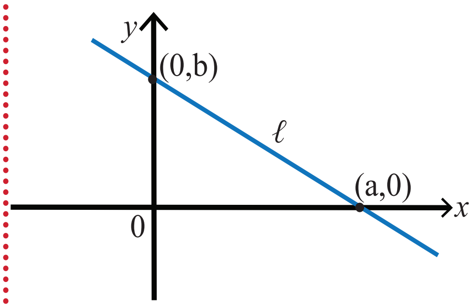
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$




and then use the point–gradient form $y - y_l = m(x - x_1)$

A special case is the line through ($a$,0) and (0,$b$), where $a,b \neq 0$.

In this case, the gradient is




$$m = \frac{b-0}{0-a} = -\frac{b}{a}$$

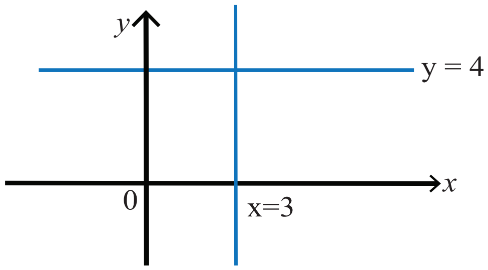
Thus the equation of the line is

$ay+bx=ab$

or, equivalently, $\frac{x}{a}+\frac{y}{b}=1$ which is easy to remember. This is called the **intercept form** of the equation of a line.

**Vertical and horizontal lines**

The equation of the vertical line through (3, 4) is $x = 3$. The equation of the horizontal line through (3, 4) is $y = 4$.



**Unit: Linear Function** | **Topic: Parallel and Perpendicular Lines**

| | |
|---|---|
| **Benchmark** | **11.3.3.7** Apply the concepts of parallel and perpendicular lines to find gradient and equation. |

**Learning Objective:** By the end of the topic, students will be able to;

- identify and explain parallel lines, and
- identify and discuss perpendicular lines.

**Essential questions:**

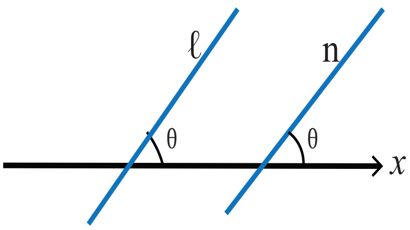
- What are parallel lines?
- What are perpendicular lines?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Confidently apply the properties of parallel and perpendicular lines to find gradient and equations. |
| **Skills** | Apply the properties of parallel and perpendicular lines to find gradient and equations. |
| **Knowledge** | Properties of parallel and perpendicular lines. |
| **Mathematical Thinking** | Think about how to use the concept of parallel and perpendicular lines to find gradient and equation. |

## Content Background

### Parallel Lines

If lines $l$ and $n$ are not vertical, then they are parallel if and only if they have the same gradient $m = \tan\theta$




Clearly, two horizontal lines are parallel. Also, any two vertical lines are parallel. If lines $l$ and n are not parallel, then their point of intersection can be found by solving the equations of the two lines simultaneously.

### Perpendicular lines

The line $y=m_1x + c_1$ is perpendicular to the line $y = m_2x + c_2$ if and only if $m_1 m_2 = -1$

***Example***

Find the equation of the line $l$ through (1, 3) perpendicular to the line $2x + 3y = 12$. Find the equation of the line through (4, 5) parallel to $l$ .

*Solution*

The gradient of the line $2x + 3y = 12$ is $-\frac{2}{3}$, so l has gradient $\frac{2}{3}$. The equation of $l$ is

$$y - 3 = \frac{3}{2}(x-1)$$
$$2y - 6 = 3x - 3$$
$$3x - 2y + 3 = 0$$

The other line has equation $y - 5 = \frac{3}{2}(x - 4)$ or, equivalently, $3x - 2y - 2 = 0$.

| Unit: Linear Function | Topic: Distance of a point from a Line |
|---|---|
| **Benchmark** | **11.3.3.8** Calculate the distance of a point from a line, distance between two points and intersection of two straight lines. |

**Learning Objective:** By the end of the topic, students will be able to;
- recognise and calculate the distance of a point from a line, and
- recognise and calculate the distance between two points and also the intersection of two straight lines.

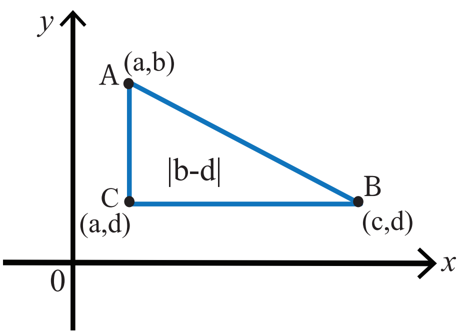
**Essential questions:**

- How is the distance between a point and a line calculated?
- How can the distance between two points be calculated?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Appreciating that distance from a point or between points are effective ways of describing different locations in real life. |
| **Skills** | Calculating distances between points, lines and intersecting lines. |
| **Knowledge** | Distances between points, lines and intersecting lines. |
| **Mathematical Thinking** | Think about how to calculate distance between two points and inter-section of two straigth lines. |

## Content Background

### The distance between two points




Distances in geometry are always positive, except when the points coincide.

The distance from A to B is the same as the distance from B to A. In order to derive the formula for the distance between two points in the plane, we consider two points A(a,b) and B(c,d). We can construct a right-angled triangle ABC, as shown in the following diagram, where the point C has coordinates (a,d).

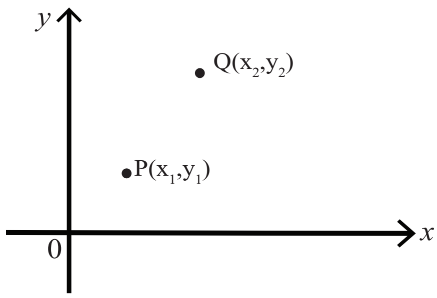
Now, using Pythagoras' theorem, we have

So $$AB^2 = |b-d|^2 - |a-c|^2 \\ = (a-c)^2 - (b-d)^2$$

A similar formula applies to three-dimensional space.

$$AB = \sqrt{(a-c)^2 - (b-d)^2}$$

### The distance formula




Suppose $P(x_1,y_1)$ and $Q(x_2,y_2)$ are two points in the number plane. Then

$PQ^2=(x_2-x_1)^2+(y_2-y_1)^2$

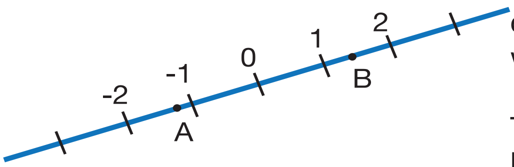
And also

$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

It is clear from the distance formula that:

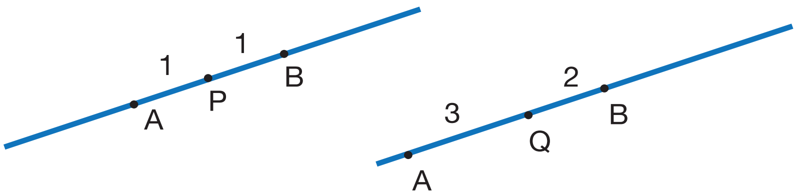
- $PQ = QP$
- $PQ = 0$ if and only if $P=Q$.

### Midpoints and division of an interval




Given two points A,B in the plane, it is clearly possible to create a number line on AB so as to label each point on AB with a (real) number.

There are (infinitely) many ways in which this can be done, but it turns out not to be particularly useful for geometrical purposes.

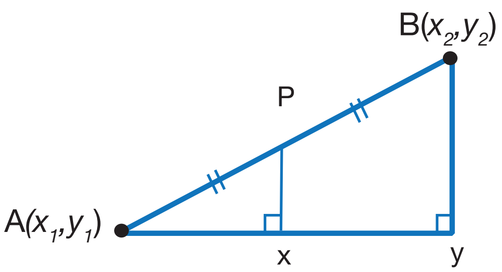



On the other hand, in the following diagrams, we can say that the point P divides the interval AB in the ratio 1:1 and the point Q divides the interval AB in the ratio 3:2.

### Midpoint of an interval

The midpoint of an interval AB is the point that divides AB in the ratio 1:1.

Assume that the point A has coordinates $(x_1,y_1)$and the point B has coordinates $(x_2,y_2)$ It is easy to see, using either congruence or similarity, that the midpoint P of AB is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$



**Unit: Functions and Graphs** | **Topic: Relations and Functions**

| | |
|---|---|
| **Benchmark** | **11.3.3.9** Define and describe a function using the vertical line test. |

**Learning Objective:** By the end of the topic, students will be able to;

- identify and define functions and its features,
- examine the behaviour of function displayed on a graph, and
- demonstrate and explain the vertical line test.

**Essential questions:**

- What is a function?
- What are the features to a function?
- How can the behaviours to a function be described when displayed on a graph?

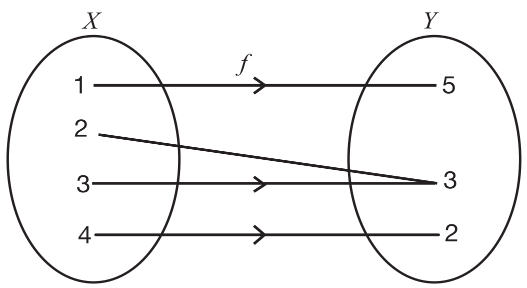
| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Discuss confidently the relationship of elements within a functions displayed on a diagram. |
| **Skills** | Analysis and solve Problem involving functions. |
| **Knowledge** | Functions and vertical line test. |
| **Mathematical Thinking** | Think about how to explain relations and functions and vertical line test using diagrams or graphs. |

## Content Background

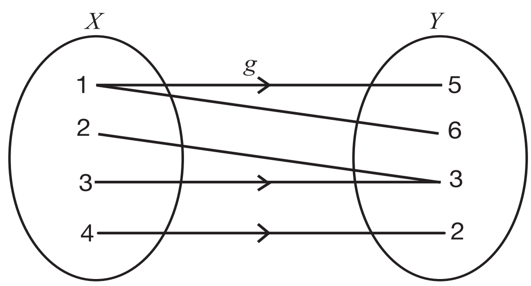
### What is a Function?

Definition of a function: A function $f$ from a set of elements $X$ to a set of elements $Y$ is a rule that assigns to each element $x$ in $X$ exactly one element $y$ in $Y$

One way to demonstrate the meaning of this definition is by using arrow diagrams.




$f: X \rightarrow Y$ is a function. Every element in $X$ has associated with it exactly one element of $Y$.




g: $X \rightarrow Y$ is not a function. The element 1 in set $X$ is assigned two elements, 5 and 6 in set $Y$.

A function can also be described as a set of ordered pairs $(x,y)$ such that for any $x$-value in the set, there is only one y-value. This means that there cannot be any repeated $x$ values with different $y$-values.

The examples above can be described by the following sets of ordered pairs.

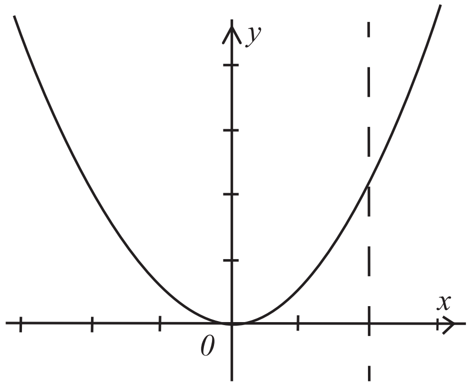
$F$ = {(1,5),(3,3),(2,3),(4,2)} is a function. $G$ = {(1,5),(4,2),(2,3),(3,3),(1,6)} is not a function.

The definition we have given is a general one. While in the examples we have used numbers as elements of $X$ and $Y$ , there is no reason why this must be so. However, in these notes we will only consider functions where $X$ and $Y$ are subsets of the real numbers.

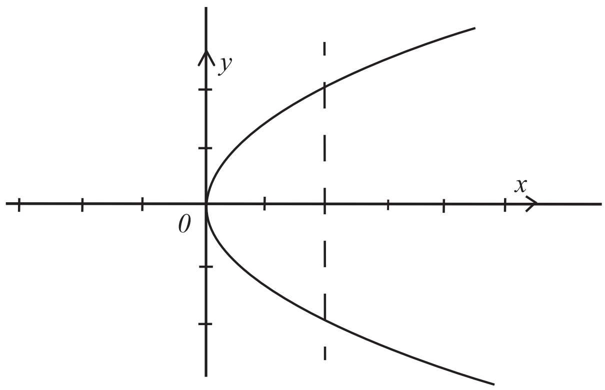
In this setting, we often describe a function using the rule, $y = f(x)$, and create a graph of that function by plotting the ordered pairs $(x,f(x))$ on the Cartesian plane. This graphical representation allows us to use a test to decide whether or not we have the graph of a function: The Vertical Line Test.

### The Vertical Line Test

The Vertical Line Test states that if it is not possible to draw a vertical line through a graph so that it cuts the graph in more than one point, then the graph is a function.




This is the graph of a function. All possible vertical lines will cut this graph only once.




This is not the graph of a function. The vertical line we have drawn cuts the graph twice.

| Unit: Functions and Graphs | Topic: Domain and Range of a Function |
|---|---|
| **Benchmark** | **11.3.3.10** Recognize and explain different functions and their features and calculate their range and domain. |

**Learning Objective:** By the end of the topic, students will be able to;

- discuss the domain and range of a function.

**Essential questions:**

- What is the definition to range in terms of a function?
- What is the definition to domain in terms of a function?
- How can the domain and range to a function be described?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Discuss with enthusiasm, the domain and range of a function. |
| **Skills** | Analysing and describe the domain and range of a function. |
| **Knowledge** | Definition of domain and range of a function. |
| **Mathematical Thinking** | Think about how to describe the domain and range of a given function. |

## Content Background

### Domain of a Function

For a function $f : X \quad Y$ the domain of f is the set $X$.

This also corresponds to the set of $x$-values when we describe a function as a set of ordered pairs $(x, y)$. If only the rule $y = f(x)$ is given, then the domain is taken to be the set of all real $x$ for which the function is defined. For example, $y = \sqrt{x}$ has domain; all real $x \geq 0$. This is sometimes referred to as the natural domain of the function.

### Range of a Function

For a function $f: X \rightarrow Y$ the range of $f$ is the set of $y$-values such that $y = f(x)$ for some $x$ in $X$. This corresponds to the set of $y$-values when we describe a function as a set of ordered pairs $(x,y)$. The function $y = \sqrt{x}$ has range; all real y ≥ 0.

***Example 1***

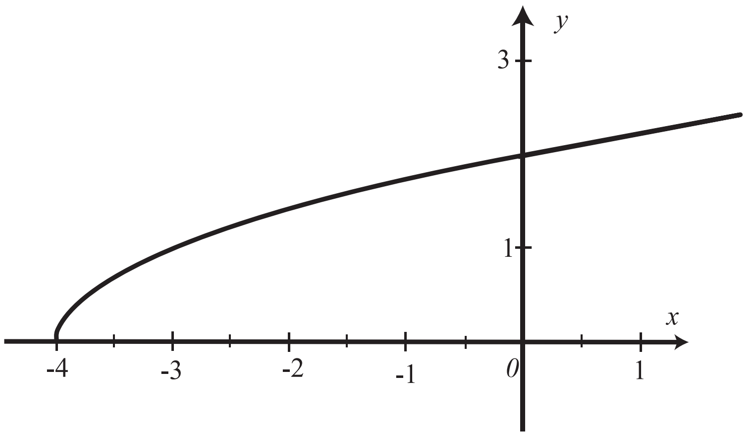
**a.** State the domain and range of $y = \sqrt{x+4}$

**b.** Sketch, showing significant features, the graph of $y = \sqrt{x+4}$

***Solution***

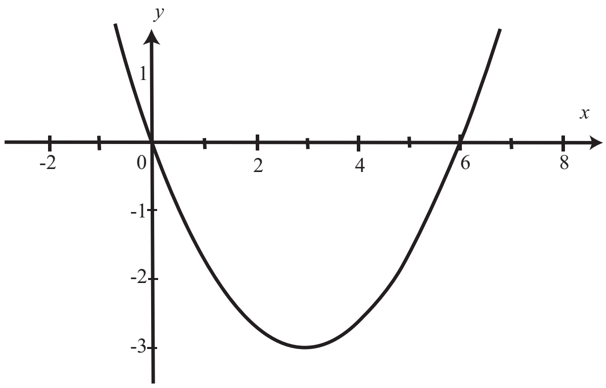
**a.** The domain of $y = \sqrt{x+4}$ is all real $x \geq -4$. We know that square root functions are only defined for positive numbers so we require that $x + 4 \geq 0$, *i.e.* $x \geq -4$. We also know that the square root functions are always positive so the range of $y = \sqrt{x+4}$ is all real y ≥ 0.

**b.** The graph of $y = \sqrt{x+4}$




***Example 2***

(i) State the equation of the parabola sketched below, which has vertex (3,-3).
(ii) Find the domain and range of this function.




**Solution**

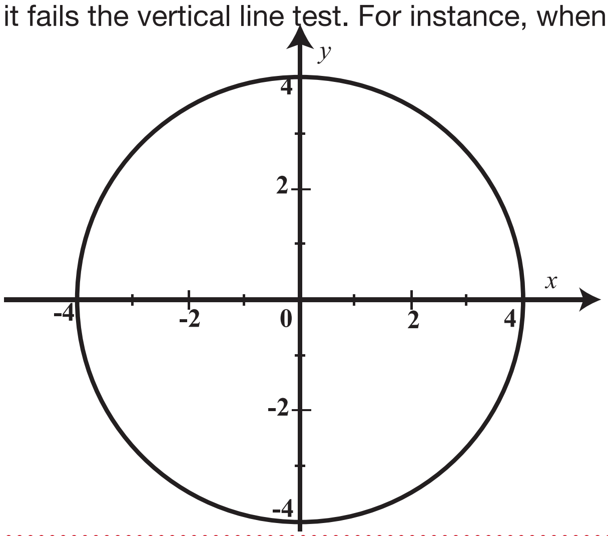
(i) The equation of the parabola is $y = \frac{x^2 - 6x}{3}$
(ii) The domain of this parabola is all real $x$. The range is all real $y \geq -3$

***Example 3***

Sketch $x^2+y^2=16$ and explain why it is not the graph of a function.

***Solution***

$x^2+y^2=16$ is not a function as it fails the vertical line test. For instance, when $x = 0$ $y = \pm 4$.



| Unit: Functions and Graphs | Topic: Absolute Value Functions |
|---|---|
| **Benchmark** | **11.3.3.11** Define absolute value of a number and graph absolute value functions. |

**Learning Objective:** By the end of the topic, students will be able to;

- identify absolute value functions and its features, and
- solve problems involving absolute functions and inequalities.

**Essential questions:**

- What is an Absolute Value Function?
- What is significant about Absolute Value Functions?
- How is an Absolute Value Function solved?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Show confident to explain and discuss features of an absolute value function. |
| **Skills** | Analysis and solve Problem involving absolute functions and inequalities. |
| **Knowledge** | Solve problems with absolute values and inequalities through graphs or simultaneously. |
| **Mathematical Thinking** | Think about how to obtain solutions to problems involving absolute values and inequalities. |

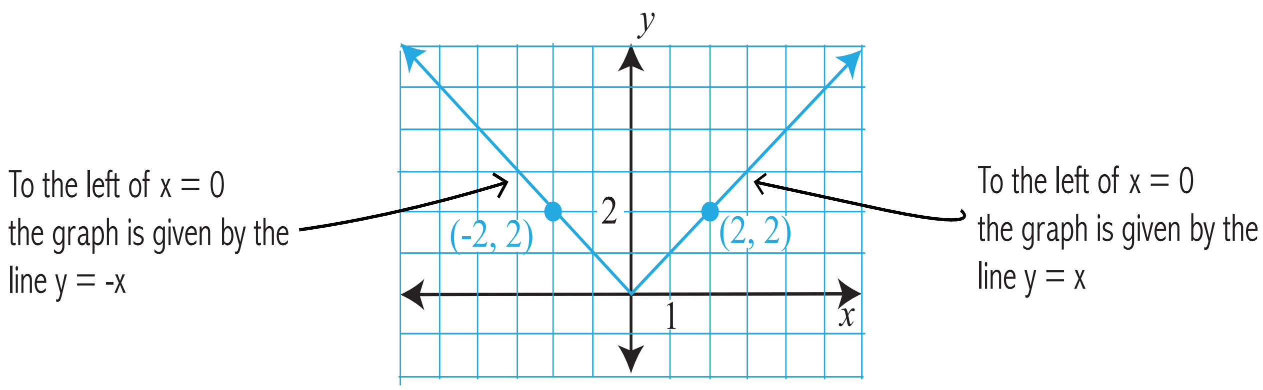
## Content Background

### 1. The Absolute Value

The absolute value of a number is its distance from 0 on the number line. Since distance is nonnegative, the absolute value of a number is always nonnegative. The symbol $|x|$ is used to represent he absolute value of a number $x$.

