General Mathematics

Senior High

Grade 12

Teacher Guide

Standards-Based

Department of Education

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Contents

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Acronyms

AAL	Assessment As Learning
AFL	Assessment For Learning
AOL	Assessment of Learning
SSBoS	Secondary School Board of Studies
CDD	Curriculum Development Division
CRS	Classroom Response System
DA	Diagnostic Assessment
HOD	Head of Department
IHD	Integral Human Development
MTDG	Medium Term Development Goal
NGO	Non-Government Organisations
PBA	Performance Based Assessments
PNG	Papua New Guinea
SAC	Subject Advisory Committee
SBC	Standards Based Curriculum
SBE	Standards Based Education
SCG	Subject Curriculum Group
STEAM	Science, Technology, Engineering, Arts and Mathematics
STEM	Science, Technology, Engineering, and Mathematics

Secretary's Message

The aims and goals of SBC is to identify the important knowledge, skills, values and attitudes that all students are expected to acquire and master in order to effectively function in society and actively function in society and actively contribute to its development, student's welfare and enable them to acquire and apply 21st Century.

The aims of teaching and learning mathematics are to encourage and enable students to recognise that mathematics permeates the world around us. Students should be encouraged to appreciate the usefulness, power and beauty of mathematics and become confident in using mathematics to analyse and solve problems both in school and in real-life situations.

A variety of teaching and learning activities provides students with ideas to motivate students to learn, and make learning relevant, interesting and enjoyable. Teachers should provide students opportunity to develop mathematical curiosity and use inductive and deductive reasoning when solving problems and develop the knowledge, skills and attitudes necessary to pursue further studies in Mathematics.

Learning Mathematics enable students to develop abstract, logical and critical thinking and the ability to reflect critically upon their work and the work of others, develop a critical appreciation of the use of information and communication technology in mathematics appreciate the international dimension of mathematics and its multicultural and historical perspectives.

Teachers are encouraged to integrate Mathematics activities with other subjects, where appropriate, so that students can see the interrelationship between subjects and that the course they are studying provides a holistic education and a pathway for the future.

I commend and approve this Grade 12 General Mathematics Teacher Guide to be used in all Senior High Schools throughout Papua New Guinea.

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UKE W. KOMBRA, (PhD)
Secretary for Education

Introduction

The aims of teaching and learning mathematics are to encourage and enable students to recognise that mathematics permeates the world around us. Students should be encouraged to appreciate the usefulness, power and beauty of mathematics and become confident in using mathematics to analyse and solve problems both in school and in real-life situations.

The curriculum is designed to ensure that students build a solid foundation in mathematics by connecting and applying mathematical concepts in a variety of ways and situations. To support this process, teachers should provide students opportunity to develop mathematical curiosity and use inductive and deductive reasoning when solving problems and develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics.

Mathematics aims to provide a meaningful pedagogical framework for teaching and learning essential and in demand knowledge, skills, values, and attitudes that are required for the preparation of students for careers, higher education and citizenship in the 21st Century.

Students should be prepared to gather and understand information, analyse issues critically, learn independently or collaboratively, organize and communicate information, draw and justify conclusions, create new knowledge, and act ethically.

Students' employability will be enhanced through the study and application of STEAM principles. STEAM is an integral component of the core curriculum. All students are expected to study STEAM and use STEAM related skills to solve problems relating to both the natural and the physical environments. The aim of STEAM education is to create a STEAM literate society. It is envisioned that the study of STEAM will motivate students to pursue and take up academic programs and careers in STEAM related fields. STEAM has been embedded in the Mathematics curriculum. Equal opportunities should be provided for all students to learn, apply and master STEAM principles and skills.

Time allocation for General Mathematics is **320** minutes for Grade 12.

Structure of the Teacher Guide

There are four main components to this teacher guide. They provide essential information on what all teachers should know and do to effectively implement the Mathematics curriculum.

Part 1 provides generic information to help the teachers to effectively use the teacher guide and the syllabus to plan, teach and assess students' performance and proficiency on the national content standards and grade-level benchmarks. The purpose of the teacher guide, syllabus and teacher guide alignment, and the four pillars of PNG SBC, which are, morals and values education, cognitive and high level thinking, and 21st Century thinking skills, STEAM, and core curriculum. These are explained to inform as well as guide the teachers so that they align SBE/SBC aims and goals, overarching and SBC principles, content standards, grade-level benchmarks, learning objectives and best practice when planning lessons, teaching, and assessing students.

Part 2 provides information on the strands, units, topics and learning objectives. How topics and learning objectives are derived is explained to the teachers to guide them to use the learning objectives provided for planning, instruction and assessment. Teachers are encouraged to develop additional topics and learning objectives to meet the learning needs of their students and communities where necessary.

Part 3 provides information on SBC planning to help guide the teachers when planning SBC lessons. Elements and standards of SBC lesson plans are described as well as how to plan for underachievers, use evidence to plan lessons, and use differentiated instruction, amongst other teaching and learning strategies.

Part 4 provides information on standards-based assessment, inclusive of performance assessment and standards, standards-based evaluation, standards-based reporting, and standards-based monitoring. This information should help the teachers to effectively assess, evaluate, report and monitor demonstration of significant aspects of a benchmark.

The above components are linked and closely aligned. They should be connected to ensure that the intended learning outcomes and the expected quality of education standards are achieved. The close alignment of planning, instruction and assessment is to the attainment of learning standards.

Purpose of the Teacher Guide

This teacher guide describes what all teachers should know and do to effectively plan, teach, and assess the Grade 12 General Mathematics content to attain the required learning and proficiency standards. The overarching purpose of this teacher guide is to help teachers to effectively plan, teach, assess, evaluate, report and monitor students' learning and mastery of national and grade-level expectations.
That is, the essential knowledge, skills, values and attitudes (KSVAs) described in the content standards and grade-level benchmarks, and their achievement of the national and grade-level proficiency standards.

Ample information with thorough guidelines is provided for the teacher.

Thus, the teacher is expected to;

- understand the significance of aligning all the elements of Standards-Based Curriculum (SBC) as the basis for achieving the expected level of education quality,
- effectively align all the components of SBC when planning, teaching, and assessing students' learning and levels of proficiency,
- effectively translate and align the Biology syllabi and teacher guide to plan, teach and assess different Biology units and topics, and the KSVAs described in the grade-level benchmarks,
- understand the Biology national content standards, grade-level benchmarks, and evidence outcomes,
- effectively make sense of the content (KSVAs) described in the Biology national content standards and the essential components of the content de scribed in the grade-level benchmarks,
- effectively guide students to progressively learn and demonstrate proficiency on a range of Scientific skills, processes, concepts, ideas, principles, practices, values and attitudes,
- confidently interpret, translate and use Biology content standards and benchmarks to determine the learning objectives and performance standards, and plan appropriately to enable all students to achieve these standards,
- embed the core curriculum in their Biology lesson planning, instruction, and assessment to permit all students to learn and master the core KSVAs required of all students,
- provide opportunities for all students to understand how STEAM has and continues to shape the social, political, economic, cultural, and environment contexts and the consequences, and use STEAM principles, skills, processes, ideas and concepts to inquire into and solve problems relating to both the natural and physical (man-made) worlds as well as problems created by STEAM,
- integrate cognitive skills (critical, creative, reasoning, decision-making, and problem-solving skills), high level thinking skills (analysis, synthesis and evaluation skills), values (personal, social, work, health, peace, relationship, sustaining values), and attitudes in lesson planning, instruction and assessment,

- meaningfully connect what students learn in Biology with what is learnt in other subjects to add value and enhance students' learning so that they can integrate what they learn and develop in-depth vertical and horizontal understanding of subject content,
- formulate effective SBC lesson plans using learning objectives identified for each of the topics,
- employ SBC assessment approaches to develop performance assessments to assess students' proficiency on a content standard or a component of the content standard described in the grade-level benchmark,
- effectively score and evaluate students' performance in relation to a core set of learning standards or criteria, and make sense of the data to ascertain students' expected proficiency status of progress towards me standards, and use evidence from the assessment of students' performance to develop effective evidence-based intervention strategies to help students' making inadequate or slow progress towards meeting the grade-level and national expectations to improve their learning and performance.