The absolute value of $x$ is defined by: $|x| = \begin{cases} x, & if\ x > 0 \\ 0, & if\ x = 0 \\ -x & , x < 0 \end{cases}$

The graph of this piece wise function consists of two rays, is V-shaped, and opens up. The corner point of the graph, called the vertex occurs at the origin.




Notice that the graph of $y = |x|$ is symmetric in the $y$-axis because for every point $(x, y)$ on the graph, the point $(-x, y)$ is also on the graph.

## 2. Graphing Absolute Value Functions

The graph of $y = a|x - h| + k$ has the following characteristics.

- The graph has vertex $(h, k)$ and is symmetric in the line $x = h$.
- The graph is V-shaped. It opens up if $a > 0$ and down if $a < 0$.
- The graph is wider than the graph of $y = |x|$ if $|a| < 1$.
- The graph is narrower than the graph of $y = |x|$ if $|a| > 1$.

To graph an absolute value function you may find it helpful to plot the vertex and one other point. Use symmetry to plot a third point and then complete the graph.

***Example:*** Graph $y = -|x + 2| + 3$.

*Solution*
To graph $y = -|x + 2| + 3$, plot the vertex at (-2, 3). Then plot another point on the graph, such as (-3, 2). Use symmetry to plot a third point, (-1, 2). Connect these three points with a V-shaped graph. Note that $a = -1 < 0$ and $|a| = 1$, so the graph opens down and is the same width as the graph of $y = |x|$.

## 3. Absolute Value Equations

1. Isolate the absolute value.
2. Identify what the isolated absolute value is set equal to..

- If the absolute value is set equal to zero, remove absolute value symbols & solve the equation to get one solution.
- If the absolute value is set equal to a negative number, there is no solution.
- If the absolute value is set equal to a positive number, set the argument (expression within the absolute value) equal to the number and set it equal to the opposite of the number, using an 'or' statement in between the two equations. Then solve each equation separately to get two solutions.

***Example***

**a.** $|3x + 12| + 7 = 7$
$|3x + 12| = 0$

*Solution*
Because this equals 0, there is ONE solution.
$3x + 12 = 0$
$x = -4$

**b.** $|3x - 7| + 7 = 2$
$|3x - 7| = -5$

Because this equals a negative number, there is NO solution.

**c.** $|3x - 7| + 7 = 9$
$|3x - 7| = 2$

Because this equals A positive number, there are TWO solutions

$3x - 7 = 2$, $3x = 9$, $x = 3$

$3x - 7 = -2$, $3x = 5$, $x = \frac{5}{3}$

## 4. Solving Absolute Value Inequalities

When absolute value inequalities are written to describe a set of values, like the inequalit $y|x - 5| \leq 4$ , it is sometimes desirable to express this set of values without the absolute value, either using inequalities, or using interval notation.

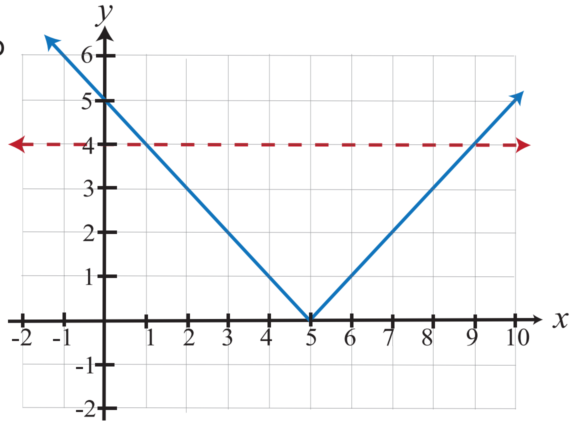
***Example:*** Solve $|x - 5| \leq 4$

*Solution:* We will need to know first where the corresponding equality is true. In this case, we first will find where $|x -5| \leq 4$. We do this because the absolute value is a nice friendly function with no breaks, so the only way the function values can switch from being less than 4 to being greater than 4 is by passing through where the values equal 4.

$|x -5| \leq 4$.
$x -5 = 4$ or $x -5 =-4$
$x = 9$ $x = 1$

To use a graph, we can sketch the function $f(x) =| x - 5|$ . To help us see where the outputs are 4, the line $g(x) = 4$ could also be sketched.

On the graph, we can see that indeed the output values of the absolute value are equal to 4 at $x = 1$ and $x = 9$. Based on the shape of the graph, we can determine the absolute value is less than or equal to 4 between these two points, when $1\leq x \leq 9$. In interval notation, this would be the interval [1,9].




***Example :*** Given the function $f(x) = -\frac{1}{2}|4x - 5| + 3$ , determine for what $x$ values the function values are negative.

*Solution*
We are trying to determine where $f(x) < 0$, which is when 4. We begin by isolating the absolute value:
$-\frac{1}{2}|4x - 5| < -3$ ……… when we multiply both sides by -2, it reverses the inequality

$\Rightarrow |4x - 5| > 6$

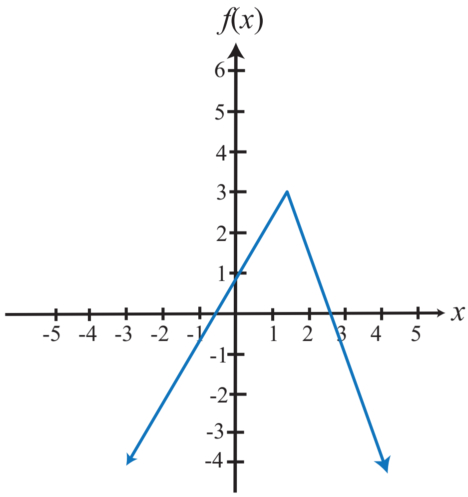
Next we solve for the equality $|4x - 5|>6$

$4x - 5 = 6$ or $4x - 5 = -6$

$x = \frac{11}{4}$ $x = -\frac{1}{4}$

We can sketch a graph of the function to determine on which intervals the original function values are negative. Notice that it is not even really important exactly what the graph looks like, as long as we know that it crosses the horizontal axis at $x = \frac{11}{4}$ and $x = -\frac{1}{4}$, and that the graph has been reflected vertically.
From the graph of the function, we can see the function values are negative to the left of the first horizontal intercept at $x = -\frac{1}{4}$ and negative to the right of the second intercept at $x = \frac{11}{4}$ This gives us the solution to the inequality: $x < -\frac{1}{4}$ *or* $x > \frac{11}{4}$

In interval notation, this would be $\left(-\infty, -\frac{1}{4}\right) \cup \left(\frac{11}{4}, \infty\right)$



| Unit: Functions and Graphs | Topic: Linear, Quadratic and Exponential equations and Inequalities |
|---|---|
| **Benchmark** | **11.3.3.12** Solve problems involving linear, quadratic, and exponential and inequality equation involving absolute values, individually or simultaneously using algebra or graphs. |

**Learning Objective:** By the end of the topic, students will be able to;

- describe linear, quadratic and exponential equations, and
- examine the inequalities that involve linear, quadratic and exponential equations

**Essential questions:**

- How can you differentiate between a linear, quadratic and exponential equation?
- How can inequalities involving linear, quadratic and exponential equations be solved?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Clearly explain or discuss features of an absolute value function. |
| **Skills** | Analysis and solve Problem involving absolute functions and inequalities. |
| **Knowledge** | Solve problems with absolute values and inequalities through graphs or simultaneously. |
| **Mathematical Thinking** | Think about how to obtain solutions to problems involving absolute values and inequalities. |

## Content Background

**Graphing and solving quadratic inequalities**

Four types of quadratic inequalities in two variables.

$y<ax^2+bx+c$ , $y>ax^2+bx+c$ , $y\leq ax^2+bx+c$, $y\geq ax^2+bx+c$

The graph of any such inequality consists of all solutions $(x,y)$ of the inequality. The steps used to graph a quadratic inequality are very much like those used to graph a linear inequality.

To graph one of the four types of quadratic inequalities shown above, follow these steps:

(i) Draw the parabola with equation $y = ax^2 + bx + c$. Make the parabola dashed for inequalities with < or > and solid for inequalities with ≤ or ≥.
(ii) Choose a point $(x,y)$ inside the parabola and check whether the point is a solution of the inequality.
(iii) If the point from Step 2 is a solution, shade the region inside the parabola. If it is not a solution, shade the region outside the parabola.

***Example***

Graph $y > x^2- 2x - 3$

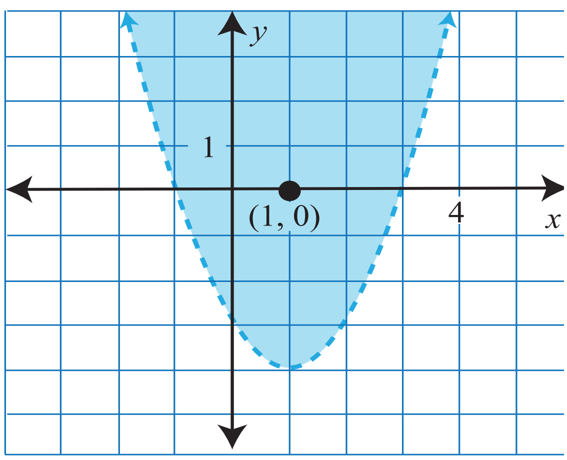
*Solution:* Follow the above three steps

(i) Graph $y =x^2 -2x -3$.
Since the inequality symbol is >, make the parabola dashed.

(ii) Test a point inside the parabola, such as (1,0).

$$y > x^2 - 2x - 3 \Leftrightarrow 0 > 1^2 - 2(1) - 3 \Leftrightarrow 0 > -4$$

So, (1,0) is a solution of the inequality.
Shade the region inside the parabola.



Graphing a system of quadratic inequalities is similar to graphing a system of linear inequalities. First graph each inequality in the system. Then identify the region in the coordinate plane common to all the graphs. This region is called the graph of the system.

***Example:*** Graph the system of quadratic inequalities.

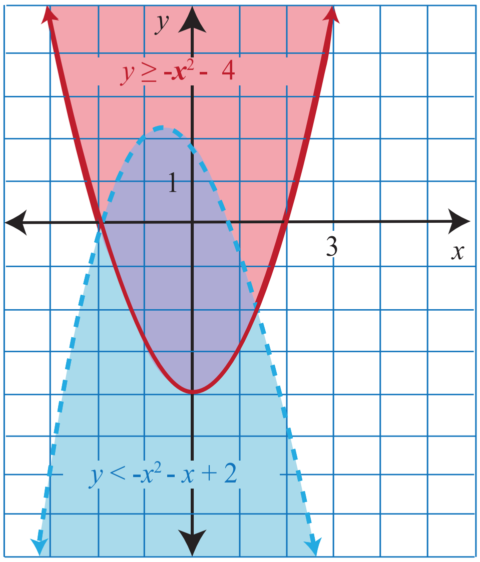
$y \geq x^2 - 4$ ..................... Inequality 1
$y < -x^2 - x + 2$ .....................Inequality 2

*Solution*

Graph the inequality $y \geq x^2 - 4$. The graph is the red region inside and including the parabola $y = x^2 - 4$.

Graph the inequality $y < -x^2 - x + 2$. The graph is the blue region inside (but not including) the parabola $y = -x^2 - x + 2$.

Identify the purple region where the two graphs overlap. This region is the graph of the system.



| Unit: Functions and Graphs | Topic: Sketches of Hyperbolic, exponential, logarithmic functions and asymptotes |
|---|---|
| **Benchmark** | **11.3.3.13** Derive and sketch graphs of hyperbolic, exponential, logarithmic functions and discuss their asymptotes and applications. |

**Learning Objective:** By the end of the topic, students will be able to;

- recognise and discuss hyperbolic, exponential and logarithmic functions and their features,
- sketch the graph to the hyperbolic, exponential and logarithmic functions, and
- explain the meaning to asymptotes.

**Essential questions:**

- What are the significances of exponential, hyperbolic and logarithmic functions?
- How are these functions solved and sketched?
- What is the definition to assyptotoes?

**Key Concepts(ASK-MT)**

| | |
|---|---|
| **Attitudes/Values** | Appreciate and enjoy discussing the features of hyperbolic, exponentials and logarithmic functions and their applications. |
| **Skills** | Sketching the graphs of hyperbolic, exponential and logarithmic functions. |
| **Knowledge** | Features of hyperbolic, exponential, logarithmic functions and the assyptotes. |
| **Mathematical Thinking** | Think about how to describe the features of hyperbolic, exponential, logarithmic functions and the assyptotes. |

## Content Background

### 1. Exponential Function

Exponential functions are one of the most important functions in mathematics. Exponential functions have many scientific applications, such as population growth and radioactive decay. Exponential function are also used in finance, so if you have a credit card, bank account, car loan, or home loan it is important to understand exponential functions and how they work.

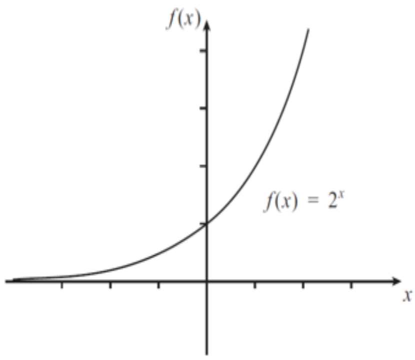
Exponential functions are function where the variable x is in the exponent. Some examples of exponential functions are $f(x)=2^x, f(x)=5^{x-2}$ $or$ $f(x)=9^{2x+1}$. In each of the three examples the variable $x$ is in the exponent, which makes each of the examples exponential functions.

An Exponential Function is a function of the form $f(x)=b^x$ $or$ $y=b^x$ where b is called the "base" and b is a positive real number other than 1(b > 0 and b ≠ 1). The domain of an exponential function is all real numbers, that is, $x$ can be any real number.

### 2. Graphing Exponential Functions

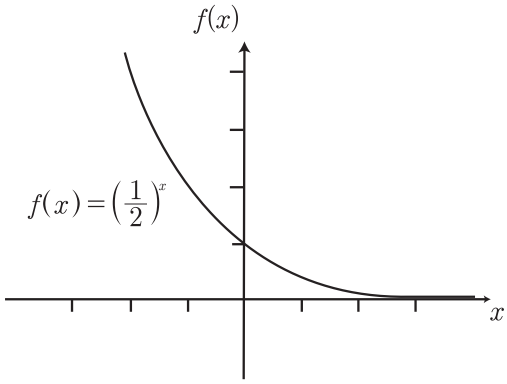
To begin graphing exponential functions we will start with two examples. We will graph the exponential functions $f(x)=2^x$ by making a table of values and plotting the points.

When creating a table of values, start with the numbers $x$ =-3,–2,–1,0,1,2 and 3 because it is important to have different types of numbers, some negative, some positive, and zero.



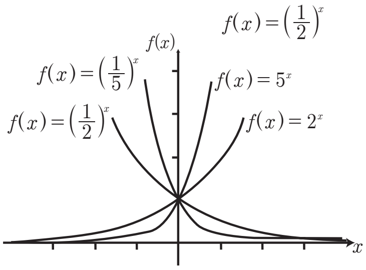

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x)=2^x$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 | 8 |

Notice that as the $x$-values get smaller, $x = -1, -2$, etc. the graph of the function gets closer and closer to the $x$-axis, but never touches the $x$-axis. This means that there is a horizontal asymptote at the $x$-axis or $y = 0$. A horizontal asymptote is a horizontal line that the graph gets closer and closer to.



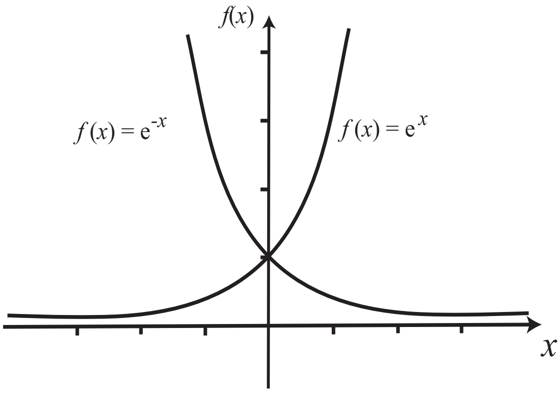

The example above demonstrates the general shape for graphs of functions of the form $f(x) = b^x$ when $b > 1$.
What happens if $0 < b < 1$? To examine this case, take another numerical example. Suppose that $b = \frac{1}{2}$ *i.e.* $f(x)=\left(\frac{1}{2}\right)^x$

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x)=(\frac{1}{2})^x$ | 8 | 4 | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |




Notice that these properties are the same as when $b > 1$. One interesting thing that you might have spotted is that $f(x)=\left(\frac{1}{2}\right)^x = 2^{-2}$ is a reflection of $f(x) = 2^x$ in the $f(x)$ axis and that $f(x)=\left(\frac{1}{5}\right)^x = 5^{-x}$ is a reflection of $f(x) = 5^x$ in the $f(x)$ axis.
In general, $f(x)=\left(\frac{1}{b}\right)^x = b^{-x}$ is a reflection of $f(x)=b^x$ in the $f(x)$ axis.

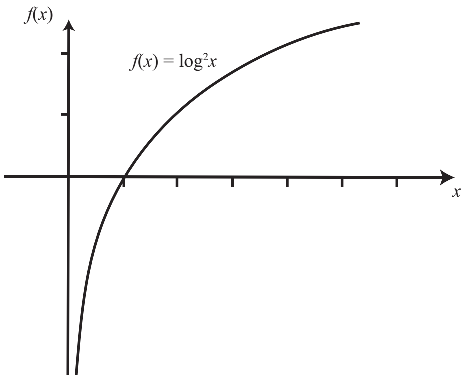
A particularly important example of an exponential function arises when $b = e$. You might recall that the number $e$ is approximately equal to 2.718. The function $f(x) = e^x$ is often called 'the' exponential function. Since $e > 1$ and $1/e < 1$, we can sketch the graphs of the exponential functions $f(x) = e^x$ and $f(x)=e^{-x}=\left(\frac{1}{e}\right)^x$.




## 2. Logarithm Functions

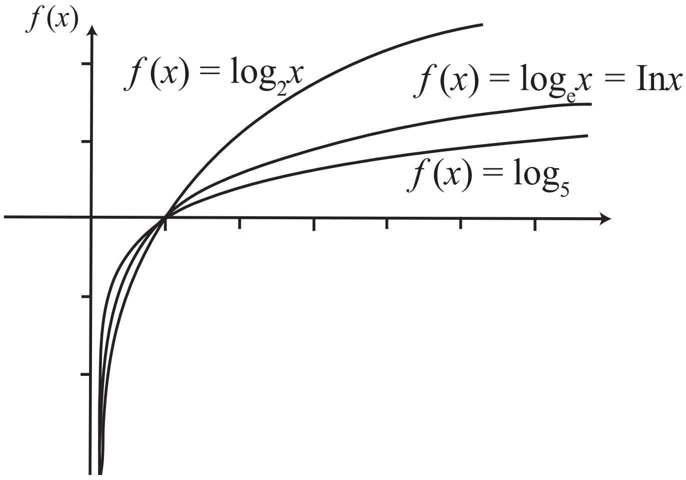
We shall now look at logarithm functions. These are functions of the form $f(x) = \log_a x$ where $a > 0$.

Let examine the case $f(x) = \log_2 x$ means $2^{f(x)} = x$. An important point to note here is that, regardless of the argument, $2^{f(x)} > 0$. So we shall consider only positive arguments.

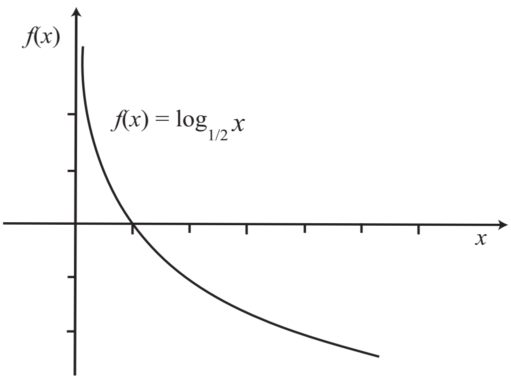



| $x$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 1 | 2 | 4 |
|---|---|---|---|---|---|
| $f(x)=\log_2 x$ | -2 | -1 | 0 | 1 | 2 |

This example demonstrates the general shape for graphs of functions of the form $f(x) = \log_2 x$ when a > 1.
What is the effect of varying a? We can see by looking at sketches of a few graphs of similar functions. For the special case where $a = e$, we often write ln $x$ instead of loge $x$.



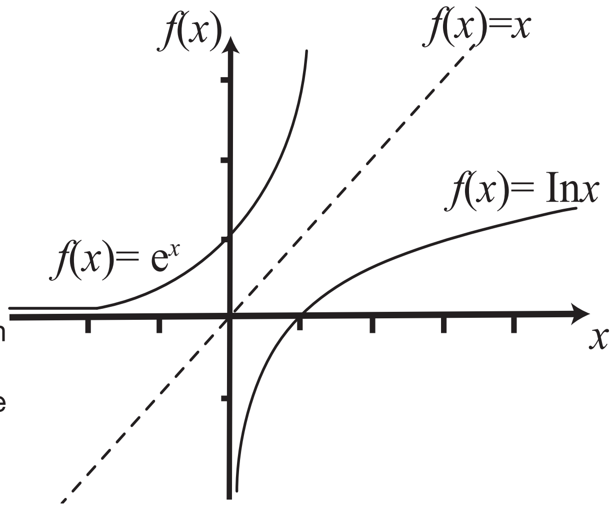

What happens if 0 < a < 1? To examine this case, take another numerical example. Suppose that a =1/2 .Then f(x) = $\log_{1/2} x$ means $(1/2)^{f(x)} = x$.




This example demonstrates the general shape for graphs of functions of the form $f(x) = \log_{1/2} x$ when 0 < a < 1.

**3. The Relationship between Exponential Functions and Logarithm Functions**

We can see the relationship between the exponential function $f(x) = e^x$ and the logarithm function $f(x) = \ln x$ by looking at their graphs.

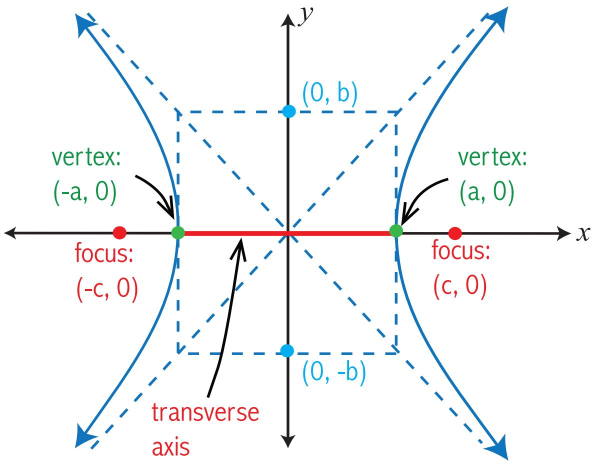



You can see straight away that the logarithm function is a reflection of the exponential function in the line represented by $f(x) = x$. In other words, the axes have been swapped: $x$ becomes $f(x)$, and $f(x)$ becomes $x$.

Key Point: The exponential function $f(x) = e^x$ is the inverse of the logarithm function $f(x) = \ln x$

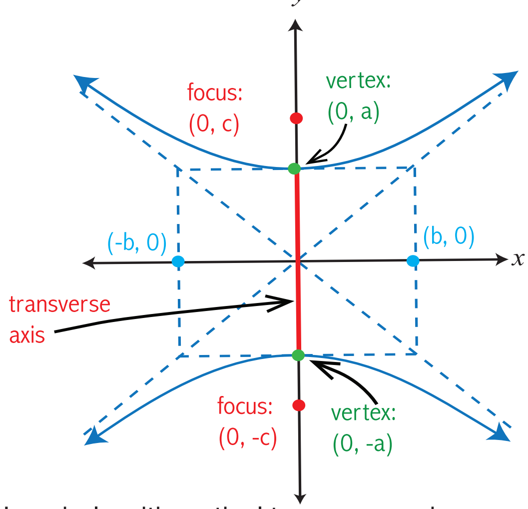
**Hyperbola**

A Hyperbola is the set of all points P such that the difference of the distances from P to two fixed points, called the foci, is constant. The line through the foci intersects the hyperbola at two points, the vertices. The line segment joining the vertices is the transverse axis and its midpoint is the centre of the hyperbola. A hyperbola has two branches and two asymptotes. The asymptotes contain the diagonals of a rectangle centered at the hyperbola's center, as shown below.




Hyperbola with horizontal transverse axis

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$




Hyperbola with vertical transverse axis

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

**Characteristics of a Hyperbola (Center At Origin)**

The standard form of the equation of a hyperbola with center at (0, 0) is as follows.

| *EQUATION* | *TRANSVERSE AXIS* | *ASYMPTOTES* | *VERTICES* |
|---|---|---|---|
| $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ | Horizontal | $y=\pm\frac{b}{a}x$ | $(\pm a, 0)$ |
| $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ | Vertical | $y=\pm\frac{a}{b}x$ | $(0 \pm a)$ |
| The foci of the hyperbola lie on the transverse axis, c units from the center where $c^2=a^2+b^2$ | | | |

**Graphing an Equation of a Hyperbola**

***Example:*** Draw the hyperbola given by $4x^2 - 9y^2 = 36$

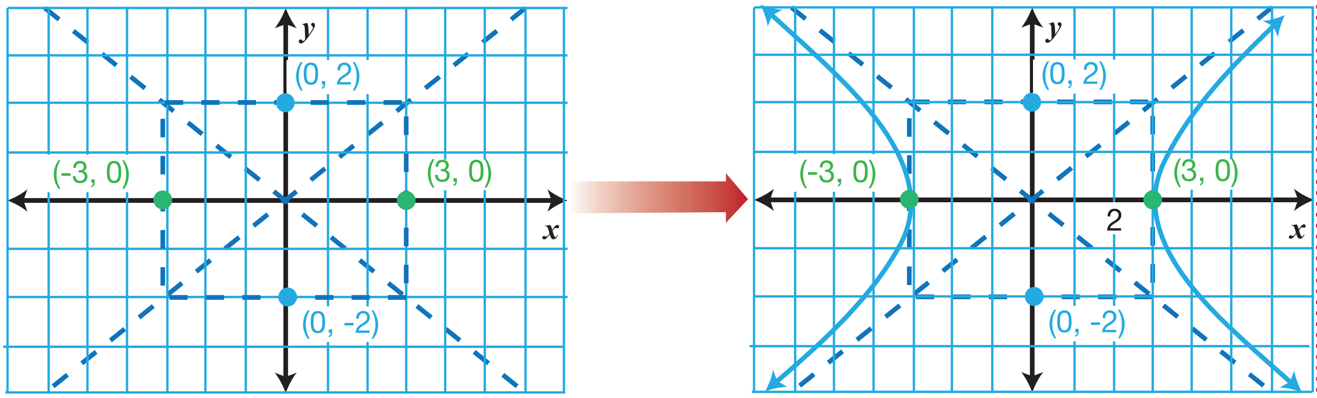
*Solution*
First rewrite the equation in standard form.

$4x^2 - 9y^2 = 36$ …….Write original equation

$\frac{4x^2}{36} - \frac{9y^2}{36} = \frac{36}{36}$ …….Divide each side by 36

$\frac{x^2}{9} - \frac{y^2}{4} = 1$ …… simplifying

Note from the equation that $a^2 = 9$ and $b^2 = 4$, so $a = 3$ and $b = 2$. Because the $x^2$-term is positive, the transverse axis is horizontal and the vertices are at (-3,0) and (3,0). To draw the hyperbola, first draw a rectangle that is centered at the origin, $2a = 6$ units wide and $2b = 4$ units high. Then show the asymptotes by drawing the lines that pass through opposite corners of the rectangle. Finally, draw the hyperbola.



| Unit: Functions and Graphs | Topic: Circles on Cartesian Plane |
|---|---|
| **Benchmark** | **11.3.3.14** Derive and sketch graphs of circles. |

**Learning Objective:** By the end of the topic, students will be able to;

- identify and discuss the features of a circle, and
- sketch circles on cartesian planes.

**Essential questions:**

- What are the main features to a circle?
- What is a unit circle?
- What is the general equation to a circle?

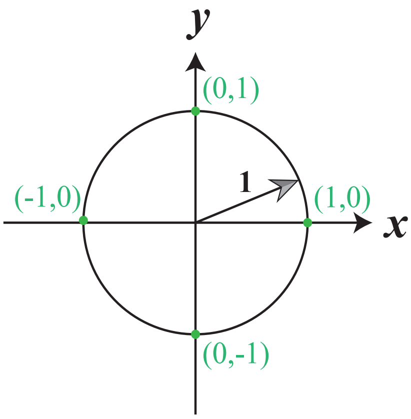
| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Confidently and carefully sketch circles onto the cartesian planes and discuss the features. |
| **Skills** | Sketching circles onto the cartesian planes. |
| **Knowledge** | Sketches of circles and their equations. |
| **Mathematical Thinking** | Think about how to sketch circles onto cartesian planes given the equations. |

## Content Background

A circle is the set of points in a plane that are a fixed distance, called the radius, from a fixed point, and called the center. Because all of the points on a circle are the same distance from the center of the circle, you can use the Distance Formula to find the equation of a circle.

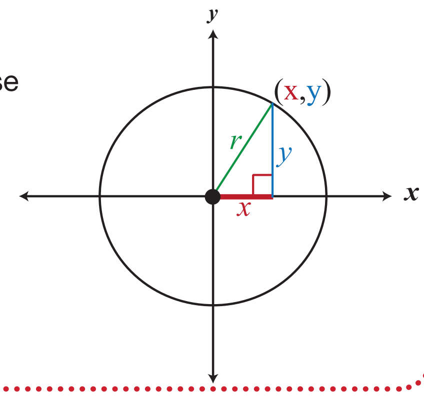
### 1. What is a 'Unit Circle?'

A "unit" circle has a radius of 1. In other words, the distance from the center of the circle to any part of the edge (circumference) is always 1. The unit of measurement doesn't really matter, because the most important thing about the unit circle is that it makes many equations and calculations much simpler.




It also serves as a useful basis for looking at the definitions of angles. Imagine that the center of the circle sits at the center of a coordinate system with an $x$-axis running horizontal and a $y$-axis running vertically. The circle crosses the $x$-axis at $x = 1$, $y = 0$. Scientists and mathematicians define the angle from that point moving in a counter-clockwise direction. So the point $x = 1$, $y = 0$ on the circle is at an angle of 0°.

### 2. Circles with Centre the Origin




In the circle below, let point $(x,y)$ represent any point on the circle whose centre is at the origin. Let $r$ represent the radius of the circle.

In the right triangle

$r$ = length of the hypotenuse

$x$ = length of leg

$y$ = length of a leg

By Pythagorean Theorem, you can write

$x^2+y^2=r^2$

This is an equation of a circle with centre at the origin.

In general: Two quantities are needed to find the equation of a circle: (1) Centre and (2) Radius.
If the centre is is (0,0), the equation of the circle will be in the form $x^2 + y^2 = r^2$.

***Example:***
Find the equation of the following circles, each of the centre (0, 0).

**(i)** Which has the radius $\sqrt{13}$ **(ii)** which contains the point (4,-1)

*Solution*
Centre (0,0) is of the form $x^2 + y^2 = r^2$

$$x^2 + y^2 = (\sqrt{13})^2$$

$$x^2 + y^2 = 13$$

Thus the equation of the circle is $x^2 + y^2 = 13$

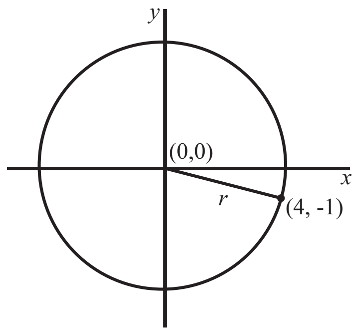
Centre (0,0) is of the form $x^2+y^2=r^2$

$(4)^2 + (-1)^2 = r^2$

$16 + 1 = r^2$

$17 = r^2$

Thus the equation of the circle is $x^2 + y^2 = 17$




Alternatively, the radius is the distance from (0,0) to (4,-1).
Using the distance formula, the radius

$$= \sqrt{(4-0)^2 - (-1-0)^2}$$

$$\sqrt{16+1}$$

$$\sqrt{17}$$

Thus the equation of the circle is $x^2 + y^2 = 17$ $\quad (\sqrt{17})^2 = 17)$

| Unit: Functions and Graphs | Topic: Equation of Circles in Standard Form |
|---|---|
| **Benchmark** | **11.3.3.15** Covert and write equation of circles in standard form $(x-h)^2+(x-k)^2=r^2$. |

**Learning Objective:** By the end of the topic, students will be able to;

- state the conditions under which the general equation of second degree in two variables represents a circle, and
- derive and find the centre and radius of a circle whose equation is given in general form.

**Essential questions:**

- Can we find a mathematical expression for a given circle?
- How can you sketch a circle onto the cartesian plane?
- How does the equation of a circle look like?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Confidently express equations to circles in the standard forms. |
| **Skills** | Converting equations of circles to the standard forms. |
| **Knowledge** | Standard form for equations to circles. |
| **Mathematical Thinking** | Think about how to derive equations of circles given certain points. |

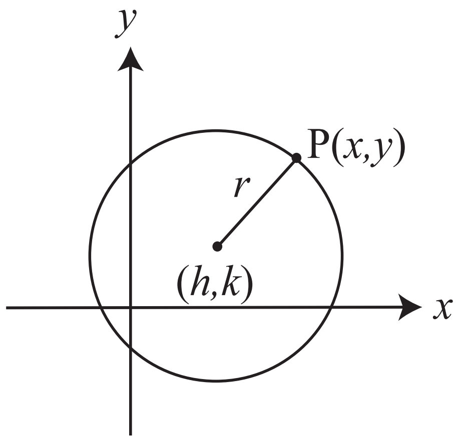
## Content Background

### 1. Circle

A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point in the same plane remains constant. The fixed point is called the centre of the circle and the constant distance is called the radius of the circle.

An equation of a circle is an equation in x and y which is satisfied by the coordinates of a point if and only if the point is on the circle.

### 2. Standard Form of the Equation of a Circle




Let $P(x, y)$ be a point in a plane which moves so that it is always a constant distance, called the radius r, from the fixed point $(h,k)$, called the centre of the circle. Then by distance formula

$(x-h)^2+(y-k)^2=r^2$ ......................(1)

Equation (1) is called standard form of the circle, with centre $(h,k)$ and radius $r$.
If the centre is at the origin (0, 0), equation (1) reduces to

$(x)^2+(y)^2=r^2$ ......................(2)

Also if the centre is at the origin (0,0) and radius is 1 (one), then the equation (1) reduces to the unit circle i.e;
$(x)^2+(y)^2=1$
Note that any equation equivalent to equation (1) is also an equation of the circle. We may reduce the equation (1) to the form.

$x^2+y^2-2ky+(h^2+k^2-r^2)=0$ ......................(3)

we observe that

(i) The equation (3) is second degree in $x$ and $y$.
(ii) The coefficients of $x^2$ and $y^2$ are equal.
(iii) There is no product term $xy$.

***Example:*** (i) Find the equation of the circle with centre at (–2, 3) and radius 6.
*Solution:* From the standard form

$$(x-h)^2+(y-k)^2=r^2 \text{ ,here } (h,k)=(-2,3), r=6$$
$$(x+2)^2+(y-3)^2=36$$

(i) State the coordinates of the centre and give the radius for the circle

$$x^2+y^2+4x-6y-3=0$$
$$x^2+4x+y^2-6y=3$$
$$(x^2+4x+4)+(y^2-6y+9)=3+4+9$$
$$(x+2)^2+(y-3)^2=16$$

complete the squares write as standard equation therefore, the centre is at (-2,3) and the radius is 4

# Strand 4: Statistics and Probability

**Content Standard:** Students will be able to investigate how to interpret data using methods of exploratory data analysis, develop and evaluate inferences, predictions and arguments that are based on data and understand and apply basic notions of chance and probability.

| Unit | Benchmark | Topic | Lesson Title |
|---|---|---|---|
| **Data Analysis** | **11.4.4.1** Explore data by constructing stem and leaf plots, frequency polygons and histogram, cumulative frequency distribution. | Collecting and Organizing Data | Selecting sample data |
| | | | Frequency Tables for ungrouped Data |
| | | | Frequency Tables for grouped Data |
| | | | Frequency polygon |
| | **11.4.4.2** Interpret left and right skewed distribution, interpolate and extrapolate data, calculate percentile, quartile and inter-quartile ranges. | Measure of central tendency | Review of Measure of central tendency |
| | | Measure of dispersion | Review of Measure of dispersion |
| | | | Coefficient of variation |
| | **11.4.4.3** Calculate mean, variance and standard deviation. | Shape of distributions | Normal and skewed distributions |
| | | | Calculate the skewness |
| **Sets and Probability** | **11.4.4.4** Define sets and elements, equality of sets, subsets and Venn diagram. | Sets and Elements | Definition and Notations of Sets and Elements |
| | **11.4.4.5** Identify and explain various sets and represent sets on a Venn diagram. | Subsets and Venn Diagrams | Field laws of Sets on Intersection and Union |
| | | | Subsets and Equality of sets |
| | **11.4.4.6** Apply universal sets and complement of a set, discussing some algebraic laws, and solve related problems. | Universal and complimentary sets | The Universal Sets involving algebraic laws |
| | | | The Complementary Sets involving algebraic laws |
| | | | Problem Solving with universal and complementary sets. |
| | **11.4.4.7** Explore, analyse, and interpret data by calculating probabilities of events, from experimental data and life data tables, compound events of two-dimensional grids and tree diagrams. | Theoretical and Experimental Probabilities | Theoretical Probability |
| | | | Experimental probability |
| | | Mutually and non-mutually exclusive events | Mutually Exclusive events |
| | | | Non-mutually Exclusive Events |
| | | Independent and dependent Events and Conditional Probability | Independent Events |
| | | | Dependent Events |
| | **11.4.4.8** Explore, analyse and interpret data by calculating probabilities of mutually and non-mutually from the Venn diagrams and calculate using the addition and multiplication rules of probability. | | Conditional Probability |
| | | | Addition Rule for Probability ('OR') |
| | | | Multiplication Rule for Probability ('AND') |

**Unit: Data Analysis** | **Topic: Collecting and Organizing Data**

| | |
|---|---|
| **Benchmark** | **11.4.4.1** Explore data by constructing stem and leaf plots, frequency polygons and histogram, cumulative frequency distribution. |

**Learning Objective:** By the end of the topic, students will be able to;

- identify types of data and variables collected,
- categorize variables at various measurement levels, and
- organize raw data into tables and present the data in various graphical forms.

**Essential questions:**

- Why is there collection and organising of data?
- How is data collected and organised?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Display, and express confidence in collection and organizing data techniques. |
| **Skills** | Identify, organize types of data and variables, categorize variables into various levels of measurement and present data into tables and graphs. |
| **Knowledge** | Presentation of Qualitative and Quantitative data in a bar or pie chart a histogram, frequency polygon, stem and leaf plot. |
| **Mathematical Thinking** | Think about how data is measured when collecting and organised in different ways. |

## Content Background

### Stem and Leaf Diagrams

Stem and leaf diagrams, or stemplots, are used to represent raw data, that is, individual observations, without loss of information. The ‘leaves’ in the diagram are actually the last digits of the values (observations) while the ‘stems’ are the remaining part of the values. For example, the value 117 would be split as ‘11’, the stem, and ‘7’, the leaf. By splitting all the values and distributing them appropriately, we form a stemplot. The example below would be a better illustration of the above explanation.

***Example***

The following are the marks (out of 100) obtained by 20 students in an assignment:

| | | | | |
|---|---|---|---|---|
| 84 | 17 | 38 | 45 | 47 |
| 53 | 76 | 54 | 75 | 22 |
| 66 | 65 | 55 | 54 | 51 |
| 44 | 39 | 19 | 54 | 72 |

In the first instance, the data is classified in the order that it appears on a stemplot .The leaves are then arranged in ascending order.

| Stem | Leaf |
|---|---|
| 1 | 7 9 |
| 2 | 2 |
| 3 | 8 9 |
| 4 | 5 7 4 |
| 5 | 3 4 5 4 1 4 |
| 6 | 6 5 |
| 7 | 6 5 2 |
| 8 | 4 |

Key: 1|7 means 17

Note: A stemplot must always be accompanied by a key in order to help the reader interpret the values.

**Frequency Distribution**

Statistical data obtained by means of census, sample surveys or experiments usually consist of raw, unorganized sets of numerical values. Before these data can be used as a basis for inferences about the phenomenon under investigation or as a basis for decision, they must be summarized and the pertinent information must be extracted.

***Example 1***

A traffic inspector has counted the number of automobiles passing a certain point in 100 successive 20-minute time periods. The observations are listed below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 23 | 20 | 16 | 18 | 30 | 22 | 26 | 15 | 5 | 18 |
| 14 | 17 | 11 | 37 | 21 | 6 | 10 | 20 | 22 | 25 |
| 19 | 19 | 19 | 20 | 12 | 23 | 24 | 17 | 18 | 16 |
| 27 | 16 | 28 | 26 | 15 | 29 | 19 | 35 | 20 | 17 |
| 12 | 30 | 21 | 22 | 20 | 15 | 18 | 16 | 23 | 24 |
| 15 | 24 | 28 | 19 | 24 | 22 | 17 | 19 | 8 | 18 |
| 17 | 18 | 23 | 21 | 25 | 19 | 20 | 22 | 21 | 21 |
| 16 | 20 | 19 | 11 | 23 | 17 | 23 | 13 | 17 | 26 |
| 26 | 14 | 15 | 16 | 27 | 18 | 21 | 24 | 33 | 20 |
| 21 | 27 | 18 | 22 | 17 | 20 | 14 | 21 | 22 | 19 |

A useful method for summarizing a set of data is the construction of a frequency table, or a frequency distribution. That is, we divide the overall range of values into a number of classes and count the number of observations that fall into each of these classes or intervals. The general rules for constructing a frequency distribution are;

(i) There should not be too few or too many classes.