How to use the Teacher Guide

Teacher Guide provides essential information about what the teacher needs to know and do to effectively plan, teach and assess students learning and proficiency on learning and performance standards. The different components of the teacher guide are closely aligned with SBC principles and practice, and all the other components of PNG SBC. It should be read in conjunction with the syllabus in order to understand what is expected of teachers and students to achieve the envisaged quality of education outcomes.

The first thing teachers should do is to read and understand each of the sections of the teacher guide to help them understand the key SBC concepts and ideas, alignment of PNG SBC components, alignment of the syllabus and teacher guide, setting of content standards and grade-level benchmarks, core curriculum, STEAM, curriculum integration, essential knowledge, skills, values and attitudes, strands, units and topics, learning objectives, SBC lesson planning, and SBC assessment. A thorough understanding of these components will help teachers meet the teacher expectations for implementing the SBC curriculum, and therefore the effective implementation of the Grade 12 General Mathematics Curriculum. Based on this understanding, teachers should be able to effectively use the teacher guide to do the following.

Identifying topics from benchmarks

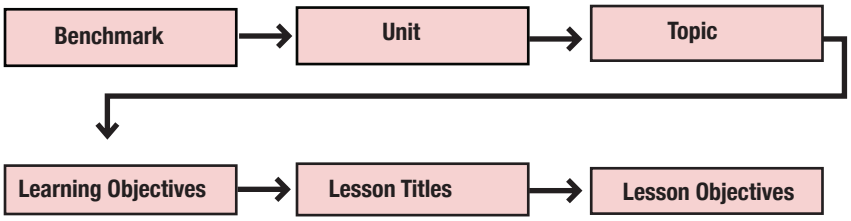
In order to identify the topic from the benchmark, the benchmark needs to be unpack. When unpacking a benchmark, identify what students will be able to know and do in order to mastered the benchmark.

Below is a description of how topics and learning objectives were derived from the grade-level benchmarks.

1. Write out the benchmark that you want to unpack.
2. Write the verbs (skills/actions) – Higher order thinking skills.
3. Underline or highlight the big idea (content) in the benchmark. The big idea (content) is the topic derived from the benchmark.
4. Write essential questions that would be engaging for students.
5. Develop sub-topics from the big idea (topic).
6. Write learning objectives according to the sub-topics.
7. Write lesson topics from the learning objectives.

Determine Lesson Objectives and Lesson Titles

Topics and learning objectives have been identified and described in the Teacher Guide. Learning objectives are derived from topics that are extracted from the grade-level benchmarks. Lesson titles are deduced from the learning objectives. Teachers should familiarise themselves with this process as it is essential for lesson planning, instruction and assessment. However, depending on the context and students' learning abilities, teachers would be required to determine additional lesson objectives and lesson titles. Teachers should use the examples provided in this teacher guide to formulate additional lesson objectives and lesson titles to meet the educational or learning needs of their students.



Identify and Teach Grade Appropriate Content

Grade appropriate content has been identified and scoped and sequenced using appropriate content organisation principles. The content is sequenced using the spiraling sequence principles. This sequencing of content will enable students to progressively learn the essential knowledge, skills, values and attitudes as they progress further into their schooling. What students learn in previous grades is reinforced and deepens in scope with an increase in the level of complexity and difficulty in the content and learning activities. It is important to understand how the content is organised so that grade appropriate content and learning activities can be selected, if not already embedded in the benchmarks and learning objectives, to not only help students learn and master the content, but ensure that what is taught is rigorous, challenging, and comparable.

Integrate the Core Curriculum in Lesson Planning, Instruction and Assessment

Teachers should use this teacher guide to help them integrate the core curriculum – values, cognitive and high level skills, 21st Century skills, STEAM principles and skills, and reading, writing, and communication skills in their lesson planning, instruction and assessment. All students in all subjects are required to learn and master these skills progressively through the education system.

Integrate Cognitive, High Level, and 21st Century Skills in Lesson Planning, Instruction and Assessment

Teachers should integrate the cognitive, high level and 21st Century skills in their annual teaching programs, and give prominence to these skills in their lesson preparation,teaching and learning activities, performance assessment, and performance standards for measuring students' proficiency on these skills.
Mathematics addresses the skills and processes of solving problems arising in everyday life, society and the workplace. Thus,students will be able to make informed decisions, problem-solving and management knowledge, skills, values and attitudes in Mathematics. This enables them to function effectively in the work and higher education environments as productive and useful citizens of a culturally diverse and democratic society in an interdependent world.

In addition, it envisaged all students attaining expected proficiency levels in these skills and will be ready to pursue careers and higher education academic programs that demand these skills, and use them in their everyday life after they leave school at the end of Grade 12. Teachers should use the teacher guide to help them to effectively embed these skills, particularly in their lesson planning and in the teaching and learning activities as well as in the assessment of students' application of the skills.

Ingrate Mathematics Values and Attitudes in Lesson Planning, Instruction and Assessment

In Mathematics, students are expected to learn, promote and use work, relationship, peace, health, social, personal, family, community, national and global values in the work and study environments as well as in their conduct as community, national and global citizens. Teachers should draw from the information and suggestions provided in the syllabus and teacher guide to integrate values and attitudes in their lesson planning, instruction, and assessment. They should report on students' progression towards internalizing different values and attitudes and provide additional support to students who are yet to reach the internalization stage to make positive progress towards this level.

Integrate Science, Technology, Engineering, Arts and Mathematics (STEAM)

Teachers should draw from both the syllabus and teacher guide in order to help them integrate STEAM principles and skills, and methodologies in their lesson planning, instruction and assessment. STEAM teaching and learning happens both inside and outside of the classroom. Effective STEAM teaching and learning requires both the teacher and the student to participate as core investigators and learners, and to work in partnership and collaboration with relevant stakeholders to achieve maximum results. Teachers should use the syllabus, teacher guides and other resources to guide them to plan and implement this and other innovative and creative approaches to STEAM teaching and learning to make STEAM principles and skills learning fun and enjoyable and, at the same time, attain the intended quality of learning outcomes.

Identify and Use Grade and Context Appropriate, Innovative, Differentiated and Creative Teaching and Learning Methodologies

SBC is an eclectic curriculum model. It is an amalgam of strengths of different curriculum types, including behavioural objectives, outcomes, and competency. Its emphasis is on students attaining clearly defined, measurable, observable and attainable learning standards, i.e., the expected level of education quality. Proficiency (competency) standards are expressed as performance standards/criteria and evidence outcomes, in real life or related situations) to indicate that they are meeting, have met or exceeded the learning standards. The selection of grade and contextually appropriate teaching and learning methodologies is critical to enabling all students to achieve the expected standard or quality of education. Teaching and learning methodologies must be aligned to the content, learning objective, and performance standard in order for the teacher to effectively teach and guide students towards meeting the performance standard for the lesson. They should be equitable and socially inclusive, differentiate, student-centred, and lifelong. They should enable STEAM principles and skills to be effectively taught and learned by students. Teachers should use the teacher guide to help them make informed decisions when selecting the types of teaching and learning methodologies to use in their teaching of the subject content, including STEAM principles and skills.