(ii) As much as possible, equal class intervals are preferred. But the first and last classes can be open-ended to cater for extreme values.

(iii) Each class should have a class mark to represent the classes. It is also named as the class midpoint of the ith class. It can be found by taking simple average of the class boundaries or the class limits of the same class.

**1. Setting up the classes**

Choose a class width of 5 for each class, then we have seven classes going from 5 to 9, from 10 to 14, …, and from 35 to 39.

**2. Tallying and counting**

| Clases | Tally Marks | Count |
|---|---|---|
| 5 - 9 | | 3 |
| 10 - 14 | | 9 |
| 15 - 19 | | 36 |
| 20 - 24 | | 35 |
| 25 - 29 | | 12 |
| 30 - 34 | | 3 |
| 35 - 39 | | 2 |

**3. Illustrating the data in tabular form**

Frequency Distribution for the Traffic Data

| Number of autos per period | Number of periods |
|---|---|
| 5 - 9 | 3 |
| 10 - 14 | 9 |
| 15 - 19 | 36 |
| 20 - 24 | 35 |
| 25 - 29 | 12 |
| 30 - 34 | 3 |
| 35 - 39 | 2 |

In this example, the class marks of the traffic-count distribution are 7, 12, 17, …, 32 and 37.

**Histogram**

A histogram is usually used to present frequency distributions graphically. This is constructed by drawing rectangles over each class. The area of each rectangle should be proportional to its frequency.

**Frequency Polygon**

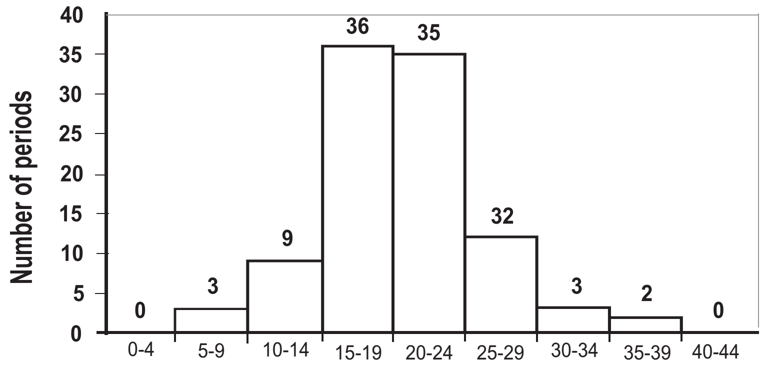
Another method to represent frequency distribution graphically is by a frequency polygon. As in the histogram, the base line is divided into sections corresponding to the class-interval, but instead of the rectangles, the points of successive class marks are being connected. The frequency polygon is particularly useful when two or more distributions are to be presented for comparison on the same graph.

***Example 2***

Construct a histogram and a frequency polygon for the traffic data in Example 1.

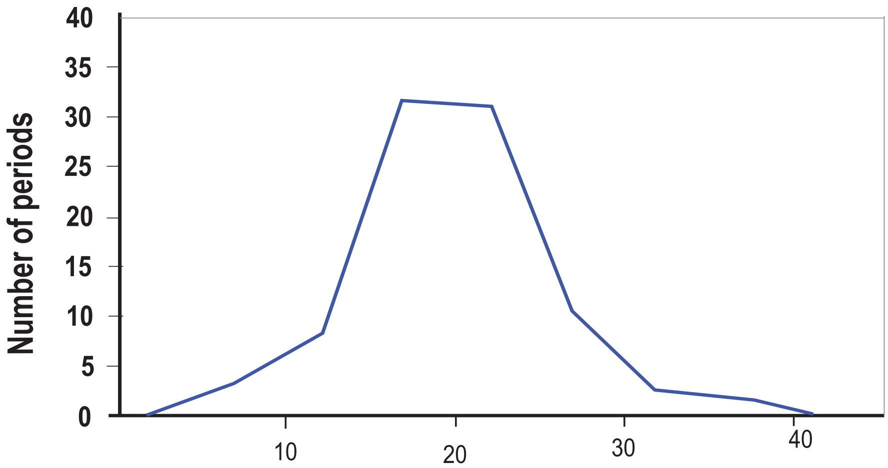
Histogram is only for continuous numerical data. This histogram is for the traffic data. Please note the difference between a bar graph and a histogram. The adjacent bars in a histogram have no gaps.

**Histogram of the traffic data**




The frequency polygon is graph where the midpoints of the bars in a histogram are connected by straight lines.

**Histogram of the traffic data**




**Frequency Curve**

A frequency curve can be obtained by smoothing the frequency polygon.

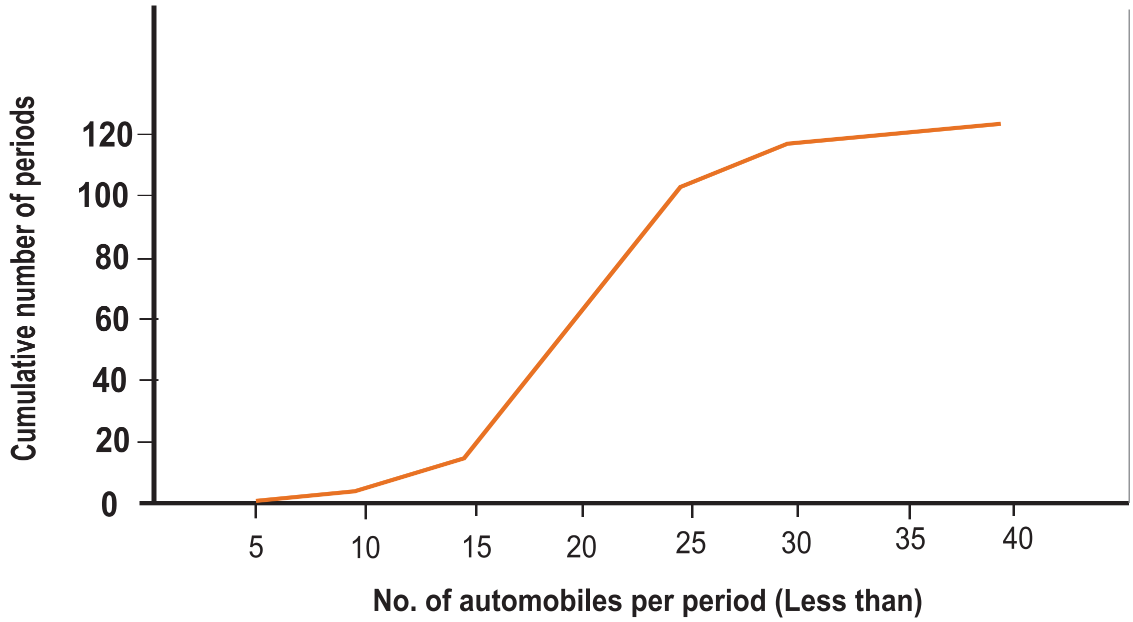
**Cumulative Frequency Distribution and Cumulative Polygon**

Sometimes it is preferable to present data in a cumulative frequency distribution, which shows directly how many of the items are less than, or greater then, various values.

| Less than | Cumulative frequency |
|---|---|
| 4.5 | 0 |
| 9.5 | 3 |
| 14.5 | 12 |
| 19.5 | 48 |
| 24.5 | 83 |
| 29.5 | 95 |
| 34.5 | 98 |
| 39.5 | 100 |

***Example 3***

Construct a "Less-than" ogive of the distribution of traffic data.




**Cumulative Frequency Curve**

A cumulative frequency curve can similarly be drawn.

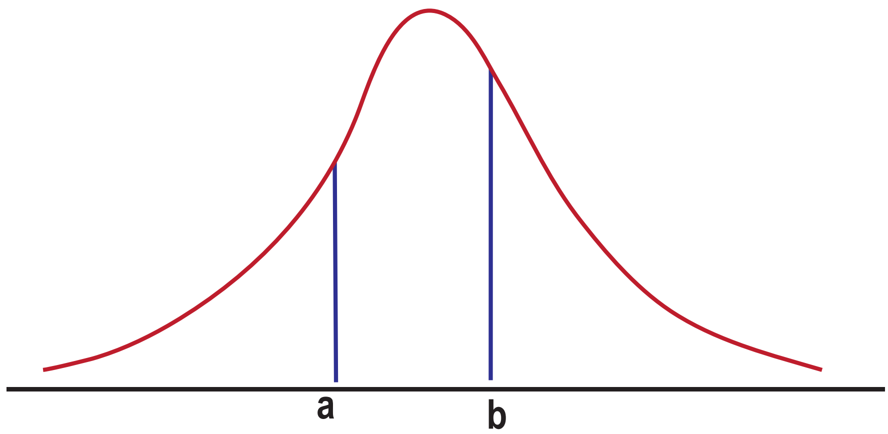
**Relative Frequency**

Relative frequency of a class is defined as:

$$\frac{\textit{Frequency of the class}}{\textit{Total Frequency}}$$

If the frequencies are changed to relative frequencies, then a relative frequency histogram, a relative frequency polygon and a relative frequency curve can similarly be constructed.

Relative frequency curve can be considered as probability curve if the total area under the curve be set to 1. Hence the area under the relative frequency curve between a and b is the probability between interval $a$ and $b$.



**Unit: Data Analysis** | **Topic: Measure of Central Tendency**

| | |
|---|---|
| **Benchmark** | **11.4.4.2.** Explore, interpret, and analyse data by calculating the various measures of central tendency, and the measures of spread. |

**Learning Objective:** By the end of the topic, students will be able to;

- calculate and interpret mean, mode and median, and
- calculate and interpret range and the interquartile range.

**Essential questions:**

- What is mean, mode, median, range, variance and standard deviation?
- How significant are these measures of tendency?
- How are these measures of tendency calculated?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Confidently calculate and interpret data using the measures Central Tendency. |
| **Skills** | Calculate the numerical measures from both the ungrouped and the grouped data and, interpret, and analyse data from these numerical measures. |
| **Knowledge** | Mean, mode, median, interquartile range. |
| **Mathematical Thinking** | Think about how to use these statistics calculated to explore, interpret, and analyse. |

## Content Background

### Central Tendency

When we work with numerical data, it seems apparent that in most set of data there is a tendency for the observed values to group themselves about some interior values; some central values seem to be the characteristics of the data. This phenomenon is referred to as central tendency. For a given set of data, the measure of location we use depends on what we mean by middle; different definitions give rise to different measures. We shall consider some more commonly used measures, namely arithmetic mean, median and mode. The formulas in finding these values depend on whether they are grouped data or ungrouped data.

### Arithmetic Mean

The arithmetic population mean, μ, or simply called mean, is obtained by adding together all of the measurements and dividing by the total number of measurements taken. Mathematically it is given as

$$\mu = \frac{\sum x_i}{N}$$

Arithmetic mean can be used to calculate any numerical data and it is always unique. It is obvious that extreme values affect the mean. Also, arithmetic mean ignores the degree of importance in different categories of data.

$$\overline{x} = \frac{\sum x_i}{n}$$

***Example***

Given the following set of ungrouped data: 20, 18, 15, 15, 14, 12, 11, 9, 7, 6, 4, 1

Find the mean of the ungrouped data.

$$\bar{x} = \frac{\sum x_i}{n} = \frac{20+18+15+15+14+12+11+9+7+6+4+1}{12} = \frac{132}{12} = 11$$

### Median

Median is defined as the middle item of all given observations arranged in order. For ungrouped data, the median is obvious. In case of the number of measurements is even, the median is obtained by taking the average of the middle.

***Example***
The median of the ungrouped data: 20, 18, 15, 15, 14, 12, 11, 9, 7, 6, 4, 1 is
$\frac{12+11}{2} = 11.5$

### Mode

Mode is the value which occurs most frequently. The mode may not exist, and even if it does, it may not be unique.

For ungrouped data, we simply count the largest frequency of the given value. If all are of the same frequency, no mode exits. If more than one values have the same largest frequency, then the mode is not unique. The value for the mode of the data above is 15 (unimodal).

***Example***
{2, 2, 2, 4, 5, 6, 7, 7, 7}
Mode = 2 or 7 (Bimodal)
Note that the mode is independent of extreme values and it may be applied in qualitative data.

### Grouped Data

***Example:*** The following table gives the frequency distribution of the number of orders received each day during the past 50 days at the office of a mail-order company. Calculate the mean.

| Number of order | $f$ |
|---|---|
| 10 - 12 | 4 |
| 13 - 15 | 12 |
| 16 - 18 | 20 |
| 19 - 21 | 14 |
| | $n$= 50 |

*Solution:* x is the midpoint of the class. It is adding the class limits and divides by 2.

| Number of order | $f$ | $x$ | $fx$ |
|---|---|---|---|
| 10 - 12 | 4 | 11 | 44 |
| 13 - 15 | 12 | 14 | 168 |
| 16 - 18 | 20 | 17 | 340 |
| 19 - 21 | 14 | 20 | 280 |
| | n=50 | | = 832 |

**Median and Interquartile Range –Grouped Data**

**Step 1:** Construct the cumulative frequency distribution.
**Step 2:** Decide the class that contain the median. Class Median is the first class with the value of cumulative frequency equal at least n/2.
**Step 3:** Find the median by using the following formula:

$$\text{Median} = L_m + \frac{\frac{n}{2} - F}{f_m} i$$

Where:

$n$ = the total frequency
$F$ = the cumulative frequency before the class median
$f_m$ = the frequency of the class median
$i$ = the class width
$L_m$ = the lower boundary of the class median

***Example:*** based the ungrouped data below, find the median;

| Time to travel to work | Frequency |
|---|---|
| 1 - 10 | 8 |
| 11 - 20 | 14 |
| 21 - 30 | 12 |
| 31 - 40 | 9 |
| 41 - 50 | 7 |

*Solution*
1st Step: Construct the cumulative frequency distribution

| Time to travel to work | Frequency | Cumulative Frequency |
|---|---|---|
| 1 - 10 | 8 | 8 |
| 11 - 20 | 14 | 22 |
| 21 - 30 | 12 | 34 |
| 31 - 40 | 9 | 43 |
| 41 - 50 | 7 | 50 |

$\frac{n}{2} = \frac{50}{2} = 25 \longrightarrow$ class median is the 3rd class
So, $F$= 22, $f_m$=12, $Lm$=20.5 and $i$=10

Therefore,

$$\text{Median} = L_m + \left(\frac{\frac{n}{2} - F}{f_m}\right) i = 21.5 + \left(\frac{25 - 22}{12}\right) 10 = 24$$

Thus, 25 persons take less than 24 minutes to travel to work and another 25 persons take more than 24 minutes to travel to work.

**Quartiles**

Using the same method of calculation as in the medium, we can get $Q_1$ and $Q_3$ equations as follows:

$$Q_1 = L_{Q1} + \left(\frac{\frac{n}{4} - F}{f_{Q1}}\right)i$$

$$Q_3 = L_{Q3} + \left(\frac{\frac{3n}{4} - F}{f_{Q3}}\right)i$$

***Example:*** Based on the grouped data below, find the Interquartile Range

| Time to travel to work | Frequency |
|---|---|
| 1 - 10 | 8 |
| 11 - 20 | 14 |
| 21 - 30 | 12 |
| 31 - 40 | 9 |
| 41 - 50 | 7 |

*Solution:*

1st Step: Construct the cumulative frequency distribution

| Time to travel to work | Frequency | Cumulative Frequency |
|---|---|---|
| 1 - 10 | 8 | 8 |
| 11 - 20 | 14 | 22 |
| 21 - 30 | 12 | 34 |
| 31 - 40 | 9 | 43 |
| 41 - 50 | 7 | 50 |

2nd Step: Determine the $Q_1$ and $Q_3$

Class $Q_1 = \frac{n}{4} = \frac{50}{4} = 12.5$

Class $Q_1$ is the 2nd class $Q_1 = L_{Q1} + \left(\frac{\frac{n}{4} - F}{f_{Q1}}\right)i$

Therefore, $= 10.5 + \left(\frac{12.5 - 8}{14}\right)10$

Class $Q_3 = \frac{3n}{4} = \frac{3(50)}{4} = 37.5$ $\quad Q_3 = L_{Q3} + \left(\frac{\frac{n}{4} - F}{f_{Q1}}\right)i$

Class $Q_1$ is the 4th class $= 30.5 + \left(\frac{37.5 - 34}{9}\right)10$

Therefore, $= 34.3889$

**Interquartile Range**

**IQR = $Q_3$-$Q_1$**

Calculate the IQ

IQR = $Q_3$- $Q_1$=34.3889 -13.7143=20.6746

**Mode- Grouped Data**

- Mode is the value that has the highest frequency in a data set.
- For grouped data, class mode ( or, modal class) is the class with the highest frequency.
- To find mode for grouped data, use the following formula:

$$Mode = L_{mo} + \left(\frac{\Delta f}{\Delta_1 + \Delta_2}\right)i$$

Where:

$i$ is the class width

$\Delta_1$ is the difference between the frequency of the class mode and the frequency of the class after the class mode

$\Delta_2$ is the difference between the frequency of the class mode and frequency of the class before the class mode

$L_{mo}$ is the lower boundary of the class mode

**Calculation of Grouped Data – Mode**

***Example:*** Based on the grouped data below, find the mode

| Time to travel to work | Frequency |
|---|---|
| 1 - 10 | 8 |
| 11 - 20 | 14 |
| 21 - 30 | 12 |
| 31 - 40 | 9 |
| 41 - 50 | 7 |

*Solution*

Based on the table,

$$L_{mo} = 10.5, \Delta_1 = (14 - 8) = 6, \Delta_2 = (14 - 12) = 2 \text{ and}$$

$$Mode = 10.5 + \left(\frac{6}{6 + 12}\right)10 = 17.5$$

| Unit: Data Analysis | Topic: Measure of Dispersion |
|---|---|
| **Benchmark** | **11.4.4.3** Calculate mean, variance and standard deviation. |

**Learning Objective:** By the end of the topic, students will be able to;

- calculate, and interpret range, and the interquartile range,
- calculate, and interpret variance and standard deviation, and
- calculate, and interpret the coefficient of variation.

**Essential questions:**

- What are the measures of dispersion?
- How significant are the measures of dispersion?
- What is the difference between measures of tendency and measures of dispersion?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Confidently calculate and interpret data using the measures of central tendencies. |
| **Skills** | Calculate the numerical measures from both the ungrouped and the grouped data and, interpret, and analyse data from these numerical measures. |
| **Knowledge** | Mean, mode, median, trimmed mean, range, interquartile range, variance, standard deviation, coefficient of variation. |
| **Mathematical Thinking** | Think about how to calculate and interpret data using measures of central tendencies. |

## Content Background

### Measures of Dispersion

In the previous topic, you have studied measures of central tendency such as mean, mode, and median of ungrouped and grouped data. In addition to these measures, we often need to calculate a second type of measure called a measure of dispersion which measures the variation in the observations about the middle value – mean or median etc. This topic is concerned with some important measures of dispersion such as mean deviation, variance, standard deviation etc., and finally analysis of frequency distributions.

### Range

The measure of dispersion which is easiest to understand and easiest to calculate is the range. Range is defined as: Range = Largest observation – Smallest observation

### Deciles, Percentile, and Fractile

Decile divides the distribution into ten equal parts while percentile divides the distribution into one hundred equal parts. There are nine deciles such that 10% of the data are $\leq D_1$; 20% of the data are $\leq D_2$; and so on. There are 99 percentiles such that 1% of the data are $\leq P_1$; 2% of the data are $\leq P_2$; and so on. Fractile, even more flexible, divides the distribution into a convenient number of parts.

### Quartiles

Quartiles are the most commonly used values of position which divides distribution into four equal parts such that 25% of the data are $\leq Q_1$; 50% of the data are $\leq Q_2$; 75% of the data are $\leq Q_3$. It is also denoted the value $(Q_3 - Q_1) / 2$ as the Quartile Deviation, $Q_{D,}$ or the semi-interquartile range.

**Mean Absolute Deviation**

Mean absolute deviation is the mean of the absolute values of all deviations from the mean. Therefore it takes every item into account. Mathematically it is given as:

$$\frac{\sum |x_i - \mu}{N}$$

where: $x_i$ is the value of the *i*th item;
μ is the population arithmetic mean;
$N$ is the population size.

**Variance and Standard Deviation**

The variance and standard deviation are two very popular measures of variation. Their formulations are categorized into whether to evaluate from a population or from a sample.

The population variance, $\sigma^{2,}$ is the mean of the square of all deviations from the mean. Mathematically it is given as:

$$\sigma = \frac{\sum (x_i - \mu)^2}{N}$$

where: $x_i$ is the value of the ith item;
μ is the population arithmetic mean;
$N$ is the population size.

The population standard deviation σ is defined as $\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$
The sample variance, denoted as $s^2$ gives:

$$\frac{\sum (x_i - x)^2}{n-1}$$

The sample standard deviation, $s$, is defined as $s = \sqrt{s^2}$ .

For ungrouped data, $s = \sqrt{\frac{\sum (x_i - x)^2}{n-1}} = \sqrt{\frac{\sum x_i^2 - (\sum x_i)^2 / n}{n-1}}$

where: $x_i$ is the value of the ith item;
$\bar{x}$ is the sample arithmetic mean;
$n$ is the sample size.

Note that when calculating the sample variance, we have to subtract 1 from the sample size which appears in the denominator.

**Measures of Grouped Data**

| Gas Consumption | Frequency $(f_i)$ | Class boundary | Class mark $(x_i)$ | $f_i x_i$ | $f_i x^2_i$ |
|---|---|---|---|---|---|
| 10-19 | 1 | 9.5 - 19-5 | 14.5 | 14.5 | 210.25 |
| 20-29 | 0 | 19.5 - 29.5 | 24.5 | 0 | 0 |
| 30-39 | 1 | 29.5 - 39.5 | 34.5 | 34.5 | 1190.25 |
| 40-49 | 4 | 39.5 - 49.5 | 44.5 | 178 | 7921 |
| 50-59 | 7 | 49.5 - 59.5 | 54.5 | 381.5 | 20791.75 |
| 60-69 | 16 | 59.5 - 69.5 | 64.5 | 1032 | 66564 |
| 70-79 | 19 | 69.5 - 79.5 | 74.5 | 1415.5 | 105454.8 |
| 80-89 | 20 | 79.5 - 89.5 | 84.5 | 1690 | 142805 |
| 90-99 | 17 | 89.5 - 99.5 | 94.5 | 1606.5 | 151814.3 |
| 100-119 | 11 | 99.5 - 109.5 | 104.5 | 1149.5 | 120122.8 |
| 110-119 | 3 | 109.5 - 119.5 | 114.5 | 343.5 | 39330.75 |
| 120-129 | 1 | 119.5 - 129.5 | 124.5 | 124.5 | 15500.25 |
| | 100 | | | 7970 | 671705 |

1. $\overline{x} = \frac{\sum x_i f_i}{n}, \quad n = \sum f_i$

$$= \frac{1 \text{ x } 14.5 + 0 \text{ x } 24.5 + \cdots + 1 \text{ x } 124.5}{100} = 79.9$$

2. For grouped data, the median can be found by first identifying the class containing the median, then apply the formula:

$$\text{median } = L_1 + \frac{\frac{n}{2} - c}{f_m}(L_2 - L_1)$$

Where:

$L_1$ is the lower class boundary of the medain class;
$n$ is the total frequency (*i.e.* the sample size);
$C$ is the cumulative frequency just before the median class;
$f_m$ is the frequency of the median class;
$L_2$ is the upper class boundary containing the median.

It is obvious that the median is affected by the total number of data but is independent of extreme values. However if the data is ungrouped and numerous, finding the median is tedious. Note that median may be applied in qualitative data if they can be ranked.

$$\text{Median} = 79.5 + \frac{50 - 48}{20} \text{ x } 10 = 80.5$$

$$Q_1 = 59.5 + \frac{25 - 13}{16} \text{ x } 10 \approx 67$$

$$Q_3 = 89.5 + \frac{75 - 68}{17} \text{ x } 10 \approx 93.6$$

3. For grouped data, the mode can be found by first identify the largest frequency of that class, called the modal class, then apply the following formula on the modal class:

$$\text{Mode} = L_1 + \frac{d_1}{d_1 + d_2}\ L_2 - L_1$$

Where:
$L_1$ is the lower class boundary of the modal class;
$d_1$ is the difference of the frequencies of the modal class with the previous class and is always positive;
$d_2$ is the difference of the frequencies of the modal class with the following class and is always positive;
$L_2$ is the upper class boundary of the modal class.

$$\text{Mode} = 79.5 + \frac{20-9}{(20-19)+(20-17)} \text{x} 10 = 82$$

$$s = \sqrt{\frac{\sum f(x-x)^2}{\sum f - 1}} = \sqrt{\frac{\sum fx^2 - (\sum fx)^2 / \sum f}{\sum f - 1}}, \text{ where } \sum f = n$$

4. $Sample\ s.d...s = \sqrt{\frac{n(\sum x_i^2 fi) - (\sum x_i f_i)^2}{n(n-1)}}$

$$= \sqrt{\frac{100(671705) - (7970)^2}{100(100-1)}}$$

$$= 19.2$$

**Coefficient of Variation**

The coefficient of variation is a measure of relative importance. It does not depend on unit and can be used to make comparison even two samples differ in means or relate to different types of measurements.

The coefficient of variation gives:

$$\frac{\text{Standard Deviation}}{\text{Mean}} \text{ x } 100\%$$

***Example***

| | $\overline{x}$ | $s$ |
|---|---|---|
| Salesman Salary | K916.76/month | K286.70 |
| Clerical Salary | K98.50/week | K20.55 |

$$CVs = \frac{286.70}{916.76} \text{ x } 100\% = 31\%$$

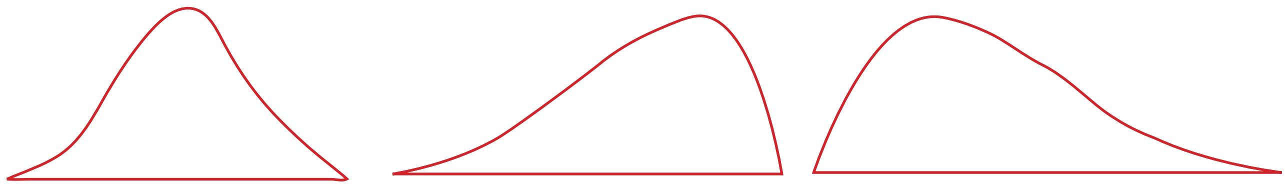
$$CVc = \frac{20.55}{98.50} \text{x} 100\% = 21\%$$

**Skewness of data distribution**

Skewness measures the shape of the distribution of numerical data. The sample skewness is computed as

$$SK = \frac{(\text{median} - \text{mean})}{s}$$

If the measure of skewness is zero, the distribution is normally distributed, as in the first figure below, a case where the mean is equal to the median. If mean is greater than the median, the distribution is negatively skewed, or skewed to the left, that is the longer tail of the distribution is to the left as in the second figure below. If mean is less than the median, the distribution is positively skewed, or skewed to the right, that is the longer tail of the distribution is to the right as in the third figure below.



***Example***

A given  data set records median of 58.6 and mean of 58. Find the skewness of these data set and described its distribution?
Solution

$$SK = \frac{(58.6 - 58)}{7.3} = 0.08$$

It is a positive skewness; therefore, the distribution is skewed to the right. Equivalently, it can be said that the distribution is symmetrical or normally distributed because the skewness is almost zero.

| Unit: Sets and Probability | Topic: Sets and Elements |
|---|---|

| | |
|---|---|
| **Benchmark** | **11.4.4.4** Define sets and elements, equality of sets, subsets and Venn diagram<br>**11.4.4.5** Identify and explain various sets and represent sets on a Venn diagram. |

**Learning Objective:** By the end of the topic, students will be able to;

- define a set and elements of a set,
- give examples of sets,
- define a subset, an empty set and equal sets,
- use different notations to describe and represent sets.

**Essential questions:**

- What is a ‘set’?
- What are elements of a set?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Show confident to explain sets and its elements. |
| **Skills** | Explain sets and its elements. |
| **Knowledge** | Sets, subsets, elements, equal sets and venn diagrams. |
| **Mathematical Thinking** | Think about how to explain sets, subsets, elements, equal sets and use venn diagrams to represent sets. |

## Content Background

### 1. Sets and Elements

A set is a collection of objects, numbers, ideas, etc… different objects etc… are called the **elements** or **members** of the set. A set may be defined by using one of the following methods:

- A = {3, 5, 7, 9, 11},
- B = {2, 4, 6, 8…},
- S = {all odd numbers},
- C = {x: $2 \le x \le 7$, x is an integer}

The symbol $\in$ means ‘is a member of a set’ Thus because 7 is a member of the set S = {3, 5, 7, 9, 11}, we write $7 \in S$. The symbol $\notin$ means ‘is not a member of the set’. Because 3 is not a member of the S we write $3 \notin S$.

The order of a set is the number of elements contained in the set. If a set A has 5 members we write $n$ (A) = 5

### 2. Empty Set, Equal Set, Finite and Infinite Sets

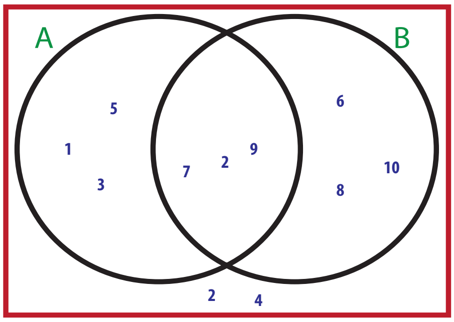
A set with no element is called an **empty** (or **null**) **set**. It can be denoted by {} or $\phi$. The number of elements in an empty set is 0, i.e., $n(\phi) = 0$.
Sets that contain exactly the same elements are **equal sets**. For example, if A = {2, 3, 5, 8} and B = {3, 8, 2, 5} then A = B. The order in which the elements of a set are written does not matter.

A **finite** set is one in which all the elements are listed such as {3, 7, 9}. An **infinite** set is one in which it is impossible to list all the members. For instance, {1, 3, 5, 7…} where the dots mean ‘and so on’.

### 3. Universal Set and Venn Diagram

The **Universal set** for any particular problem is the set which contains all the available elements for that problem. Thus if the universal set is all the odd numbers up to and including 11, we write
£ = {1, 3, 5, 7, 9, 11}.

A **Venn diagram** is a diagrammatical way of representing relationships between sets. On a Venn diagram the universal set is shown as a rectangle and subsets are shown as circles.




***Example:*** Write down the elements of the sets £, A and B.
The elements of £ are each number in the diagram.
£ = {1,2,3,4,5,6,7,8,9,10}

Any number inside the A circle is considered an element in set A. Set A contains the numbers 1,3,5,7,9 (the 7 and 9 are also part of the B set) A= {1, 3, 5, 7, 9}
Any number inside the B circle is considered an element of set B. Set B contains the numbers 6,7,8,9,10 (the 7 and 9 are also part of set A) B = {6, 7, 8, 9, 10}

### 4. Subsets

If all the members of a set A are also members of set B then A is said to be a subset of B. Thus if A = {p, q, r} and B = {p, q, r, s} we write . Every set has at least two subsets, itself and the null set.

### 5. Complementary of a Set

The **complement** of set A is the set of elements of £, which do not belong to A.
Thus if A = {2, 4,6 } and £ = {1, 2,3,4,5,6,7} the complement of A is A = {1,3,5,7}.

### 6. Intersections and Union of Sets

**Union and the word 'or'**
The word 'or' tells us that there is a union of two sets. ***For example:***

{singers} ∪ {instrumentalists} = {people who sing or play an instrument}
{vowels} ∪ {letters in 'dingo'} = {letters that are vowels or are in 'dingo'}
The word 'or' in mathematics always means 'and/or', so there is no need to add 'or both' to these descriptions of the unions. ***For example,***
If A = { 0, 2, 4, 6, 8, 10, 12, 14 } and B = { 0, 3, 6, 9, 12 }, then
A ∪ B = { 0, 2, 3, 4, 6, 8, 9, 10, 12, 14 }.
Here the elements 6 and 12 are in both sets A and B

**Disjoint Sets**
Two sets are called disjoint if they have no elements in common.
***For example:*** The sets S = { 2, 4, 6, 8 } and T = { 1, 3, 5, 7 } are disjoint.

**Benchmark** **11.4.4.6** Apply universal sets and complement of a set, discussing some algebraic laws, and solve related problems.

**Learning Objective:** By the end of the topic, students will be able to;

- solve problems with number of elements of a sets, and
- solve problems with three sets venn diagram.

**Essential questions:**

- What does a venn diagram represent?
- How convenient is a venn diagram in terms of solving real life problems?

**Key Concepts(ASK-MT)**

| | |
|---|---|
| **Attitudes/Values** | Creatively construct venn diagrams and illustrate the elements represented in the problems correctly. |
| **Skills** | Construct venn diagrams to represent information to solve real life problems. |
| **Knowledge** | Venn diagrams and three sets venn diagrams. |
| **Mathematical Thinking** | Think about how to construct venn diagram to solve problems. |

## Content Background

Sets represented on Venn Diagrams can be used to solve problems. In most cases the number of elements in a set is shown on the Venn diagram, rather than the elements themselves.

**1. Problems with the number of Elements of a set**

For sets that intersect, a rule that is often used is: $n(A \cup B) = n(A) + n(B) - n(A \cup B)$
***For example:*** The rule $n(A \cup B) = n(A) + n(B) - n(A \cup B)$ can be demonstrated:
$E = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{2, 4, 6, 8\}$ , $B = \{3, 4, 5, 6, 7, 8\}$

$A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$, $A \cup B = \{4, 6, 8\}$
$n(A) = 4$; $n(B) = 6$; $n(A \cup B) = 7$; $n(A \cup B) = 3$; $n(A) + n(B) - n(A \cup B) = 4 + 6 - 3 = 7 = n(A \cup B)$

The number on the Venn diagram represents the number of elements in each of the sets. When drawing a Venn diagram containing the number of elements, it is good to start with the number of elements in the intersections first, then in all the other possible areas.

***Example:*** In a class of 25 students, 15 take History, 17 take Geography and 3 take neither subject. How many class members take both subjects?
If H = set of students who take history and G = set of students who take geography, then $n(H) = 15$, $n(G) = 17$, $n(H \cup G) = 15 - x + x + 17 - x = 32 - x$

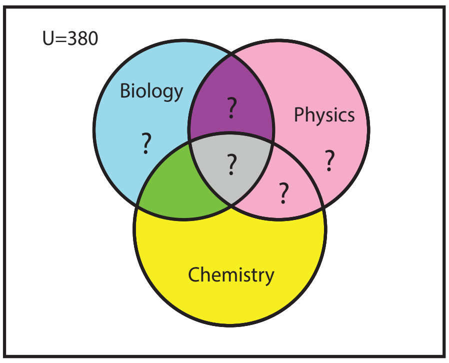
$$22 = 32 - x$$
$$x = 10$$

**2. Intersection of three sets**

***Example***

(1). How to solve three sets Venn-Diagram problem if the middle pattern number is not given?

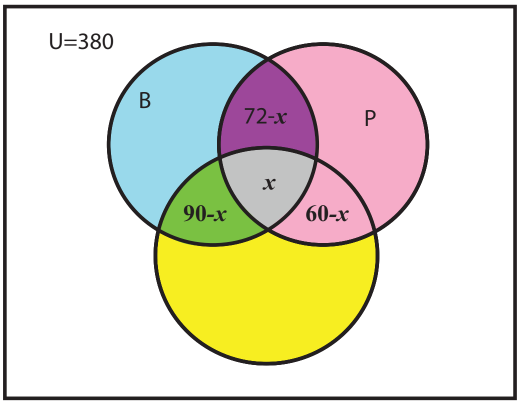
- o 380 students are taking courses:
- o 215 taking Biology, 173 taking Physics, 182 taking chemistry.
- o 72 taking Biology and Physics, 90 taking Biology and Chemistry, 60 taking Physics and Chemistry



Find the number of students in each of the following part.

B - Biology, P - Physics, C – Chemistry
Let x taking all three courses, so

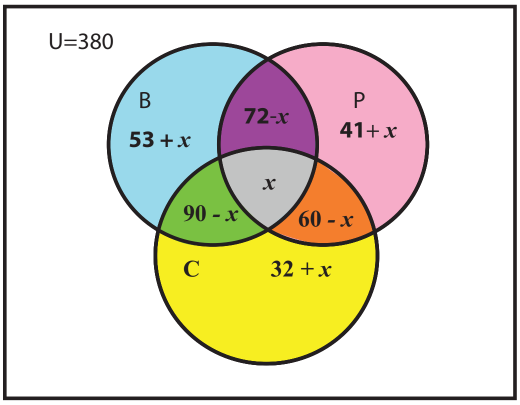
o (72 - x) only taking Biology and Physics,
o (90 - x) only taking Biology and Chemistry,
o (60 - x) only taking Physics and Chemistry,




Label these information as shown in the right Venn diagram.

Also,

(53 + x) → only taking Biology,
(41 + x) → only taking Physics,
(32 + x) → only taking Chemistry,



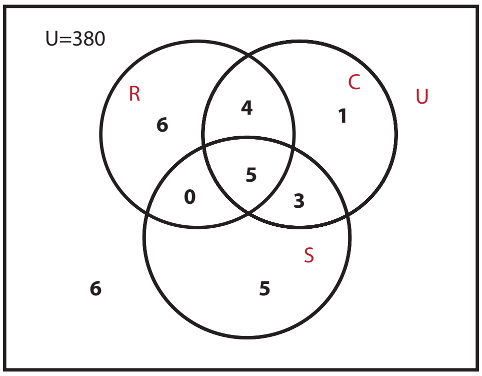

Label these information in Figure (2)
53 + x 72- x 41+ x
Because (53 + x) = 215 - (72 - x) - (90 - x) - x
(41 + x) = 173 - (72 - x) - (60 - x) - x
(32 + x) = 182 - (60 - x) - (90 - x) – x

Add all the parts:
(53 + x) + (72 - x) + (41+ x) + (90 - x) + x + (60 - x) + (32 + x) = 380,
x = 32

x = 32 → students taking all the three courses.
o (72 - x) = 40 students only taking Biology and Physics,
o (90 - x) = 58 students only taking Biology and Chemistry
o (60 - x) = 28 students only taking Physics and Chemistry,
o (53 + x) = 85 students only taking Biology
o (41 + x) = 73 students only taking Physic
o (32 + x) = 64 students only taking Chemistry

***Example (2).*** In a group of 30 people, 15 run, 13 swim, 13 cycle, 5 run and swim, 8 cycle and swim, 9 run and cycle, and 5 do all three activities. How many of the 30 people neither run nor cycle?

*Solution*
n(U) = 30; n (R) = 15; n (S) = 13; n (C) = 13.
n (R ∩ S) = 5;
n (C ∩ S) = 8;
n (R ∩C) = 9.
R ∩ C ∩ S = 5.




6 + 5 = 11
Therefore, 11 students out of the 30 students neither run nor cycle.

***Example 3:*** From 50 students taking examinations in Mathematics, Physics and Chemistry, each of the student has passed in at least one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 Mathematics and Chemistry and at most 20 Physics and Chemistry.

What is the largest possible number that could have passed all three examinations?

Solution:
Let M be the set of students passing in Mathematics
P be the set of students passing in Physics
C be the set of students passing in Chemistry

Now,

$$n(M \cup P \cup C) = 50, n(M) = 37, n(P) = 24, n(C) = 43$$
$$n(M \cap P) \leq 19, n(M \cap C) \leq 29, n(P \cap C) \leq 20 \text{(Given)}$$
$$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C)$$
$$-n(P \cap C) + n(M \cap P \cap C) \leq 50$$
$$\Rightarrow 37 + 24 + 43 - 19 - 29 - 20 + n(M \cap P \cap C) \leq 50$$
$$\Rightarrow n(M \cap P \cap C) \leq 50 - 36$$
$$\Rightarrow n(M \cap P \cap C) \leq 14$$

Thus, the largest possible number that could have passed all the three examinations is 14.

| Unit : Sets and Probability | Topic Theoretical and Experimental Probability |
|---|---|
| **Benchmark** | **11.4.4.7** Explore, analyse, and interpret data by calculating probabilities of events from experimental data, life data tables, compound events of two-dimensional grids and tree diagrams. |

**Learning Objective:** By the end of the topic, students will be able to;

- use the definition of probability to calculate probability of events and probability of complementary of an event, and
- calculate, and interpret probabilities from data, and life tables.

**Essential questions:**

- What is the difference between theoretical and experimental probability?
- What is probability?
- How is the probability of events calculated?
- What are complementary events?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Confidently identify and predict changes of an event occurring. |
| **Skills** | Predicting the occurance of events and providing reasons. |
| **Knowledge** | Theoretical and Experimental Probability. |
| **Mathematical Thinking** | Think about how to calculate and interpret the probabilities of events. |

## Content Background

### 1. Probability theory

The probability theory is the study of the chance of an events happening. Probability is used every day of our lives without knowing, when we say I will be there, we are sure or 100% sure that we will be there. An event that is impossible has 0% chance of happening or probability of 0, an event that is sure has a 100% chance of happening or probability of 1, a possible event has greater than 0% to less than 100% chance of happening or probability greater than zero but less than 1.

### 2. Sample space

A sample space is the set of all possible outcomes of an experiment.
***Example:*** The list of sample space for choosing a number between 1 and 9 is

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

The set of even numbers is {2, 4, 6, 8}, frequency of 4, and relative frequency or probability of even numbers selecting an even number is 4/9.