Plan Standards-Based Lessons

SBC lesson planning is quite difficult to do. However, this will be easier with more practice and experience over time. Effective SBC lesson plans must meet the required standards or criteria so that the learning objectives and performance standards are closely aligned to attain the expected learning outcomes. Teachers should use the guidelines and standards for SBC lesson planning and examples of SBC lesson plans provided in the teacher guide to plan their lessons. When planning lessons, it is important for teachers to ensure that all SBC lesson planning standards or criteria are met. If standards are not met, instruction will not lead to the attainment of intended performance and proficiency standards. Therefore, students will not attain the national content standards and grade-level benchmarks.

Use Standards-Based Assessment

Standards-Based Assessment has a number of components. These components are intertwined and serve to measure evaluate, report, and monitor students' achievement of the national and grade-level expectations, i.e., the essential knowledge, skills, values and attitudes they are expected to master and demonstrate proficiency on.

Teachers should use the information and examples on standards-based assessment assess, record, evaluate, report and monitor students' performance in relation to the learning standards.

Make informed Judgments About Students' Learning and Progress Towards Meeting Learning Standards

Teachers should use the teacher guide to effectively evaluate students' performance and use the evidence to help students to continuously improve their learning as well as their classroom practice.

It is important that teachers evaluate the performance of students in relation to the performance standards and progressively the grade-level benchmarks and content standards to make informed judgments and decisions about the quality of their work and their progress towards meeting the content standards or components of the standards. Evaluation should not focus on only one aspect of students' performance. It should aim to provide a complete picture of each student's performance. The context, inputs, processes, including teaching and learning processes, and the outcomes should be evaluated to make an informed judgment about each student's performance, Teachers should identify the causal factors for poor performance, gaps in students learning, gaps in teaching, teaching and learning resource constraints, and general attitude towards learning. Evidence-based decisions can then be made regarding the interventions for closing the gaps to allow students to make the required progress towards meeting grade-level and national expectations.

Prepare Students' Performance Reports

Reporting of students' performance and progress towards the attainment of learning standards is an essential part of SBC assessment. Results of students' performance should be communicated to particularly the students and their parents to keep them informed of students' academic achievements and learning challenges as well as what needs to be done to enable the students' make positive progress towards meeting the proficiency standards and achieve the desired level of education quality. Teachers should use the information on the reporting of students' assessment results and the templates provided to report the results of students' learning.

Monitor Students' Progress Towards Meeting the National Content Standards and Grade-Level Benchmarks

Monitoring of student's progress towards the attainment of learning standards is an essential component of standards-based assessment. It is an evidence-based process that involves the use of data from students' performance assessments to make informed judgements about students' learning and proficiency on the learning standards or their components, identify gaps in students' learning and the causal factors, set clear learning improvement targets, and develop effective evidence-based strategies (including preplanning and re-teaching of topics), set clear timeframes, and identify measures for measuring students' progress towards achieving the learning targets.

Teachers should use the teacher guide to help them use data from students' performance assessments to identify individual students' learning weaknesses and develop interventions, in collaboration with each student and his/her parents or guardians, to address the weaknesses and monitor their progress towards meeting the agreed learning goals.

Develop additional Benchmarks

Teachers can develop additional benchmarks using the examples in the teacher guide to meet the learning needs of their students and local communities. However, these benchmarks will not be nationally assessed as these are not comparable. They are not allowed to set their own content standards or manipulate the existing ones. The setting of national content standards is done at the national level to ensure that required learning standards are maintained and monitored to sustain the required level of education quality.

Avoid Standardisation

The implementation of Grade 12 General Mathematics curriculum must not be standardised. SBC does not mean that the content, lesson objectives, teaching and learning strategies, and assessment are standardised. This is a misconception and any attempt to standardise the components of curriculum without due consideration of the teaching and learning contexts, student's backgrounds and experiences, and different abilities and learning styles of students will be counterproductive. It will hinder students from achieving the expected proficiency standards and hence, high academic standards and the desired level of education quality. That is, they should not be applied across all contexts and with all students, without considering the educational needs and the characteristics of each context.

Teachers must use innovative, creative, culturally relevant, and differentiated teaching and learning approaches to teach the curriculum and enable their students to achieve the national content standards and grade-level benchmarks. And enable all students to experience success in learning the curriculum and achieve high academic standards.

What is provided in the syllabus and teacher guide are not fixed and can be changed. Teachers should use the information and examples provided in the syllabus and the teacher guide to guide them to develop, select, and use grade, context, and learner appropriate content, learning objectives, teaching and learning strategies, and performance assessment and standards. SBC is evidence-based hence decisions about the content, learning outcomes, teaching and learning strategies, students' performance, and learning interventions should be based on evidence. Teaching and learning should be continuously improved and effectively targeted using evidence from students' assessment and other sources.

Syllabus and Teacher Guide Alignment

A teacher guide is a framework that describes how to translate the content standards and benchmarks (learning standards) outlined in the syllabus into units and topics, learning objectives, lesson plans, teaching and learning strategies, performance assessment, and measures for measuring students' performance (performance standards). It expands the content overview and describes how this content identified in the content standards and their components (essential KSVAs) can be translated into meaningful and evidence-based teaching topics and learning objectives for lesson planning, instruction and assessment. It also describes and provides examples of how to evaluate and report on students' attainment of the learning standards, and use evidence from the assessment of students' performance to develop evidence-based interventions to assist students who are making slow progress towards meeting the expected proficiency levels to improve their performance.

Grade 12 General Mathematics comprises of the Syllabus and Teacher Guide. These two documents are closely aligned, complimentary and mutually beneficial. They are the essential focal points for teaching and learning the essential Mathematics knowledge, skills, values and attitudes.

Syllabus and Teacher Guide Alignment

Syllabus	Teacher Guide
Outlines the ultimate aim and goals, and what to teach and why teach it	Describes how to plan, teach, and assess students' performance
• Overarching and SBC principles • Content overview • Core curriculum • Essential knowledge, skills, values and attitudes • Strands and units • Evidence outcomes • Content standards and grade-level • benchmark • Overview of assessment, evaluation, and Reporting	• Determine topics for lesson planning, instruction and assessment • Formulate learning objectives • Plan SBC lesson plans • Select teaching and learning strategies • Implement SBC assessment and evaluation • Implement SBC reporting and monitoring

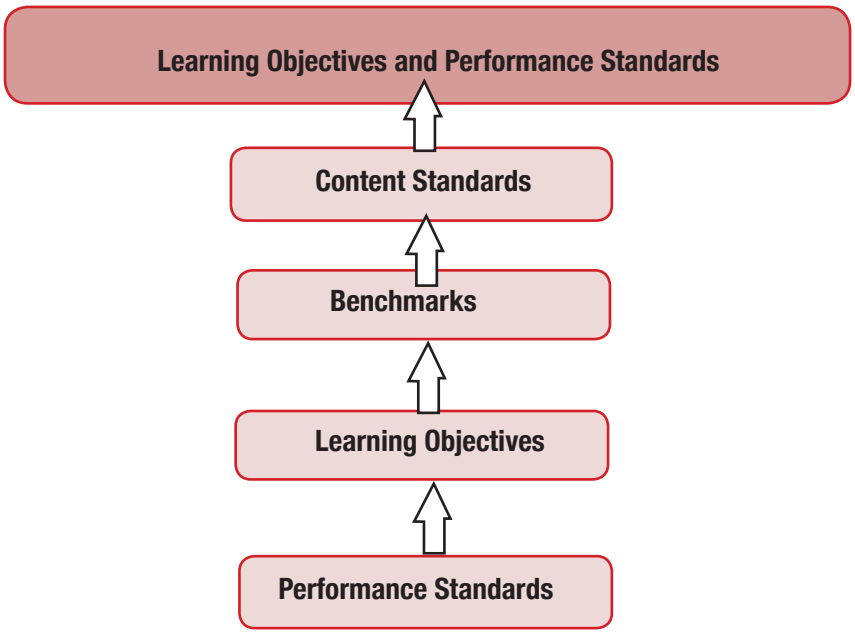
The syllabus outlines the ultimate aim and goals of SBE and SBC, what is to be taught and why it should be learned by students, the underlying principles and articulates the learning and proficiency standards that all students are expected to attain. On the other hand, the teacher guide expands on what is outlined in the syllabus by describing the approaches or the how of planning, teaching, learning, and assessing the content so that the intended learning outcomes are achieved.