### 3. Probability of a single event

The probability of an event E is a number which describes the chance of that event occurring. The probability of an event E occurring, is described and calculated using the formula;

$$\mathrm{P(E)} = \frac{number\ of\ outcomes\ in\ event\ E}{number\ of\ outcomes\ in\ the\ sample\ space}$$

$$P(E) = \frac{n\,(E)}{n\,(S)}$$

Numerator n($E$) is the number of favorable outcomes in event $E$, and the denominator n($S$) is the total number of outcomes in a sample space.

A sample space is the set of all the possible outcomes from which an event occurs. Mathematically an event is a subset of a sample space.

$P(E) = 0$, only if $n(E) = 0$, there no outcomes in event $E$; event $E$ is an impossible event.
$P(E)=1$, only if $E$ is a sure event; $n(E)$ is equivalent to the number of outcomes in the sample space.
$0<P(E)<1$, only if $E$ is a possible event, meaning there are outcomes in event $E$. Therefore, the probability of any $E$ to occur is equal to 0,1, or greater than 0, and less than 1; written as $0 \le P(E) \le 1$.

Probability Scale: (yet to insert the scale diagram of probability

Probability can be expressed as a decimal,  fraction, or as a percentage.

**4. Complementary of an event**

The complementary of an event means the outcomes that are not in this particular event. The probability of the complement of an event E denoted by  is calculated as

$$P(\overline{E}) = 1 - P(E) \qquad P(E) = 1 - P(\overline{E}) \qquad P(E) + P(\overline{E}) = 1$$

***Examples***

**(i)** The sample space, S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Probability of an even number is 5/10 = 0.5, probability of an odd number is also 5/10 = 0.5

**(ii)** Probability of a number greater than 6 is 4/10 = 0.4. Therefore, the probability of the complement or not getting a number greater than 6 is, 1 – 0.4 = 0.6

**(iii)** A box contains 8 blue marbles and 12 red marble. Probability of selecting a blue marble from this box is 8/20 = 0.4, and probability of selecting a red marble 12/20 = 0.6.

**5. Experimental probability**

The experimental probability involves trials, outcomes, and frequency. The number of trials is the total number of times, repeated, in an experiment. The outcomes are the different results possible for a trial of an experiment. The frequency of a particular outcome is the number of times this outcome is observed from an experiment.
The relative frequency of an outcome is the probability of the occurrence of this outcome. Because of the way that is determined, relative frequency is often referred to as experimental probability.

**i.e., Experimental Probability = relative frequency**

$$= \frac{f}{\sum f} \quad or \quad \frac{f}{\sum f} \text{x}100\%$$

***Example***

The students of a class combined to toss two coins 2000 times. The results are recorded in the table below.

| Results | Frequency |
|---|---|
| 2 heads | 563 |
| 1 head and 1 tail | 987 |
| 2 tails | 450 |

Use this information to predict the probability of getting "2 heads" when you toss two coins.
*Solution:*
From the data collected, 2 heads occurred 563 times out of the 2000 times, i.e., on 28.15% of the occasions. If we assume that the frequency of occurrence will be the same for future tosses, then we could predict that when two coins are tossed the chance of getting two heads is approximately 28%.
( Similarly we could predict from the data that the chance of getting 1 head and 1 tail is $\frac{987}{2000}$ , or approximately 49%, and the chance of getting 2 tails is $\frac{450}{2000}$ , or approximately 23%.)

**6. Theoretical Probability**

If all the possible outcomes are equally likely, then the theoretical probability of an event E happening is given by

$$P(E) = \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$$

The probability of an event may be expressed as fraction, a decimal or a percentage

***Example:*** When a die is rolled, what is the probability of getting
**(a)** 2?
**(b)** an even number?
*Solution*
The sample space is S={1,2,3,4,5,6} and all these outcomes are equally likely. Therefore the total number of outcomes is 6.

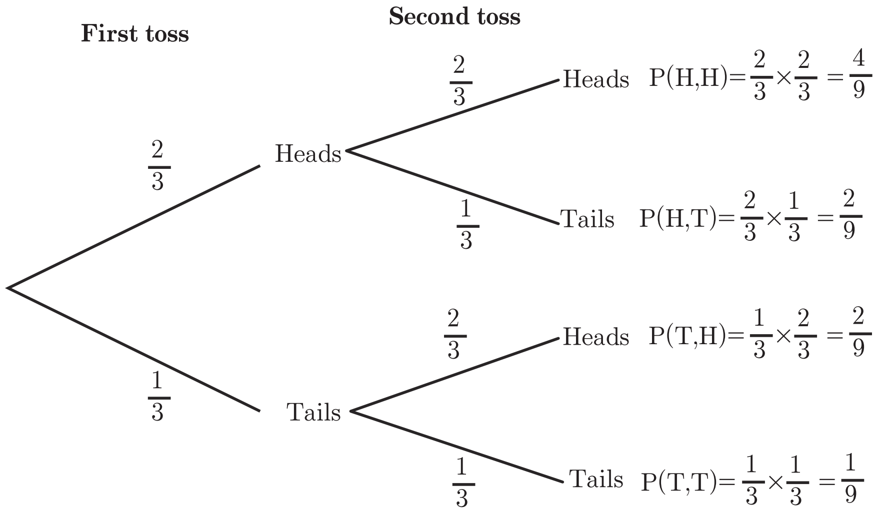
**(a)** To get a 2 there is only one favourable outcome. Hence, P(2)=$\frac{1}{6}$

**(b)** To get an even number there are three favourable outcomes, 2, 4 and 6. Hence
P(even number)=$\frac{3}{6}=\frac{1}{2}$ ( or 0.5 or 50%)

**7. Probability Tree Diagrams**

Tree diagrams show all the possibilities for combined events together with their probabilities.

***Example***
A coin is biased so that it is twice as likely to give heads than tails.
This means that whenever the coin is tossed P(H) = and P(T). The tree diagram below shows the possible outcomes when this coin is tossed twice.



### 8. Compound or multi-stage event

Multi-stage event is one which is made up of simpler events. When finding the sample space for multi-stage events, it is often useful to use systematic method such as a list, table or tree diagram.

***Example***

Tim has three pairs of trousers (blue,brown and grey) and two shirts ( one long sleeved and one short sleeved). Find all the possible combinations of trousers and shirts he can wear.

*Solution:*

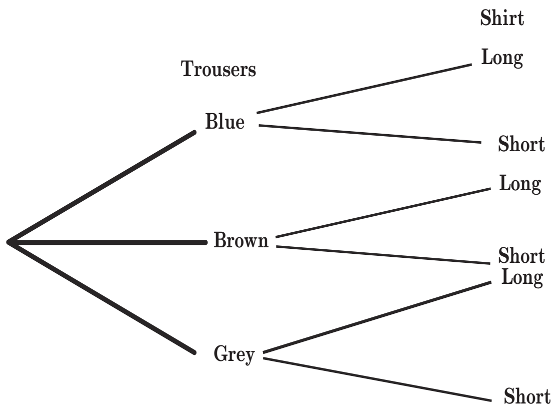
(i) Using a table

| | Trousers | | | |
|---|---|---|---|---|
| | | Blue(Bl) | Brown(Br) | Grey (G) |
| | Long (L) | Bl L | Br L | G L |
| Shirt | Short (S) | Bl S | Br S | G S |

From the table we can determine that there are six equally likely outcomes:
S= {(Bl,L),(Br,L),(G,L),(Bl,S),(Br,S),(G,S)}

(ii) Tree Diagrams

Using a tree diagram yields the same results:




S= {(Bl,L),(Br,L),(G,L),(Bl,S),(Br,S),(G,S)}
This is called a two-stage event because it is made up of two simpler events; the colour of the trousers ( with three possible outcomes) and the type of the shirt ( with two possible outcomes). The table or tree diagrams are convenient ways of identifying the sample space of two-stage events.

| Unit: Probability | Topic: Mutually and Non-Mutually exclusive events |
|---|---|
| **Benchmark** | **11.4.4.8** Explore, analyze, and interpret data by calculating probabilities of mutually and not-mutually from the Venn diagrams and calculate the addition and multiplication rules of probability. |

**Learning Objective:** By the end of the topic, students will be able to;

- identify, calculate, and interpret the probabilities of mutually, non-mutually exclusive, independent and dependent events, and
- calculate the probabilities of the union of events.

**Essential questions:**

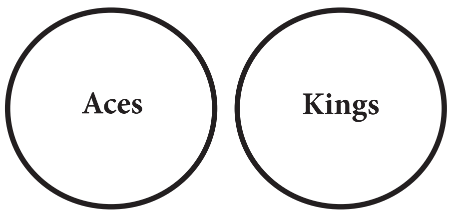
- What are mutually exclusive events?
- What are non-mutually exclusive events
- How are these exclusive events calculated?
- Why is it important to evaluate the probability to mutually exclusive and non mutually exclusive events?

| Key Concepts(ASK-MT) | |
|---|---|
| **Attitudes/Values** | Show confidence in applying the rules of probability. |
| **Skills** | Apply the routes of probability. |
| **Knowledge** | Mutually, and not-mutually exclusive events, union of events, independent and dependent events. |
| **Mathematical Thinking** | Think about how to apply the specific rules of probability. |

## Content Background

### 1. Mutually and Non-mutually Exclusive Events: (A or B)

*Mutually exclusive* means that A and B cannot both happen at the same time. Venn Diagram showing mutually exclusive events:



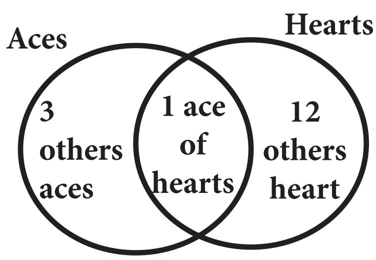

When events A and B are mutually exclusive: **P( A or B) = P(A) + P (B)**.
For example, if a card is drawn at random from a pack of 52;

$$P(Ace) = \frac{4}{52} = \frac{1}{13} \qquad P(King) = \frac{4}{52} = \frac{1}{13}$$

$$P(Ace\ or\ King) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

This can also be calculated directly using the fact that 8 out of the 52 cards are aces or kings, giving the probability $\frac{8}{52} = \frac{2}{13}$

The events 'draw an Ace' and 'draw a Heart' are **not mutually exclusive** as drawing the 'Ace' and drawing a 'Heart' can happen at the same time.



Venn Diagram showing non- mutually exclusive events:
When events A and B are not mutually exclusive, you dont add the probabilities, instead you subtract the probabilities.

$$P(A or\ B) = P(A) + P(B) - P(A \text{ and } B)$$

***Example***

(i) if a card is drawn at random from a pack of 52
P(Ace) = $\frac{4}{52}$ and P (Heart) = $\frac{13}{52}$ adding these gives $\frac{17}{52}$.
But this is not the probability of an Ace or a Heart. Since 13 cards are hearts and there are another 3 aces, there are just 16 cards out of 52 cards that are either hearts or aces so:

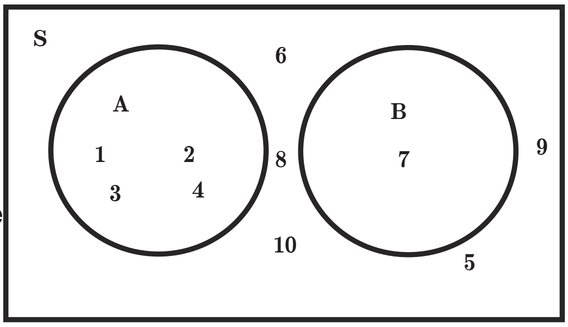
P(Ace or Heart) $= \frac{16}{52}$ **not** $\frac{16}{52}$

In this case adding gives a value that is too high because the Ace of Hearts is included twice.

(ii) From a pack of cards numbered 1 to 10, one card is drawn at random. What is the probability that the number on the card is:

(a) Less than 5 or divisible by 7?
(b) Less than 5 or divisible by 2?

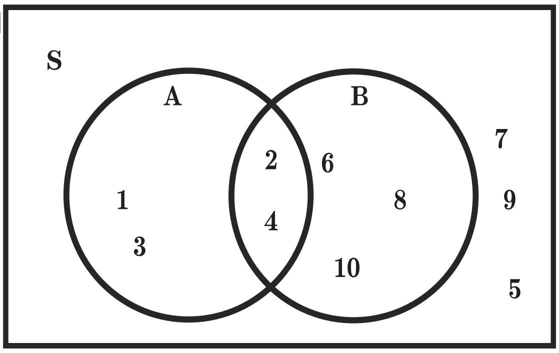
***Solution:***




(a) Let A be the event of the drawing a number less than 5 and B be the event of drawing a number divisible by 7. The events A and B are mutually exclusive, we can-not select a number less than 5 and a number divisible by 7 at the same time.

We require A and B and denote this by A ∪B ( Called the sum of A and B).
Now from the diagram

$$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) = \frac{4}{10} + \frac{1}{10} = \frac{1}{2}$$




(b) Let A be the event of drawing a number less than 5 and B be the event of drawing a number divisible by 2.
These events A and B are not mutually exclusive; drawing a number less than 5 does not exclude the possibility of drawing a number divisible by 2.
Since A and B are not mutually exclusive events, A∪B contains the points which are common to A and B, therefore these two points are A∪B.
We want P(A∪B), but if we add P(A) and P(B) the points of P(A∪B) will have their probabilities counted twice.

Thus $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - P(A \cap \mathrm{B})$

$$= \frac{4}{10} + \frac{5}{10} - \frac{2}{10}$$

$$= \frac{7}{10}$$

Note: In probability theory A∪B is called the intersection or product of the events A and B. It is written as AB and denotes the event that both A and B occur.

1. If A and B are non mutually exclusive events;
   P(A∪B) = P (A) +P(B) - P(AB)
2. If A and B are mutually exclusive events;
   P(AB) = 0.
   Hence P(A∪B) = P(A) + P(B).

**2. Independent and Dependent Events: (A and B)**

**Independent** means that A has no effect on the occurance B and vice versa.
Events A and B are indepentent events if and only if : **P(A and B) = P(A) x P(B)**

For example, if a coin is tossed and a card is taken at random from a pack of 52

P (Head) = $\frac{1}{2}$ P(King) $= \frac{4}{25} = \frac{1}{13}$

P(Head and King) $= \frac{1}{2} \text{x} \frac{1}{13} = \frac{1}{26}$

When events are **dependent**, it is necessary to use **conditional probabilities.**
The conditional probability of event B, given event A is denoted by;

***Example***

A bag contains 9 red marbles and 3 green marbles. For each case below, find the probability of randomly selecting a red marble on the first draw and a green marble on the second draw.

(a) The first marble is replaced

a. if the first marble is replaced before the second marble is selected, then the events are independent.

a $$P(\text{red}) = \frac{9}{9+3} = \frac{9}{12} = \frac{3}{4}$$

$$P(\text{green}) = \frac{3}{9+3} = \frac{3}{12} = \frac{1}{4}$$

$$P(\text{red and green}) = \frac{3}{4} \text{x} \frac{1}{4} = \frac{3}{16}$$

The probability is $\frac{3}{16}$, or 18.75%

(b) The first marble is not replaced

b. if the first marble is not replaced before the second marble is selected, then the size of the sample space for the second event changes from 12 to 11.

b $$P(\text{red}) = \frac{9}{9+3} = \frac{9}{12} = \frac{3}{4}$$

$$P(\text{green}) = \frac{3}{9+2} = \frac{3}{11}$$

$$P(\text{red and green}) = \frac{3}{4} \text{x} \frac{3}{11} = \frac{9}{44}$$

The probability is $\frac{9}{44}$, or 20.5%

# Standards-Based Lesson Planning

## What are Standards-Based Lessons?

In a Standards-Based Lesson, the most important or key distinction is that, a student is expected to meet a defined standard for proficiency. When planning a lesson, the teacher ensures that the content and the methods of teaching the content enable students to learn both the skills and the concepts defined in the standard for that grade level and to demonstrate evidence of their learning.

Planning lessons that are built on standards and creating aligned assessments that measure student progress towards standards is the first step teacher must take to help their students reach success. A lesson plan is a step-by-step guide that provides a structure for an essential learning.

When panning a standards-based lesson, teacher instructions are very crucial for your lessons. How teachers instruct the students is what really points out an innovative teacher to an ordinary teacher. Teacher must engage and prepare motivating instructional activities that will provide the students with opportunities to demonstrate the benchmarks. For instance, teacher should at least identify 3-5 teaching strategies in a lesson; teacher lectures, ask questions, put students into groups for discussion and role play what was discussed.

## Why is Standards-Based Lesson Planning Important?

There are many important benefits of having a clear and organized set of lesson plans. Good planning allows for more effective teaching and learning. The lesson plan is a guide and map for organizing the materials and the teacher for the purpose of helping the students achieve the standards. Lesson plans also provide a record that allows good, reflective teachers to go back, analyse their own teaching (what went well, what didn't), and then improve on it in the future. Standards-based lesson planning is vital because the content standards and benchmarks must be comparable, rigorous, measurable and of course evidence based and be applicable in real life that we expect students to achieve. Therefore, teachers must plan effective lessons to teach students to meet these standards. As schools implement new standards, there will be much more evidence that teachers will use to support student learning to help them reach the highest levels of cognitive complexity. That is, students will be developing high-level cognitive skills.

**Components of a Standards-Based Lesson Plan**
An effective lesson plan has three basic components;

- aims and objectives of the course;
- teaching and learning activities;
- assessments to check student understanding of the topic.

Effective teaching demonstrates deep subject knowledge, including key concepts, current and relevant research, methodologies, tools and techniques, and meaningful applications.

## Planning for under-achievers NORMA

**Who are underachieving students?**

Under achievers are students who fail or do not perform as expected. Underachievement may be caused by emotions (low self-esteem) and the environment (cultural influences, unsupportive family)

**How can we help underachievement?**

Underachievement varies between students. Not all students are in the same category of underachievement.

Given below a suggested strategies teachers may adopt to assist underachievers in the classroom.

- Examine the Problem Individually

It is important that underachieving students are addressed individually by focusing on the student's strengths.

- Create a Teacher-Parent Collaboration

Teachers and parents need to work together and pool their information and experience regarding the child. Teachers and parents begin by asking questions such as;

- In what areas has the child shown exceptional ability?
- What are the child's preferred learning styles?
- What insights do parents and teachers have about the child's strengths and problem areas?

- Help student to plan every activity in the classroom
- Help students set realistic expectations
- Encourage and promote the student's interests and passions.
- Help children set short and long-term academic goals
- Talk with them about possible goals.
- Ensure that all students are challenged (but not frustrated) by classroom activities
- Always reinforce students

## Example of Standards-Based Lesson Planning

The following sample lesson can help teachers to plan effective lessons. Teachers are encouraged to study the layout of the different components of these lessons and follow this design in their preparation and teaching of each lesson. Planning a good lesson helps the teacher in maintaining a standard teaching pattern which should not deviate students learning of the concept from the topic.

## Sample Lesson Plan

### Strand 4: Statistics and Probability

**Content Standard 4:** Students will be able to investigate how to interpret data using methods of exploratory data analysis, develop and evaluate inferences, predictions and arguments that are based on data and understand and apply basic notions of chance and probability.

### Unit: Sets and Probability

**Benchmark: 11.4.4.6** Apply universal sets and complement of a set, discussing some algebraic laws, and solve related problems.

**Topic:** Subsets and Venn diagram

**Lesson Title:** Venn Diagrams

**Objective**: By the end of the lesson, students should be able to design Venn Diagrams and display information onto Venn Diagrams in order find solutions to given problems.