This teacher guide should be used in conjunction with the syllabus. Teachers should use these documents when planning, teaching and assessing Grade 12 General Mathematics content.

Teachers will extract information from the syllabus (e.g., content standards and grade-level benchmarks) for lesson planning, instruction and is for measuring students' attainment a content standard as well as progress to the next grade of schooling.

Learning and Performance Standards Alignment

Content Standards, Benchmarks, Learning Objectives, and Performance Standards are very closely linked and aligned (see below). There is a close linear relationship between these standards. Students' performance on a significant aspect of a benchmark (KSVA) is measured against a set of performance standards or criteria to determine their level of proficiency using performance assessment. Using the evidence from the performance assessment, individual student's proficiency on the aspect of the benchmark assessed and progression towards meeting the benchmark and hence the content standard are then determined.



Effective alignment of these learning standards and all the other components of PNG SBE and SBC (ultimate aim and goals, overarching, SBC and subject-based principles, core curriculum, STEAM, and cognitive, high level, and 21st Century skills) is not only critical but is also key to the achievement of high academic standards by all students and the intended level of education quality. It is essential that teachers know and can do standards alignment when planning, teaching, and assessing students' performance so that they can effectively guide their students towards meeting the grade-level benchmarks (grade expectations) and subsequently the content standards (national expectations).

Learning and Performance Standards

Standards-Based Education (SBE) and SBC are underpinned by the notion of quality. Standards define the expected level of education quality that all students should achieve at a particular point in their schooling. Students' progression and achievement of education standard (s) are measured using performance standards or criteria to determine their demonstration or performance on significant aspects of the standards and therefore their levels of proficiency or competency. When they are judged to have attained proficiency on a content standard or benchmark or components of these standards, they are then deemed to have met the standard(s) that is, achieved the intend level of education quality.

Content standards, benchmarks, and learning objectives are called learning standards while performance and proficiency standards (evidence outcomes) can be categorised as performance standards. These standards are used to measure students' performance, proficiency, progression and achievement of the desired level of education quality. Teachers are expected to understand and use these standards for lesson planning, instruction and assessment.

Content Standards

Content standards are evidence-based, rigorous and comparable regionally and globally. They have been formulated to target critical social, economic, political, cultural, environment, and employable skills gaps identified from a situational analysis. They were developed using examples and experiences from other countries and best practice, and contextualized to PNG contexts.

Content standards describe what (content - knowledge, skills, values, and attitudes) all students are expected to know and do (how well students must learn and apply what is set out in the content standards) at each grade-level before proceeding to the next grade. These standards are set at the national level and thus cannot be edited or changed by anyone except the National Subject-Based Standards Councils.

Content Standards;

- are evidenced-based,
- are rigorous and compards,
- are set at the national level,
- state or describe the expected levels of quality or achievement,
- are clear, measurable and attainable,
- are linked to and aligned with the ultimate aim and goals of SBE and SBC and overarching and SBC principles,
- delineate what matters, provide clear expectations of what students should progressively learn and achieve in school, and guide lesson planning, instruction,assessment,
- comprise knowledge, skills, values, and attitudes that are the basis for quality education,
- provide teachers a clear basis for planning, teaching, and assessing lessons, and
- provide provinces, districts, and schools with a clear focus on how to develop and organise their instruction and assessment programs as well as the content that will include in their curriculum.

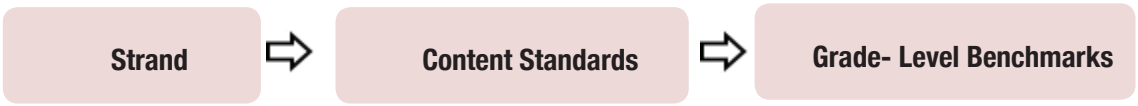
Benchmarks

Benchmarks are derived from the content standards and benchmarked at the grade-level. Benchmarks are specific statements of what students should know (i.e., essential knowledge, skills, values or attitudes) at a specific grade-level or school level. They provide the basis for measuring students' attainment of a content standard as well as progress to the next grade of schooling.

Grade-level benchmarks;

- are evidenced-based,
- are rigorous and comparable to regional and global standards;
- are set at the grade level,
- are linked to the national content standards,
- are clear, measurable, observable and attainable,
- articulate grade level expectations of what students are able to demonstrate to indicate that they are making progress towards attaining the national content standards,
- provide teachers a clear basis for planning, teaching, and assessing lessons,
- state clearly what students should do with what they have learned at the end of each school-level,
- enable students' progress towards the attainment of national content standards to be measured, and
- enable PNG students' performance to be compared with the performance of PNG students with students in other countries.

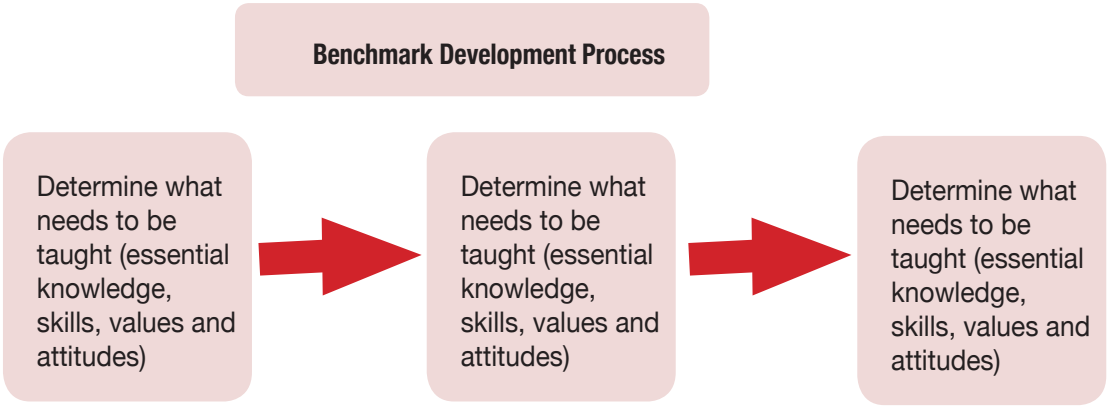
Approach for Setting National Content Standards and Grade-Level Benchmarks



Development of Additional Benchmarks

Teachers should develop additional benchmarks to meet the learning needs of their students. They should engage their students to learn about local, provincial, national and global issues that have not been catered for in the grade-level benchmarks but are important and can enhance students' understanding and application of the content. However, it is important to note that these benchmarks will not be nationally examined as they are not comparable. Only the benchmarks developed at the national level will be tested. This does not mean that teachers should not develop additional benchmarks. An innovative, reflect, creative and reflexive teacher will continuously reflect on his/her classroom practice and use evidence to provide challenging, relevant, and enjoyable learning opportunities for his/her students to build on the national expectations for students.