**Materials:** blank papers, activity handouts, rings/loops, number blocks

| ASK-MT | |
|---|---|
| **Attitudes/Values** | Appreciate and display confidence in drawing up the Venn diagrams and filling in the information from the problems to solve real life problems |
| **Skills** | Drawing up Venn diagrams and filling in the information |
| **Knowledge** | Subsets and Venn Diagrams |
| **Mathematical Thinking** | Think about how information from a real life problem can be displayed onto a Venn diagram to solve problems. |

## Lesson Procedure

| TEACHER ACTIVITIES TEACHER ACTIVITIES | STUDENT ACTIVITIES |
|---|---|
| **INTRODUCTION** | **5 minutes** |
| • Recap students' previous knowledge through the following questions.<br>(i) What is a venn diagram?<br>(ii) How can you extract information from a venn diagram? | • Use their previous knowledge on sets and venn diagrams to share their ideas on the respective teachers' questions. |
| **BODY** | **20 minutes** |
| **Modeling** | |
| • Demonstrate venn diagram using rings/loops<br>(i) Throw loop/ring in such a way that three or two loops will intersect each other.<br>• Explain that a single loop can represent a set; two loops represent two sets etc… | • Observe attentively to teachers demonstration of Venn diagram using rings/loops.<br>• Evaluate and discuss the intersection of two or three rings/loops.<br>• Discuss the number of sets represented by the rings/loops. |
| **Guided Practice** | |
| • Define Venn diagram and provide 2 or 3 example problems for class discussion.<br>• Ask focused question on the example problems to stimulate students thinking.<br>• Engage students to answer the example problems. | • Understand and take note of the definition of venn diagram.<br>• Think about how to solve example problems presented by the teacher.<br>• Discuss the solution to the example questions together with the teacher. |
| **Independent Practice** | |
| • Provide two or more exercises on real life problems for independent practice. (Allow for group discussions to solve the problems) | • Individually or in groups think about the problems given and practice to solve. |
| **CONCLUSION** | **15 minutes** |
| • Allow students to present their solution on the board for class discussion.<br>• Make corrections where necessary on the student's presentations and stress on the key points. | • Present and their solutions on the board and explain their answers.<br>• Make necessary corrections if any through consolidating the key points of the lesson highlighted by the teacher. |

## Lesson Evaluation

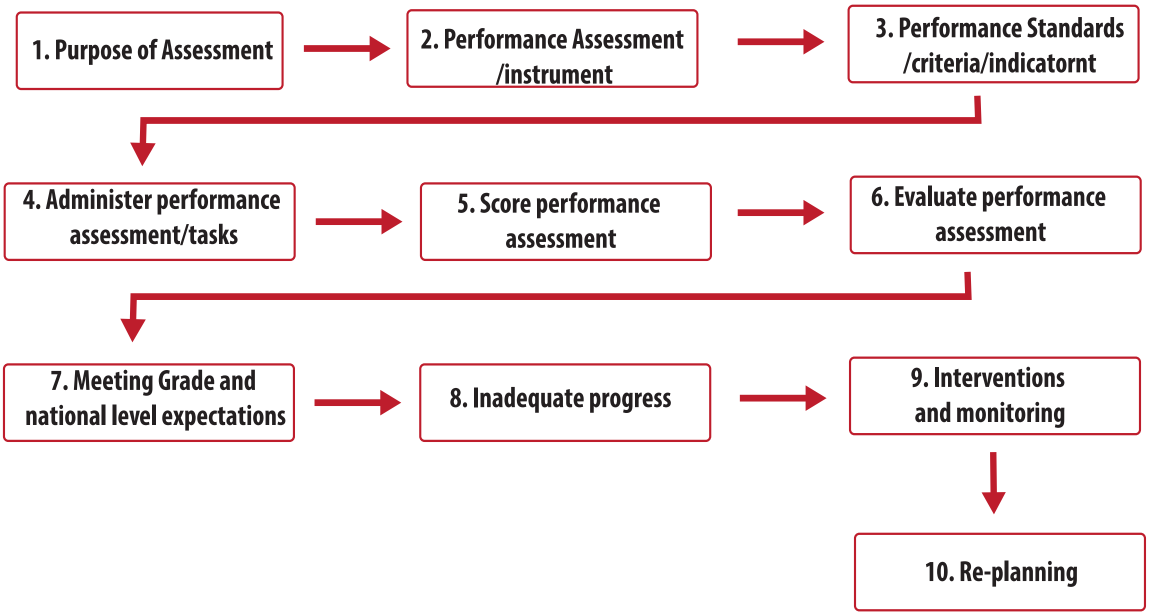
Students can;

- define a venn diagram,
- draw up a venn diagram to fill in information,
- explain the information represented in a venn diagram,
- evaluate a Venn diagram to solve real problems.

# Assessment, Monitoring and Reporting

## What is Standards-Based Assessment (SBA)?

Standards-Based Assessment is an on-going and a systematic process of **assessing, evaluating, reporting** and **monitoring** students' performance and progression towards meeting grade and national level expectations. It is the measurement of students' proficiency on a learning objective or a specific component of a content standard and progression towards the attainment of a benchmark and content standard.




## Purpose of Standards-Based Assessment

Standards-Based Assessment (SBA) serves different purposes. These include instruction and learning purposes. The primary purpose of SBA is to improve student learning so that all students can attain the expected level of proficiency or quality of learning.

Enabling purposes of SBA is to:

- measure students' proficiency on well-defined content standards, benchmarks and learning objectives
- ascertain students' attainment or progress towards the attainment of specific component of a content standard
- ascertain what each student knows and can do and what each student needs to learn to reach the expected level of proficiency
- enable teachers to make informed decisions and plans about how and what they would do to assist weak students to make adequate progress towards meeting the expected level of proficiency
- enable students to know what they can do and help them to develop and implement strategies to improve their learning and proficiency level
- communicate to parents, guardians, and relevant stakeholders the performance and progress towards the attainment of content standards or its components
- compare students' performances and the performances of other students

## Principles of Standards-Based Assessment

The principle of SBA is for assessment to be;

- emphasing on tasks that should encourage deeper learning
- be an integral component of a course, unit or topic and not something to add on afterward
- a good assessment requires clarity of purpose, goals, standards and criteria
- of practices that should use a range of measures allowing students to demonstrate what they know and can do
- based on an understanding of how students learn
- of practices that promote deeper understanding of learning processes by developing students' capacity for self-assessment
- improving performance that involves feedback and reflection
- on-going rather than episodic
- given the required attention to outcomes and processes
- be closely aligned and linked to learning objectives, benchmarks and content standards

## Standards-Based Assessment Types

In standards-Based Assessment, there are three broad assessments types.

**1. Formative Assessment**

Formative assessment includes 'assessment *for* and *as'* and is conducted during the teaching and learning of activities of a topic.

*Purposes of assessment for Learning*

- On-going assessment that allows teachers to monitor students on a day-to-day basis.
- Provide continuous feedback and evidence to the teachers that should enable them to identify gaps and issues with their teaching, and improve their classroom teaching practice.
- Helps students to continuously evaluate, reflect on, and improve their learning

*Purposes of assessment as Learning*

- Occurs when students reflect on and monitor their progress to inform their future learning goals
- Helps students to continuously evaluate, reflect, and improve their own learning
- Helps students to understand the purpose of their learning and clarify learning goals

**2. Summative Assessment**

Summative assessment focuses on 'assessment *of learning'* and is conducted after or at the conclusion of teaching and learning of activities or a topic.

*Purposes of assessment of Learning*

- Help teachers to determine what each student has achieved and how much progress he/she has made towards meeting national and grade-level expectations
- Help teachers to determine what each student has achieved at the end of a learning sequence or a unit.
- Enable teachers to ascertain each student's development against the unit or topic objectives and to set future directions for learning.
- Help students to evaluate, reflect on, and prepare for next stage of learning

**3. Authentic Assessment**

- Is performed in a real life context that approximates as much as possible, the use of a skill or concept in the real world.
- Is based on the development of a meaningful product, performance or process
- Students develop and demonstrate the application of their knowledge, skills, values and attitudes in real life situations which promote and support the development of deeper levels of understanding.

Authentic assessment refers to assessment that:

- Looks at students actively engaged in completing a task that represents the achievement of a learning objective or standard
- Takes place in real life situations
- Asks students to apply their knowledge, skills, values and attitudes in real life situations
- Students are given the criteria against which they are being assessed

**Performance Assessment**

Performance assessment is a form of testing that requires students to perform a task rather than select an answer from a ready-made list. For example, a student may be asked to explain historical events, generate scientific hypotheses, solve math problems, converse in a foreign language, or conduct research on an assigned topic. Teachers, then judge the quality of the student's work based on an agreed-upon set of criteria. It is an assessment which requires students to demonstrate that they have mastered specific skills and competencies by performing or producing something.

Types of performance assessment;

*i. Products*

This refers to concrete tangible items that students create through either the visual, written or auditory media such as;

- Creating a health/physical activity poster
- Video a class game or performance and write a broadcast commentary
- Write a speech to be given at a school council meeting advocating for increased time for health and physical education in the curriculum
- Write the skill cues for a series of skill photo's
- Create a brochure to be handed out to parents during education week
- Develop an interview for a favourite sports person
- Write a review of a dance performance
- Essays
- Projects

*ii. Process Focused Tasks*

It shows the thinking processes and learning strategies students use as they work such as;

- Survival scenarios
- Problem solving initiative/adventure/ activities
- Decision making such as scenario's related to health issues
- Event tasks such as creating a game, choreographing a dance/gymnastics routine, creating an obstacle course
- Game play analysis
- Peer assessment of skills or performances
- Self-assessment activities
- Goal setting, deciding a strategy and monitoring progress towards achievement

*iii. Portfolio*

This refers to a collection of student work and additional information gathered over a period of time that demonstrates learning progress.

*iv. Performances*

It deals with observable affective or psycho-motor behaviours put into action such as;

- Skills check during game play
- Role plays
- Officiating a game
- Debates
- Performing dance/gymnastics routines
- Teaching a skill/game/dance to peers

## Performance Standards

Performance Standards are concrete statements of how well students must learn what is set out in the content standards, often called the “be able to do” of “what students should know and be able to do.” Performance standards are the indicators of quality that specify how competent a students’ demonstration or performance must be. They include explanations of how well students must demonstrate the content, explaining how good is good enough.

Performance standards:

- measure students’ performance and proficiency (using performance indicators) in the use of a specific knowledge, skill, value, or attitude in real life or related situations
- provide the basis (performance indicators) for evaluating, reporting and monitoring students’ level of proficiency in use of a specific knowledge, skills, value, or attitude
- are used to plan for individual instruction to help students not yet meeting expectations (desired level of mastery and proficiency) to make adequate progress towards the full attainment of benchmarks and content standards
- are used as the basis for measuring students’ progress towards meeting grade-level benchmarks and content standards

## Assessment Strategies

It is important for teachers to know that, assessment is administered in different ways. Assessment does not mean a test only. There are many different ways to find out about student's strengths and weaknesses. Relying on only one method of assessing will not reflect student's achievement.

Provided in the appendices is a list of suggested strategies you can use to assess student's performances. These strategies are applicable in all the standards-based assessment types.

## How to Develop effective SBA tools

Teachers are required to use the steps outlined below when planning assessment. These steps will guide you to develop effective assessments to improve student's learning as well as evaluating their progress towards meeting national and grade –level expectations.

**Step.1**: Identify what (knowing/ skills) to assess

**Step.2**: Clearly describe the purpose of assessment(what is that you intend to do by assessing students' perormance

**Step.3:** Prepare the assessment task to assess students' performance

**Step.4**: Devise a scoring method for scoring students' performance (performnace

**Step.5**: Identify the expected level of proficiency

**Step.6**: Develop a descriptor for each performnce standard or criteria

**Step.7**: Identify qualifiers for performance lebels (Qualifiers are used to define each of the levels of performance (e.g. all for proficient, few for progressing, one for not yet or Excellent

## Sample of Assessment Types

**Formative - Sample 1**

**STRAND 4: Statistics and Probability**

**Content Standard 4 :** Students will be able to investigate how to interpret data using methods of exploratory data analysis, develop and evaluate inferences, predictions and arguments that are based on data and understand and apply basic notions of chance and probability.

**Unit:** Sets and Probability

**Benchmark: 11.4.4.6** Apply universal sets and complement of a set, discussing some algebraic laws, and solve related problems.

**Topic:** Subsets and venn diagram

**Lesson Title:** Venn diagrams

**Objective:** By the end of the lesson, students should be able to design venn diagrams and display information onto Venn Diagrams in order find solutions to given problems.

**Materials:** Blank papers, activity handouts, rings/loops, number blocks.

**What is to be assessed? - (ASK-MT)**

| ASK-MT | |
|---|---|
| **Attitudes/Values** | Appreciate and display confidence in drawing up the Venn diagrams and filling in the information from the problems to solve real life problems |
| **Skills** | Drawing up Venn diagrams and filling in the information |
| **Knowledge** | Subsets and Venn Diagrams |
| **Mathematical Thinking** | Think about how information from a real life problem can be displayed onto a Venn diagram to solve problems. |

**Purpose of the assessment**
To measure students' proficiency on the achievement of the benchmark and learning objectives.

**Expected level of proficiency**
Design Venn Diagrams and display information in order to find solutions to given problems.

**Performance Task**
Draw a Venn diagram to represent given information to solve problems.

**Assessment Strategy**
This assessment will be conducted in one lesson as an assessed lesson exercise.

**Assessment Scoring**
Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess the child's learning through the assessment tool a lesson exercise.

| Performance standards/ Criteria | A | B | C | D | Score |
|---|---|---|---|---|---|
| | Advance 10 | Proficient 9-5 | Progressing 3-4 | Not Yet 2 | __/10 Marks |
| Draw a Venn diagram to represent given information to solve related problems. | Correct sketch of the Venn Diagram and represented all information correctly and answered all the related questions with clear calculation steps | Correct sketch of the Venn Diagram and represented all information correctly and answered all the related questions. | Satisfactory sketch of the Venn Diagram and represented most information correctly and answered some of the related questions. | Poor sketch of the Venn Diagram and represented few information and answered only one of the related questions. | |

## Summative - Sample 2

### STRAND 4: Statistics and Probability

**Content Standard 4:** Students will be able to investigate how to interpret data using methods of exploratory data analysis, develop and evaluate inferences, predictions and arguments that are based on data and understand and apply basic notions of chance and probability.

**Unit:** Sets and Probability

**Benchmark 11.4.4.4- 11.4.4.6:** (refer to the benchmarks in unit: sets and probability of Strand 4)

**Topics:** (refer to the topics in unit: sets and probability of Strand 4)

**Lesson topics:** (refer to the lesson topics in unit: sets and probability of Strand 4)

**Instructional Objective (s)**: (refer to unit: sets and probability of Strand 4)

**What is to be assessed? - (VASK-MT)**

| ASK-MT | |
|---|---|
| **Attitudes/Values** | Appreciate the usefulness of Sets and problems display confidences in solving sets related problems. |
| **Skills** | Drawing up Venn diagrams and apply algebraic laws to solve related problems. |
| **Knowledge** | Sets and elements, equality of sets, subsets, universal sets, complements, algebraic laws of sets. |
| **Mathematical Thinking** | Think about how to solve Sets related questions. |

**Purpose of the assessment**

To measure students' proficiency on the achievement of the benchmarks and learning objectives in this unit. (This assessment is to be conducted after teaching the unit)

**Expected level of proficiency**

All students are expected to;

- define sets and elements, equality of sets, subsets and Venn diagram,
- apply universal sets and complement of a set, and use algebraic laws to solve related problems.

**Performance Task**

Students will do an assignment out of 20 marks. You can use other assessment tools (assignment, projects, etc.) assess student's proficiency on these benchmarks.
Task: Students will be given two week to complete this assignment. They are to;

1. Survey a group of students within the school to determine which of the three types of sports; soccer, rugby and basketball they like. A student can choose more than one sports they like.
2. Draw a Venn diagram and explain each of the number in the diagram.

**Assessment Strategy**

An assignment will be used to measure students' proficiency.

**Assessment Scoring**

Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess the child's learning through the assessment tool an assignment.

| Performance standards/ Criteria | A | B | C | D | Score |
|---|---|---|---|---|---|
| | **Advance 20** | **Proficient 13-19** | **Progressing 6-12** | **Not Yet 2-5** | **__/20 Marks** |
| Constraints | Assignment was completed with all constraints and criteria met or exceeded. Reflects attention to detail and quality. | Assignment was completed with some of the constraints and criteria met. Reflects some attention to detail, but quality is minimal. | Assignment was completed with a few of the constraints and criteria met. Reflects minimal effort and lacks detail or quality. | Assignment was not completed and does not reflect the adherence to the constraints or criteria. | |
| Presentation of venn diagram | Correct sketch of the Venn Diagram and represented all information correctly and answered all the related questions with clear calculation steps | Correct sketch of the Venn Diagram and represented all information correctly and answered all the related questions. | Satisfactory sketch of the Venn Diagram and represented most information correctly and answered some of the related questions. | Shows poor knowledge of the person or persons involved in these major events | |
| Analysis | Student carefully analysed the information collected and drew appropriate and inventive conclusions supported by the evidence. | Student shows good effort in analysing the evidence collected. | Student conclusions could have be supported by stronger evidence. Level of analysis could have been deeper | Student conclusions simply involved restating information. Conclusions were not supported by evidence. | |
| Time Management | Assignment completed and turned in on time. Student worked diligently when assignment time was available. Student was on task most of the time. | Assignment was completed, but had notable errors. Student utilized assignment time somewhat efficiently, but spent time socializing. Student was on task 70% - 80% of the time | Assignment was not turned in on time and/or complete. The student was on task less than 60% of the time. | Assignment was not turned in on time and was not completed. Student wasted Assignment time and at times was disruptive to others. | |

## Authentic Assessment- Sample 3

### Strand 2: Geometry, Measurement and Transformation

**Content Standard:** Students will be able to comprehend the meaning and significance of geometry, measurements and spatial relationship including units and system of measurement and develop and use techniques, tools, and formulas for measuring the properties of objects and relationships among the properties and use transformations and symmetry to analyze

**Unit: Trigonometry**

**Benchmark: 11.2.2.3 – 11.2.2.5** (refer to the benchmarks in unit: Trigonometry, Strand 2)

**Topics:** (refer to the Topics in Unit: Trigonometry, Strand 2)

**Lesson topics:** (refer to the topics in unit: Trigonometry, Strand 2)

**Instructional Objective (s)**: (refer to the topics in Unit: Trigonometry, Strand 2)

What is to be assessed?  - (ASK-MT)

| ASK-MT | |
|---|---|
| **Attitudes/Values** | Enjoy solving real life problems using the application of trigonometry |
| **Skills** | Calculate unknown length or angle of a triangle and use the trigonometric ratio to solve everyday situations. |
| **Knowledge** | Application of Trigonometry |
| **Mathematical Thinking** | Thinking about how to solve real life problems using the application of trigonometry |

### Purpose of the assessment

To measure students proficiency on the achievement of the benchmarks and learning objectives in this unit. This assessment is to be conducted after teaching this unit.

### Expected level of proficiency

All students are expected to;

- Use right angle trigonometric ratios to determine an unknown length of a side or the measure of angle.
- Apply the concepts of trigonometry to real world situations.

### Performance Task

Students will do an assignment out of 20 marks. You can use other assessment tools (assignment, projects, etc.) assess student's proficiency on these benchmarks.
*Task*
Your task is to visit a tour, building, tall tree or a peak and measure its height using the concept of the 'Angle of Elevation and Depression'

In group of Students will be given two week to complete this assignment.

*Task Details:* In groups of four (4) or five (5), students are to:

- Visit a tour, building, or peak and measure its height using the concept of the 'Angle of Elevation and Depression'.
- Demonstrate a sketch/diagram showing all the necessary measurements.
- Show all necessary calculations involved to find the height.

**Assessment Strategy**

An assignment will be used to measure students' proficiency.

**Assessment Scoring**

Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess the child's learning through the assessment tool an assignment.

| Criteria | Model/Exemplar | Proficient | Developing | Beginning | Score |
|---|---|---|---|---|---|
| | (20 points | (13-19 points) | (6-12 points) | (2-5 points) | ___/20 |
| Quality | Maximum effort was put forth to complete the project in a professional manner. Project demonstrates a high degree of quality and attention to detail. Workmanship is excellent.. | Some effort was made to com-plete the project to a level that was sufficient for grading, but does not meet a professional level of quality or appearance. Workmanship is of acceptable quality. | Minimal effort was made to complete the project and the quality and workmanship is sub-par, but still meets the minimal standard. | Little or no effort was made to produce a quality project. Project obviously does not meet minimal standards. | |
| Mathematical Calculations | All calculations are very clear, organized, and neatly completed with no inaccuracies. | All calculations are clear, organized, and neatly completed with 1-2 inaccuracies. | Most calculations are clear, organized, and neatly completed with 3-4 inaccuracies. | Calculations are unclear and disorganized and 5 or more inaccuracies may be present. | |
| Diagrams | All diagrams are neatly produced; appropriately scaled and labeled. | All diagrams are neatly produced; not appropriately scaled and labeled. | All diagrams are not neatly produced; not appropriately scaled and labeled. | All diagrams are not neatly produced; not appropriately scaled and labeled. | |
| Effort and Collaboration | An exceeding amount of time and effort are present and the task responsibilities were shared equitably among group partners. | A substantial amount of effort is present and the task responsibilities were shared equitably among group partners | An average amount of effort is present, and the task responsibilities were not shared equitably among group partners. | A poor amount of effort is present, and the task responsibilities were not shared equitably among group partners. | |

## STEAM Assessment

### Sample 4: Integrated Strands in relation to the project from integrated subjects

**Unit:** (Integrated Units from all Subjects in this project)

**Content Standard:** (Integrated Content Standard from all Subjects in project)

**Benchmark:** (Integrated Benchmarks from all Subjects in this project)

**Topic:** (Integrated Topics from all Subjects in this project)

**Lesson topic:** (Integrated Topics from all Subjects in concern)

**Instructional Objective** (s): Students will be able to;

- Create a STEAM project "building a prototype model of a catapult launching system" to enhance their understand of this concept

| ASK-MT | |
|---|---|
| **Values/Attitudes** | Appreciate the beauty of the application of mathematics during the designing process of the project. |
| **Skills** | Calculating size and space Time management and efficiency, Linear measurement and scaling techniques, Calculating mechanical advantage |
| **Knowledge** | Size and space Time management and efficiency, Linear measurement and scaling techniques |
| **Mathematical Thinking** | Think about how to integrate and apply the mathematical knowledge in the project |

## What is to be assessed? - (KSAVs)

Integrated subjects concepts used designing the projects.

## Purpose of the assessment

To measure students proficiency on the achievement of the benchmarks and learning objectives for integrated subjects in the project. (STEAM Project)

### Expected level of proficiency

All students are expected to;

- "Build a prototype model of a catapult launching system" through integrating concepts learned in other subjects.

## Performance Task

Student will carry out a project worth 30 marks that should contribute to the School Learning Improvement Program (SLIP). This project will assess students proficiency on the mentioned benchmarks. In order for this assessment type to attain its intended purpose the following must be done carefully;

**Task:** Students will be given a month to complete this project.

(1) all grade 11 Mathematics teachers discuss the STEAM project with their HOD

(2) the Mathematics HOD brings this project to the attention of the Head Teacher hence it will involve the learning of all grade 11 classes in the school.

(3) once approved by the Head Teacher, the Mathematics HOD now convenes a meeting with all other subject HOD to integrate this project into their learning. HOD for Mathematics will have developed criteria already and will discuss around that.

4) the HOD for other subjects meet with their respective subject teachers to gauge their views and write up criteria's with reference to the theme of the project, "STEM Design and Engineering Challenge" bringing out the essence of their subjects in this project.

(5) the Head Teacher then convenes a meeting with all teachers as they are now aware of the project. HOD for respective subjects give feedback from their meetings. Issues concerning this project must be ironed out and all subjects now carry out this assessment, starting with Mathematics.

The grade 11 Mathematics teacher will now do the following;

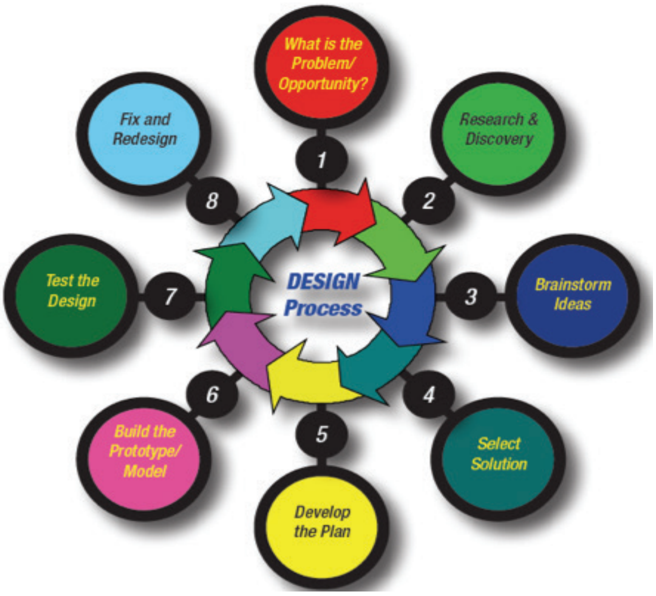
(i) Group the students into groups of 6 to design ( drawing and manual) a tangible technology that will enhance the notion of "building a prototype model of a catapult launching system"

(ii) The teacher then assesses their designs and the best designs now compete with the other best designs from other grade 11 classes.

(iii) All the best designers now create models of their designs with assistance from their class members. At this stage the other subjects now carry forward this assessed projects theme, 'building a prototype model of a catapult launching system" however in the context of their subjects. STEAM is an integrated approach of teaching. All subjects must incorporate the theme put forward by Mathematics. They develop criteria that should address this theme. For instance; Technology and Industrial Arts (TIA) will develop criteria that will engage the students to construct the models. Science teachers will develop criteria to test students' knowledge of the Science process of Engineering Design thinking when they create the models around the theme of "prototype model of a catapult launching system". The English subject teachers will set criteria and guidelines for students on how to write reports so they write to tell others what they have learned and experienced. They must also be given guidelines to writing report. Students get to write report of how they designed this technology. The Mathematics teacher will provide criteria for the students in terms of the measurements, angles and operations used to work out the size and shape of the technology.

Task: Students will be given 6 weeks to complete this project. They are to;

- Design and build a prototype model of a catapult launching system that is easy to use and easy to transport.
- Follow the Design Process to prepare their prototype model in time.
- Write and prepare a short presentation to explain the catapult that was built and the process of building it.



**Design Specification:**

The catapult should be designed to launch a golf ball at least fifteen feet, to a 18cm x 18cm target.

- The catapult should include a system for determining range, reliability, and accuracy.
- The catapult should be mobile, yet stable. Outriggers or other support systems need to be included to maintain stability when the launcher is used.
- The catapult should be no larger than 30cm long x 30 cm deep x 90cm tall.
- The catapult should feature a locking pin or trigger that activates the catapult to launch.
- Your team should prepare to deliver a presentation about the merits of your catapult model and design.

## Assessment Strategy

Design Project will be used to measure student's proficiency.

The students will be reinforced in the following STEAM concepts.

**Science**

- Applications of simple machines, including wheels and axles, levers, and pulleys
- Balance and equilibrium
- Energy transformations, such as rotary motion to linear motion
- Mechanical advantage

**Technology and Engineering**

- Prototyping and modelling
- Invention and innovation
- Structural integrity/strength
- Brainstorming and problem solving
- Trial and error engineering concepts

**ARTS**

- Perspective drawing (3D)
- Critical Thinking Process
- Applying the Principles of Graphic design
  - Balance
  - proximity
  - Repetition
  - colour
  - negative/positive space
  - Applying creative process

**Math**

- Calculating size and space
- Time management and efficiency
- Linear measurement and scaling techniques
- Calculating mechanical advantage

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# Appendices

## APPENDIX 1: BLOOM'S TAXONOMY

| LEVEL OF UNDERSTANDING | KEY VERBS |
|---|---|
| **CREATING**<br>Can the student create a new product or point of view? | Construct, design, and develop, generate, hypothesize, invent, plan, produce, compose, create, make, perform, plan, produce, assemble, formulate, |
| **EVALUATING**<br>Can the student justify a stand or decision? | Appraise, argue, assess, choose, conclude, critique, decide, defend, evaluate, judge, justify, predict, prioritize, provoke, rank, rate, select, support, monitor, |
| **ANALYSING**<br>Can the student distinguish between the different parts? | Analysing, characterize, classify, compare, contrast, debate, criticise, deconstruct, deduce, differentiate, discriminate, distinguish, examine, organize, outline, relate, research, separate, experiment, question, test, |
| **APPLYING**<br>Can the student use the information in a new way | Apply, change, choose, compute, dramatize, implement, interview, prepare, produce, role play, select, show, transfer, use, demonstrate, illustrate, interpret, operate, sketch, solve, write, |
| **UNDERSTANDING**<br>Can the student comprehend ideas or concepts? | Classify, compare, exemplify, conclude, demonstrate, discuss, explain, identify, illustrate, interpret, paraphrase, predict, report, translate, describe, classify, |
| **REMEMBERING**<br>Can the student recall or remember the information? | Define, describe, draw, find, identify, label, list, match, name, quote, recall, recite, tell, write, duplicate, memorise, recall, repeat, reproduce, state, |

## APPENDIX 2: 21ST CENTURY SKILLS

| | |
|---|---|
| **WAYS OF THINKING** | **Creativity and innovation**<br>• Think creatively<br>• Work creatively with others<br>• Implement innovations<br>**Critical thinking, problem solving and decision making**<br>• Reason effectively and evaluate evidence<br>• Solve problems<br>• Articulate findings<br>**Learning to learn and meta-cognition**<br>• Self-motivation<br>• Positive appreciation of learning<br>• Adaptability and flexibility |
| **WAYS OF WORKING** | **Communication**<br>• Competency in written and oral language<br>• Open minded and preparedness to listen<br>• Sensitivity to cultural differences<br>**Collaboration and teamwork**<br>• Interact effectively with others<br>• Work effectively in diverse teams<br>• Prioritise, plan and manage projects |
| **TOOLS FOR WORKING** | **Information literacy**<br>• Access and evaluate information<br>• Use and manage information<br>• Apply technology effectively<br>**ICT literacy**<br>• Open to new ideas, information, tools and ways of thinking<br>• Use ICT accurately, creatively, ethically and legally<br>• Be aware of cultural and social differences<br>• Apply technology appropriately and effectively |
| **LIVING IN THE WORLD** | **Citizenship – global and local**<br>• Awareness and understanding of rights and responsibilities as a global citizen<br>• Preparedness to participate in community activities<br>• Respect the values and privacy of others<br>**Personal and social responsibility**<br>• Communicate constructively in different social situations<br>• Understand different viewpoints and perspectives<br>**Life and career**<br>• Adapt to change<br>• Manage goals and time<br>• Be a self-directed learner<br>• Interact effectively with others |

## APPENDIX 3: TEACHING AND LEARNING STRATEGIES

| STRATEGY | TEACHER | STUDENTS |
|---|---|---|
| **CASE STUDY**<br>**Used to extend students' understanding of real life issues** | Provide students with case studies related to the topic of the lesson and allow them to analyse and evaluate. | Study the case study and identify the problem addressed. They analyse the problem and suggest solutions supported by conceptual justifications and make presentations. This enriches the students' existing knowledge of the topic. |
| **DEBATE**<br>**A method used to increase students' interest, involvement and participation** | Provide the topic or question of debate on current issues affecting a bigger population, clearly outlining the expectations of the debate.<br>Explain the steps involved in debating and set a criteria/standard to be achieved. | Conduct researches to gather supporting evidence about the selected topic and summarising the points.<br>They are engaged in collaborative learning by delegating and sharing tasks to group members.<br>When debating, they improve their communication skills. |
| **DISCUSSION**<br>**The purpose of discussion is to educate students about the process of group thinking and collective decision.** | The teacher opens a discussion on certain topic by asking essential questions.<br>During the discussion, the teacher reinforces and emphasises on important points from students responses. Teacher guide the direction to motivate students to explore the topic in greater depth and the topic in more detail.<br>Use how and why follow-up questions to guide the discussion toward the objective of helping students understand the subject and summarise main ideas. | Students ponder over the question and answer by providing ideas, experiences and examples.<br>Students participate in the discussion by exchanging ideas with others. |
| **GAMES AND SIMULATIONS**<br>**Encourages motivation and creates a spirit of competition and challenge to enhance learning** | Being creative and select appropriate games for the topic of the lesson.<br>Give clear instructions and guidelines. The game selected must be fun and build a competitive spirit to score more than their peers to win small prices. | Go into groups and organize.<br>Follow the instructions and play to win |
| **OBSERVATION**<br>**Method used to allow students to work independently to discover why and how things happen as the way they are. It builds curiosity.** | Give instructions and monitor every activity students do | Students possess instinct of curiosity and are curious to see the things for themselves and particularly those things which exist around them. A thing observed and a fact discovered by the child for himself becomes a part of mental life of the child. It is certainly more valuable to him than the same fact or facts learnt from the teacher or a book.<br>Students<br>Observe and ask essential questions<br>Record<br>Interpret |

| | | |
|---|---|---|
| **PEER TEACHING & LEARNING** *(power point presentations, pair learning)* Students teach each other using different ways to learn from each other. It encourages; team work, develops confidence, feel free to ask questions, improves communication skills and most importantly develop the spirit of inquiry. | Distribute topics to groups to research and teach others in the classroom.<br>Go through the basics of how to present their peer teaching. | Go into their established working groups.<br>Develop a plan for the topic.<br>Each group member is allocated a task to work on.<br>Research and collect information about the topic allocated to the group. Outline the important points from the research and present their findings in class. |
| PERFORMANCE-RELATED TASKS (dramatization, song/lyrics, wall magazines) Encourages creativity and take on the overarching ideas of the topic and are able to recall them at a later date | Students are given the opportunity to perform the using the main ideas of a topic.<br>Provide the guidelines, expectations and the set criteria | Go into their established working groups.<br>Being creative and create dramas, songs/lyrics or wall magazines in line with the topic. |
| PROJECT (individual/group) Helps students complete tasks individually or collectively | Teacher outline the steps and procedures of how to do and the criteria | Students are involved in investigations and finding solutions to problems to real life experiences. They carry out researches to analyse the causes and effects of problems to provide achievable solutions. Students carefully utilise the problem-solving approach to complete projects. |
| USE MEDIA & TECHNOLOGY to teach and generate engagement depending on the age of the students | Show a full movie, an animated one, a few episodes form documentaries, you tube movies and others depending on the lesson.<br>Provide questions for students to answer before viewing | Viewing can provoke questions, debates, critical thinking, emotion and reaction.<br>After viewing, students engage in critical thinking and debate |

# APPENDIX 4 : ASSESSMENT STRATEGIES

| STRATEGY | DESCRIPTION |
|---|---|
| **ANALOGIES** | Students create an analogy between something they are familiar with and the new information they have learned. When asking students to explain the analogy, it will show the depth of their understanding of a topic. |
| **CLASSROOM PRESENTATIONS** | A classroom presentation is an assessment strategy that requires students to verbalize their knowledge, select and present samples of finished work, and organize their thoughts about a topic in order to present a summary of their learning. It may provide the basis for assessment upon completion of a student's project or essay. |
| **CONFERENCES** | A conference is a formal or informal meeting between the teacher and a student for the purpose of exchanging information or sharing ideas. A conference might be held to explore the student's thinking and suggest next steps; assess the student's level of understanding of a particular concept or procedure; and review, clarify, and extend what the student has already completed |
| **DISCUSSIONS** | Having a class discussion on a unit of study provides teachers with valuable information about what the students know about the subject. Focus the discussions on higher level thinking skills and allow students to reflect their learning before the discussion commences. |
| **ESSAYS** | An essay is a writing sample in which a student constructs a response to a question, topic, or brief statement, and supplies supporting details or arguments. The essay allows the teacher to assess the student's understanding and/or ability to analyse and synthesize information. |
| **EXHIBITIONS/ DEMONSTRATIONS** | An exhibition/demonstration is a performance in a public setting, during which a student explains and applies a process, procedure, etc., in concrete ways to show individual achievement of specific skills and knowledge. |
| **INTERVIEWS** | An interview is a face-to-face conversation in which teacher and student use inquiry to share their knowledge and understanding of a topic or problem, and can be used by the teacher to explore the student's thinking; assess the student's level of understanding of a concept or procedure and gather information, obtain clarification, determine positions, and probe for motivations. |
| **LEARNING LOGS** | A learning log is an ongoing, visible record kept by a student and recording what he or she is doing or thinking while working on a particular task or assignment. It can be used to assess student progress and growth over time. |
| **OBSERVATION** | Observation is a process of systematically viewing and recording students while they work, for the purpose of making programming and instruction decisions. Observation can take place at any time and in any setting. It provides information on students' strengths and weaknesses, learning styles, interests, and attitudes. |
| **PEER ASSESSMENT** | Assessment by peers is a powerful way to gather information about students and their understanding. Students can use set criteria to assess the work of their classmates. |
| **PERFORMANCE TASKS** | During a performance task, students create, produce, perform, or present works on "real world" issues. The performance task may be used to assess a skill or proficiency, and provides useful information on the process as well as the product. |
| **PORTFOLIOS** | A portfolio is a collection of samples of a student's work, and is focused, selective, reflective, and collaborative. It offers a visual demonstration of a student's achievement, capabilities, strengths, weaknesses, knowledge, and specific skills, over time and in a variety of contexts. |

| | |
|---|---|
| **QUESTIONS AND ANSWERS (ORAL)** | In the question–and-answer strategy, the teacher poses a question and the student answers verbally, rather than in writing. This strategy helps the teacher to determine whether students understand what is being, or has been, presented, and helps students to extend their thinking, generate ideas, or solve problems. |
| **QUIZZES, TESTS, EXAM-INATIONS** | A quiz, test, or examination requires students to respond to prompts in order to demonstrate their knowledge (orally or in writing) or their skills (e.g., through performance). Quizzes are usually short; examinations are usually longer. Quizzes, tests, or examinations can be adapted for exceptional students and for re-teaching and retesting. |
| **QUESTIONNAIRES** | Questionnaires can be used for a variety of purposes. When used as a formative assessment strategy, they provide teachers with information on student learning that they can use to plan further instruction. |
| **RESPONSE JOURNALS** | A response journal is a student's personal record containing written, reflective responses to material he or she is reading, viewing, listening to, or discussing. The response journal can be used as an assessment tool in all subject areas. |
| **SELECTED RESPONSES** | Strictly speaking a part of quizzes, tests, and examinations, selected responses require students to identify the one correct answer. The strategy can take the form of multiple-choice or true/false formats. Selected response is a commonly used formal procedure for gathering objective evidence about student learning, specifically in memory, recall, and comprehension. |
| **STUDENT SELF-ASSESSMENTS** | Self-assessment is a process by which the student gathers information about, and reflects on, his or her own learning. It is the student's own assessment of personal progress in terms of knowledge, skills, processes, or attitudes. Self-assessment leads students to a greater awareness and understanding of themselves as learners. |

## APPENDIX 5: Standard-based Lesson Plan Template

**Strand:** ..........................................................................................

**Unit:** ..........................................................................................

**Content Standard:** ..........................................................................................

**Benchmark:** ..........................................................................................

**Topic :** ..........................................................................................

**Lesson Title:** ..........................................................................................

**Lesson Objective (s):** By the end of the lesson, students will be able to;

- ..........................................................................................
- ..........................................................................................

**Materials:**

| Key Concepts(ASK-MT) | |
|---|---|
| Attitudes / Values | |
| Skills | |
| Knowledge | |
| Mathematics Thinking | |

**Lesson Procedure**

| Teacher Activity | Student Activity |
|---|---|
| Introduction | (time in minutes) |
| | |
| Body | (time in minutes) |
| Modeling | |
| | |
| Guided Practice | |
| | |
| Independent Practice | |
| | |
| Conclusion | (time in minutes) |
| | |

**Assessment/Lesson Evaluation**

..........................................................................................

..........................................................................................

# APPENDIX 6: Standard based Lesson Plan template - Integrating STEAM

**Strand:** ..........................................................................................

**Unit:** ..........................................................................................

**Content Standard:** ..........................................................................................

**Benchmark:** ..........................................................................................

**Topic :** ..........................................................................................

**Lesson Title:** ..........................................................................................

**Lesson Objective (s):** By the end of the lesson, students will be able to;

- ..........................................................................................

**Essential Questions**

- ..........................................................................................

**Materials:**

| Key Concepts(ASK-MT) | |
|---|---|
| Attitudes / Values | |
| Skills | |
| Knowledge | |
| Mathematics Thinking | |
| STEAM Knowledge and Skills | |
| Skills | |
| Knowledge | |
| STEAM Performance Indicator: | |

**Lesson Procedure**

| Teacher Activity | Student Activity |
|---|---|
| Introduction | (time in minutes) |
| | |
| Body | (time in minutes) |
| Modeling | |
| | |
| Guided Practice | |
| | |
| Independent Practice | |
| | |
| Conclusion | (time in minutes) |
| | |

**Assessment/Lesson Evaluation**

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## APPENDIX 7: Time Allocation

| Grade 9 and 10 | No. lesson/ wk | Min/week | Grade 11 and 12 | No. lessons/ wk | Min/week |
|---|---|---|---|---|---|
| English | 6 | 6 x 40 = 240 | Applied English | 6 | 6 x 40 = 240 |
| Mathematics | 5 | 5 x 40 = 200 | L & L | 6 | 6 x 40 = 240 |
| Science | 5 | 5 x 40 = 200 | Advanced Math | 5 | 5 x 80 = 400 |
| Social Science | 5 | 5 x 40 = 200 | General Math | 6 | 8 x 40 = 320 |
| PD | 5 | 5 x 40 = 200 | Physics | 6 | 6 x 40 = 240 |
| Business Studies | 5 | 5 x 40 = 200 | Biology | 6 | 6 x 40 = 240 |
| Design & Technology | 5 | 5 x 40 = 200 | Chemistry | 6 | 6 x 40 = 240 |
| Arts | 5 | 5 x 40 = 200 | Applied Science | 6 | 6 x 40 = 240 |
| CCVE | 3 | 3 x 40 = 120 | Geology | 6 | 6 x 40 = 240 |
| RI | 1 | 1 x 60 = 60 | Geography | 6 | 6 x 40 = 240 |
| Agriculture | 5 | 5 x 40 = 200 | History | 6 | 6 x 40 = 240 |
| TOTALS | 50 | 2020 min | Legal Studies | 6 | 6 x 40 = 240 |
| | | | HPE | 6 | 6 x 40 = 240 |
| | | | PE | 6 | 6 x 40 = 240 |
| | | | RE | 1 | 1 x 60 = 60 |
| | | | Business Studies | 6 | 6 x 40 = 240 |
| | | | Accounting | 6 | 6 x 40 = 240 |
| | | | Economics | 6 | 6 x 40 = 240 |
| | | | Design & Tech | 6 | 6 x 40 = 240 |
| | | | Computer Studies | 6 | 6 x 40 = 240 |
| | | | ICT | 6 | 6 x 40 = 240 |
| | | | CCVE | 2 | 3 x 40 = 120 |
| | | | ANRM | 6 | 6 x 40 = 240 |
| | | | TOTALS | 128 lessons/ wk | 5,460 min/wk |