Teachers should follow the following process when developing additional grade-level benchmarks.



Learning Objectives

Learning or instructional Objectives are precise statements of educational intent. They are formulated using a significant aspect or a topic derived from the benchmark, and is aligned with the educational goals, content standards, benchmarks, and performance standards. Learning objectives are stated in outcomes language that describes the products or behaviours that will be provided by students. They are stated in terms of measurable and observable student behaviour. For example, students will be able to explore the idea of direct and indirect proportion.

Performance Standards

Performance Standards are concrete statements of how well students must learn what is set out in the content standards, often called the "be able to do" of "what students should know and be able to do." Performance standards are the indicators of quality that specify how competent a students' demonstration or performance must be. They are explicit definitions of what students must do to demonstrate proficiency or competency at a specific level on the content standards.

Performance standards;

- measure students' performance and proficiency **(using performance indicators)** in the use of a specific knowledge, skill, value, or attitude in real life or related situations,
- provide the basis **(performance indicators)** for evaluating, reporting and monitoring students' level of proficiency in use of a specific knowledge, skills, value, or attitude,
- are used to plan for individual instruction to help students not yet meeting expectations **(desired level of mastery and proficiency)** to make adequate progress towards the full attainment of benchmarks and content standards, and
- are used as the basis for measuring students' progress towards meeting grade-level benchmarks and content standards.

Proficiency Standards

Proficiency standards describe what all students in a particular grade or school level can do at the end of a strand, or unit. These standards are sometimes called evidence outcomes because they indicate if students can actually apply or use what they have learned in real life or similar situations. They are also categorized as benchmarks because that is what all students are expected to do before exiting a grade or are deemed ready for the next grade.

Core Curriculum

A core set of common learnings (knowledge, skills, values, and attitudes) are integrated into the content standards and grade-level benchmarks for all subjects. This is to equip all students with the most essential and in-demand knowledge, skills, and dispositions they will need to be successful in modern/postmodern work places, higher-education programs and to be productive, responsible, considerate, and harmonious citizens. Common set of learning are spirally sequenced from Preparatory-Grade 12 to deepen the scope and increase the level of difficulty in the learning activities so that what is learned is reinforced at different grade levels.

The core curriculum includes:

- cognitive (thinking) skills (refer to the syllabus for a list of these skills),
- reasoning, decision-making and problem-solving skills,
- high level thinking skills (analysis, synthesis and evaluation skills),
- 21st Century skills (refer to illustrative list in the appendix 2),
- reading, writing and communication skills,
- STEAM principles and skills,
- essential values and attitudes (core personal and social values, and sustaining values), and
- spiritual values and virtues.

The essential knowledge, skills, values and attitudes comprising the core curriculum are interwoven and provide an essential and holistic framework for preparing all students for careers, higher education and citizenship.

All teachers are expected to include the core learnings in their lesson planning, teaching, and assessment of students in all their lessons. They are expected to foster, promote and model the essential values and attitudes as well as the spiritual values and virtues in their conduct, practice, appearance, and their relationships and in their professional and personal lives. In addition, teachers are expected to mentor, mould and shape each student to evolve and possess the qualities envisioned by society.

Core values and attitudes must not be taught in the classroom only; they must also be demonstrated by students in real life or related situations inside and outside of the classroom, at home, and in everyday life. Likewise, they must be promoted, fostered and modelled by the school community and its stakeholders, especially parents. A holistic of school approach to values and attitudes in teaching, promoting and modelling is critical to students and the whole school community to internalise the core values and attitudes and make them habitual in their work and school place, and in everyday life. Be it work values, relationship values, peace values, health values, personal and social values, or religious values, teachers should give equal prominence to all common learnings in their lesson planning, teaching, assessment, and learning interventions. Common learnings must be at the heart of all teaching and extra-curricular programs and activities.

Science, Technology, Engineering ,Arts and Mathematics

STEAM education is an integrated, multidisciplinary approach to learning that uses science,technology, engineering, arts and mathematics as the basis for inquiring about how STEAM has and continues to change and impact the social, political, economic, cultural and environmental contexts and identifying and solving authentic (real life) natural and physical environment problems by integrating STEAM-based principles, cognitive, high level and 21st Century skills and processes, and values and attitudes.

Mathematics is focused on both goals of STEAM rather than just the goal of problem-solving. This is to ensure that all students are provided opportunities to learn, integrate, and demonstrate proficiency on all essential STEAM principles, processes, skills, values and attitudes to prepare them for careers, higher education and citizenship.

Through STEAM education students will be able to:

(i) examine and use evidence to draw conclusions about how STEAM has and continues to change the social, political, economic, cultural and environmental contexts.

(ii) Investigate and draw conclusions on the impact of STEAM solutions to problems on the social, political, economic, cultural and environmental contexts.

(iii) Identify and solve problems using STEAM principles, skills, concepts, ideas and process.

(iv) Identify, analyse and select the best solution to address a problem.

(v) build prototypes or models of solutions to problems.

(vi) replicate a problem solution by building models and explaining how the problem was or could be solved.

(vii) test and reflect on the best solution chosen to solve a problem.

(viii) collaborate with others on a problem and provide a report on the process of problem solving used to solve the problem.

(ix) use skills and processes learnt from lessons to work on and complete STEAM projects.

(x) demonstrate STEAM principles, skills, processes, concepts and ideas through simulation and modelling.

(xi) explain the significance of values and attitudes in problem-solving.

STEAM is a multidisciplinary and integrated approach to understanding how science, technology, engineering, arts and mathematics shape and are shaped by our material, intellectual, cultural, economic, social, political and environmental contexts. And for teaching students the essential in demand cognitive, high level and 21st Century skills, values and attitudes, and empower them to effectively use these skills and

predispositions to identify and solve problems relating to the natural and physical environments as well as the impact of STEAM-based solutions on human existence and livelihoods, and on the social, political, economic, cultural, and environmental systems.

STEAM disciplines have and continue to shape the way we perceive knowledge and reality, think and act, our values, attitudes, and behaviours, and the way we relate to each other and the environment. Most of the things we enjoy and consume are developed using STEAM principles, skills, process, concepts and ideas.

Things humans used and enjoyed in the past and at present are developed by scientists, technologists, engineers, artists and mathematicians to address particular human needs and wants. Overtime, more needs were identified and more products were developed to meet the ever changing and evolving human needs. What is produced and used is continuously reflected upon, evaluated, redesigned, and improved to make it more advanced, multi-purpose, fit for purpose, and targeted towards not only improving the prevailing social, political, economic, cultural and environmental conditions but also to effectively respond to the evolving and changing dynamics of human needs and wants. And, at the same time, solutions to human problems and needs are being investigated and designed to address problems that are yet to be addressed and concurred. This is an evolving and ongoing problem-solving process that integrates cognitive, high level, and 21st Century skills, and appropriate values and attitudes.

STEAM is a significant framework and focal point for teaching and guiding students to learn, master and use a broad range of skills and processes required to meet the skills demands of PNG and the 21st Century. The skills that students will learn will reflect the demands that will be placed upon them in a complex, competitive, knowledge-based, information-age,technology- driven economy and society. These skills include cognitive (critical, synthetic, creative, reasoning, decision-making, and problem-solving) skills, high level (analysis, synthesis and evaluation) skills and 21st Century skills (see Appendix 4). Knowledge-based, information, and technology driven economies require knowledge workers not technicians. Knowledge workers are lifelong learners, are problem solvers, innovators, creators, critical and creative thinkers, reflective practitioners, researchers (knowledge producers rather than knowledge consumers), solutions seekers,outcomes oriented, evidence-based decision makers, and enablers of improved and better outcomes for all.

STEAM focuses on the skills and processes of problem solving. These skills and processes are at the heart of the STEAM movement and approach to not only problem solving and providing evidence-based solutions but also the development and use of other essential cognitive, high level and 21st Century skills. These skills are intertwined and used simultaneously to gain a broader understanding of the problems to enable creative, innovative, contextually relevant, and best solutions to be developed and implemented to solve the problems and attain the desired outcomes. It is assumed that by teaching students STEAM-based problem-solving skills and providing learning opportunities inside and outside the classroom will motivate more of them to pursue careers and academic programs in STEAM related fields thus, closing the skills gaps and providing a pool of cadre of workers required by technology, engineering, science, and mathematics-oriented industries.

Although, STEAM focuses on the development and application of skills in authentic (real life) contexts, for example the use of problem- solving skills to identify and solve problems relating to the natural and physical worlds, it does not take into account the significant influence values and attitudes have on the entire process of problem solving. Values and attitudes are intertwined with knowledge and skills. Knowledge, skills, values and attitudes are inseparable. Decisions about skills and processes of skills development and application are influenced by values and attitudes (mindset) that people hold. In the same light, the use of STEAM principles, processes and skills to solve problems in order to achieve the outcomes envisaged by society are influenced by values and the mindset of those who have identified and investigated the problem as well as those who are affected by the problem and will benefit from the outcome.

STEAM Problem-Solving Methods and Approaches

Problem-solving involves the use of problem-solving methods and processes to identify and define a problem, gather information to understand its causes, draw conclusions, and use the evidence to design and implement solutions to address it. Even though there are many different problem-solving methods and approaches, they share some of the steps of problem-solving, such as;

- identifying the problem,
- understanding the problem by collecting data,
- analyse and interpret the data,
- draw conclusions,
- use data to consider possible solutions,
- select the best solution,
- test the effectiveness of the solution by trialling and evaluating it, and
- review and improve the solution.

STEAM problem solving processes go from simple and technical to advance and knowledge-based processes. However, regardless of the type of process used, students should be provided opportunities to learn the essential principles and processes of problem solving and, more significantly, to design and create a product that addressed a real problem and meets a human need.

The following are some of the STEAM problem solving processes.

1. Engineering and Technology Problem Solving Methods and Approaches

Engineering and technology problem-solving methods are used to identify and solve problems relating to the physical world using the design process. The following are some of the methods and approaches used to solve engineering and technology related problems.

Parts Substitution
It is the most basic of the problem-solving methods. It simply requires the parts to be substituted until the problem is solved.

Diagnostics
After identifying a problem, the technician would run tests to pinpoint the fault. The test results would be used either as a guide for further testing or for replacement of a part, which also need to be tested. This process continues until the solution is found and the device is operating properly.

Troubleshooting
Troubleshooting is a form of problem solving, often applied to repair failed products or processes.

Reverse Engineering
Reverse engineering is the process of discovering the technological principles underlying the design of a device by taking the device apart, or carefully tracing its workings or its circuitry. It is useful when students are attempting to build something for which they have no formal drawings or schematics.

Divide and Conquer

Divide and conquer is the technique of breaking down a problem into sub-problems, then breaking the sub-problems down even further until each of them is simple enough to be solved. Divide and conquer may be applied to all groups of students to tackle sub-problems of a larger problem, or when a problem is so large that its solution cannot be visualised without breaking it down into smaller components.

Extreme Cases

Considering "extreme cases" – envisioning the problem in a greatly exaggerated or greatly simplified form, or testing using extreme condition – can often help to pinpoint a problem. An example of the extreme-case method is purposely inputting an extremely high number to test a computer program.

Trial and Error

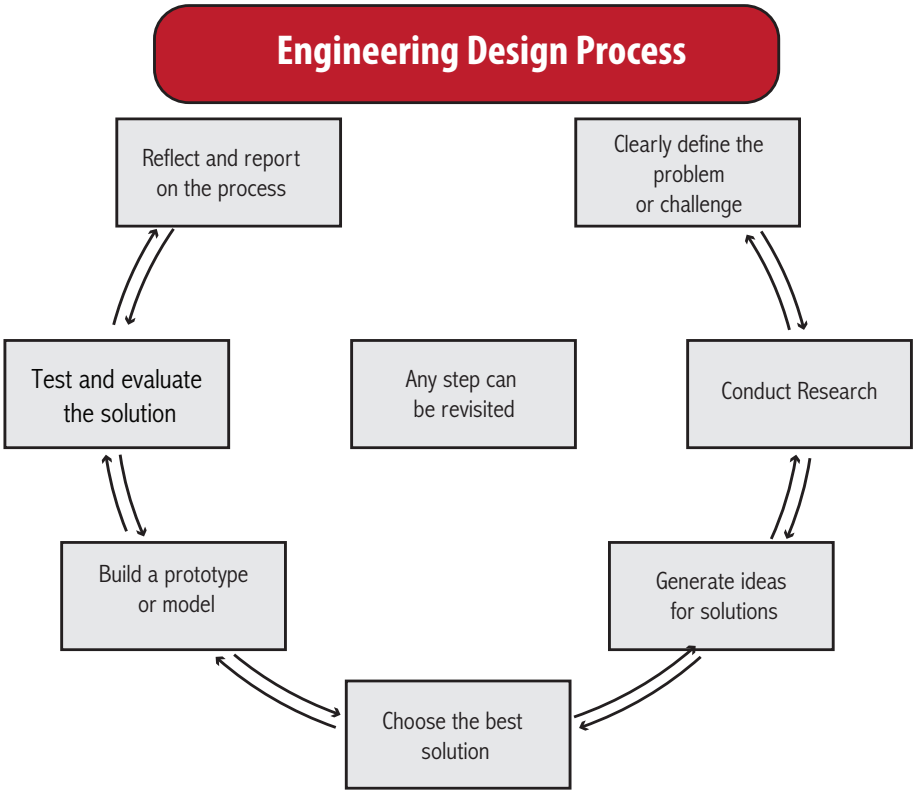
The trial and error method involve trying different approaches until a solution is found. It is often used as a last resort when other methods have been exhausted.

- Test and evaluate the solution.
- Repeat steps as necessary to modify the design or correct faults.
- Reflect and report on the process.

1. Engineering Design Process

Technological fields use the engineering design process to identify and define the problem or challenge, investigate the problem, collect and analyse data, and use the data to formulate potential solutions to the problem, analyse each of the solutions in terms of its strengths and weaknesses, and choose the best solution to solve the problem. It is an open-ended problem-solving process that involves the full planning and development of products or services to meet identified needs. It involves a sequence of steps as illustrated.

1) Analyse the context and background, and clearly define the problem.
2) Conduct research to determine design criteria, financial or other constraints, and availability of materials.
3) Generate ideas for potential solutions, using processes such as brainstorming and sketching.
4) Choose the best solution.
5) Build a prototype or model.
6) Test and evaluate the solution.
7) Repeat steps as necessary to modify the design or correct faults.
8) Reflect and report on the process.



STEAM-Based Lesson planning

Effective STEAM lesson planning is key to the achievement of expected STEAM outcomes. STEAM skills can be planed and taught using separate STEAM-based lesson plans or integrated into the standards-based lesson plans. To effectively do this, teachers should know how to write effective standards and STEAM-based lesson plans.

An example of a STEAM-based lesson plan is provided in appendix.
Teachers should use this to guide them to integrate STEAM content and teaching, learning and assessment strategies into their standards-based lesson plans.

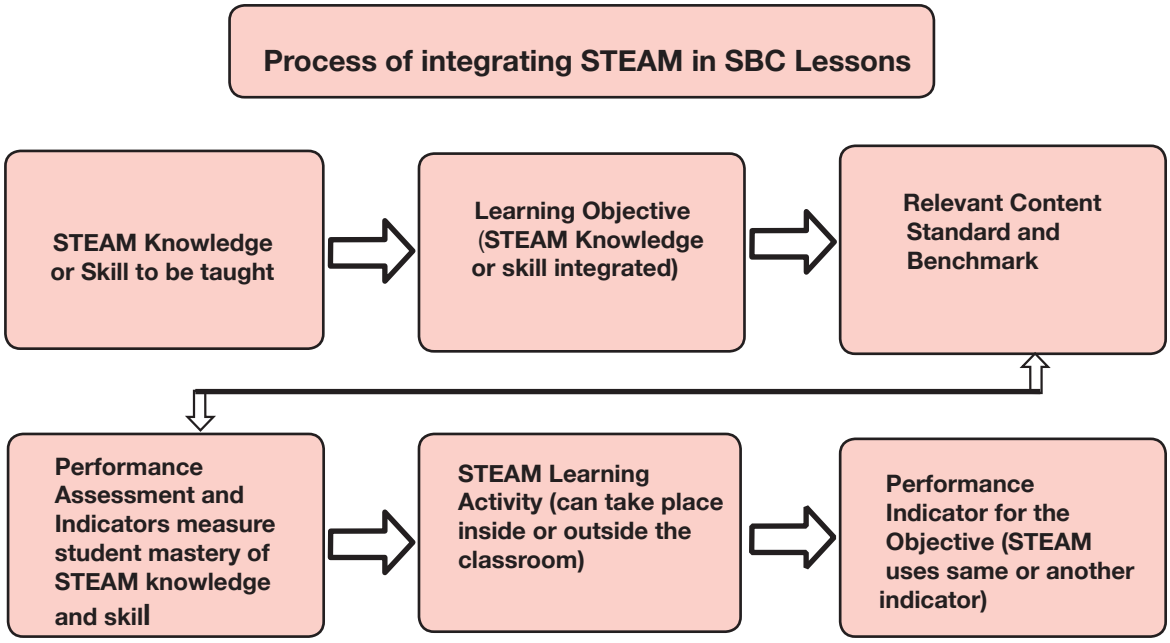
Integration of STEAM problem-solving skills into standards-based lesson plans

Knowing how to integrate STEAM problem-solving skills, principles, values and attitudes as well as STEAM teaching, learning, and assessment strategies into standards-based lesson plans is essential for achieving the desired STEAM learning outcomes. When integrating STEAM problem-solving skills into the standards-based lesson plans, teachers should ensure that these skills are not only effectively aligned to the learning objective and performance standards, they must also be effectively taught and assessed.

STEAM principles and problem-solving skills are integrated into the content standards and grade-level benchmarks. A list of these skills, including 21st Century skills, is provided in the syllabus. Teachers should ensure that these skills are integrated in their standards-based lesson plans, taught and assessed to determine students' level of proficiency on each skill or specific components of the skill.

Teachers are expected to integrate the essential STEAM principles, processes, skills, values and attitudes described in the Grade 12 benchmarks when formulating their standards-based lesson plans. Opportunities should be provided inside and outside of the classroom for students to learn, explore, model and apply what they learn in real life or related situations. These learning experiences will enable students to develop a deeper understanding of STEAM principles, processes, skills, values and attitudes and appreciate their application in real life to solve problems.

Teachers should use the following process as guide to integrate STEAM principles and problem-solving skills into the standards-based lesson plans.



Steps for integrating STEAM problem-solving principles and skills into standards-based lesson plans.

Step 1: Identify the STEAM knowledge or skill to be taught (from the table of KSVAs for each content standard and benchmark). This is captured in the learning objective stated in the standards-based lesson plan.

Step 2: Develop and include a performance standard or indicator for measuring student mastery of the STEAM knowledge or skill (e.g. level of acceptable competency or proficiency) if this is different from the one already stated in the lesson plan.

Step 3: Develop student learning activity (An activity that will provide students the opportunity to apply the STEAM knowledge or skill specified by the learning objective and appropriate statement of the standards). Activity can take place inside or outside of the classroom, and during or after school hours.

Step 4: Develop and use performance descriptors (standards or indicators) to analyse students' STEAM related behaviours and products (results or outcomes), which provide evidence that the student has acquired and mastered the knowledge or skill of the learning objective specified by the indicator (s) of the standard(s).

STEAM Teaching Strategies

STEAM education takes place in both formal and informal classroom settings. It takes place during and after school hours. It is a continuous process of inquiry, data analysis, making decisions about interventions, and implementing and monitoring interventions for improvements.

There are a variety of STEAM teaching strategies. However, teaching strategies selected must enable teachers to guide students to use the engineering and artistic design processes to identify and solve natural and physical environment problems by designing prototypes and testing and refining them to effectively mitigate the problems identified. The following are some of the strategies that could be used to utilise the STEAM approach to solve problems and coming up with technological solutions.

1. Inquiry-Based Learning
2. Problem-Based Learning
3. Project-based learning
4. Collaborative Learning

Collaborative learning involves individuals from different STEAM disciplines and expertise in a variety of STEAM problem solving approaches working together and sharing their expertise and experiences to inquire into and solve a problem. Teachers should plan to provide students opportunities to work in collaboration and partnership with experts and practitioners engaged in STEAM related careers or disciplines to learn first-hand about how STEAM related skills, processes, concepts, and ideas are applied in real life to solve problems created by natural and physical environments. Collaborative learning experiences can be provided after school or during school holidays to enable students to work with STEAM experts and practitioners to inquiry and solve problems by developing creative, innovative and sustainable solutions. Providing real life experiences and lessons, e.g., by involving students to actually solve a scientific, technological, engineering, or mathematical, or Arts problem, would probably spark their interest in a STEAM career path.

Developing STEAM partnerships with external stakeholders e.g., high education institutions, private sector, research and development institutions, and volunteer and community development organizations can enhance students' learning and application of STEAM problem solving principles and skills.

Some examples of STEAM-related partnership experiences may include:

- Participatory Learning
- Group-Based Learning
- Task Oriented Learning
- Action Learning
- Experiential Learning
- Modelling
- Simulation

STEAM Learning Strategies

Teachers should include in their lesson plans STEAM learning activities. These activities should be aligned to principle or a skill planned for students to learn and demonstrate proficiency at the end of the lesson to expose students to STEAM and giving them opportunities to explore STEAM-related concepts, they will develop a passion for it and, hopefully, pursue a job in a STEAM field.
Providing real life experiences and lessons, e.g., by involving students to actually solve a scientific, technological, engineering, or mathematical, or arts problem, would probably spark their interest in a STEAM career path. This is the theory behind STEAM education.

STEAM-Based Assessment

STEAM-based assessment is closely linked to standards-based assessment where assessment is used to assess students' level of competency or proficiency of a specific knowledge, skill, value, or attitude taught using a set of performance standards (indicators or descriptors). The link also includes the main components such as the purpose, the assessment principles and assessment strategies and tools.
In STEAM-based assessment, assessments are designed for what students should know and be able to do. In STEAM learning, students are assessed in a variety of ways including portfolios, project/problem-based assessments, backwards design, authentic assessments, or other student-centered approaches.
When planning and designing the assessment, teachers should consider the authenticity of the assessment by designing an assessment that relates to a real world task or discipline specific attributes such as simulation, role play, placement assessment, live projects and debates. These tasks should make the activity meaningful to the student, and therefore be motivating as well as developing employability skills and discipline specific attributes.

Effective STEAM-Based Assessment Strategies

The following are the six assessment tools and strategies to impact teaching and learning as well as help teachers foster 21st Century learning environment in their classrooms.

1. Rubrics
2. Performance-Based Assessments (PBAs)
3. Portfolios
4. Student self-assessment
5. Peer-assessment
6. Student Response Systems (SRS).

Although the list does not include all innovative assessment strategies, it includes what we think are the most common strategies, and ones that may be particularly relevant to the educational context of developing countries in this 21st Century. Many of the assessment strategies currently in use fit under one or more of the categories discussed. Furthermore, it is important to note that these strategies also connect in a variety of ways.

1. Rubrics

Rubrics are both a tool to measure students' knowledge and ability as well as an assessment strategy. A rubric allows teachers to measure certain skills and abilities not measurable by standardized testing systems that assess discrete knowledge at a fixed moment in time. Rubrics are also frequently used as part of other assessment strategies including; portfolios, performances, projects, peer-review and self-assessment which are also elaborated in this section.

2. Performance-Based Assessments

Performance-Based Assessments (PBA), also known as project-based or authentic assessments, are generally used as a summative evaluation strategy to capture not only what students know about a topic, but if they have the skills to apply that knowledge in a "real-world" situation.
By asking them to create an end product. PBA pushes students to synthesize their knowledge and apply their skills to a potentially unfamiliar set of circumstances that is likely to occur beyond the confines of a controlled classroom setting.
The implementation of performance-based assessment strategies can also impact other instructional strategies in the classroom.

3. Portfolio Assessment

Portfolios are a collection of student work gathered over time that is primarily used as a summative evaluation method. The most salient characteristic of the portfolio assessment is that rather than being a snapshot of a student's knowledge at one point in time (like a single standardized test), it highlights student effort, development, and achievement over a period of time; portfolios measure a student's ability to apply knowledge rather than simply regurgitate. They are considered both student-centred and authentic assessments of learning.

4. Self-assessment

While the previous assessment tools and strategies listed in this report generally function as summative approaches, self-assessment is generally viewed as a formative strategy, rather than one used to determine a student's final grade. Its main purpose is for students to identify their own strengths and weakness and to work to make improvements to meet specific criteria.
Self-assessment occurs when students judge their own work to improve performance as they identify discrepancies between current and desired performance. In this way, self-assessment aligns well with standards- based education because it provides clear targets and specific criteria against which students or teachers can measure learning.

Self-assessment is used to promote self-regulation, to help students reflect on their progress and to inform revisions and improvements on a project or paper. In order for self-assessment to be truly effective four conditions must be in place: the self- assessment criteria is negotiated between teachers and students, students are taught how to apply the criteria, students receive feedback on their self-assessments and teachers help students use assessment data to develop an action plan.

5. Peer assessment

Peer assessment, much like self-assessment, is a formative assessment strategy that gives students a key role in evaluating learning. Peer assessment approaches can vary greatly but, essentially process develops both the assessor and assessee's skills and knowledge.

The primary goal for using peer assessment is to provide feedback to learners. This strategy may be particularly relevant in classrooms with many students per teacher since student time will always be more plentiful than teacher time. Although any single student's feedback may not be rich or in-depth as tea her's feedback, the research suggests that peer assessment can improve learning.

6. Student Response System

Student response system (SRS), also known as classroom response (CRS), audience response system (ARS) is a general term that refers to a variety of technology-based formative assessment tools that can be used to gather student-level data instantly in the classroom. Through the combination of hardware, (voice recorders, PC, internet connection, projector and screen) and software.

Teachers can ask students a wide range of questions (both closed and open ended), where students can respond quickly and anonymously, and the teacher can display the data immediately and graphically. The use of technology also includes a use of video which examines how a range of strategies can be used to assess students' understanding.

The value of SRS comes from teachers analysing information quickly and then devising real-time instructional solutions to maximize student learning. This includes a suggested approach to help teachers and trainers assess learning.

Curriculum Integration

What is Curriculum Integration?

Curriculum integration is making connections in learning across the curriculum. The ultimate aim of curriculum integration is to act as a bridge to increase students' achievement and engage in relevant curriculum.

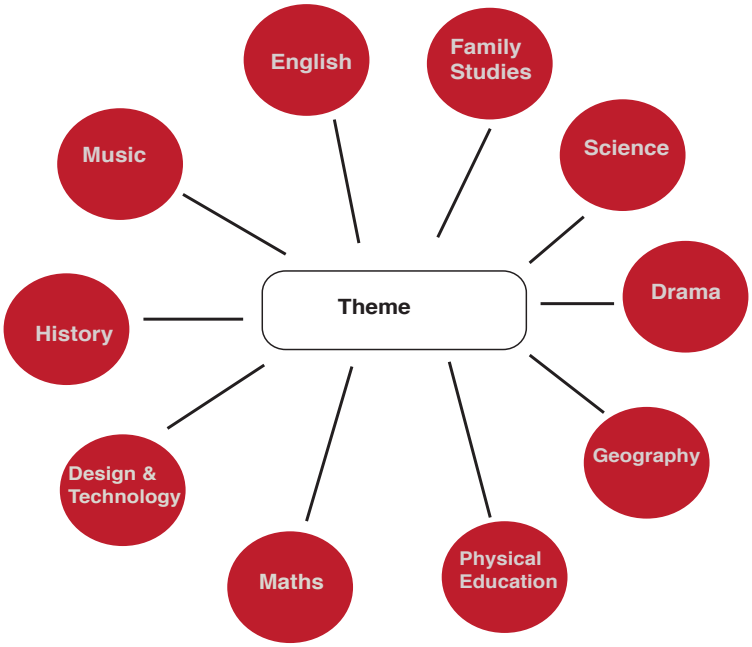
Teachers must develop intriguing curriculum by going beyond the traditional teaching of content based or fragmented teaching to one who is knowledge based and who should be perceived as a 21st Century innovative educator. Curriculum integration is a holistic approach to learning thus curriculum integration in PNG SBC will have to equip students with the essential knowledge, skills, values and attitudes that are deemed 21st Century.

There are three approaches that PNG SBC will engage to foster conducive learning for all its children whereby they all can demonstrate proficiency at any point of exit. Adapting these approaches will have an immense impact on the lives of these children thus they can be able to see themselves as catalyst of change for a competitive PNG. Not only that but they will be comparable to the world standards and as global citizens.

Engaging these three approaches in our curriculum will surely sharpen the knowledge and ability of each child who will foresee themselves as assets through their achievements thus contribute meaningfully to their country. They themselves are the agents of change. Integrated learning will bear forth a generation of knowledge based populace who can solve problems and make proper decisions based on evidence. Thus, PNG can achieve its goals like the Medium Term Development Goals (MTDG) and aims such as the Vision 2050 for a happy, healthy and wealthy society whereby, all its citizens should have access and fair distribution to income, shelter, health, education and general goods and services improving the general standard of living for PNG in the long run.

1. (i) Multidisciplinary Approach

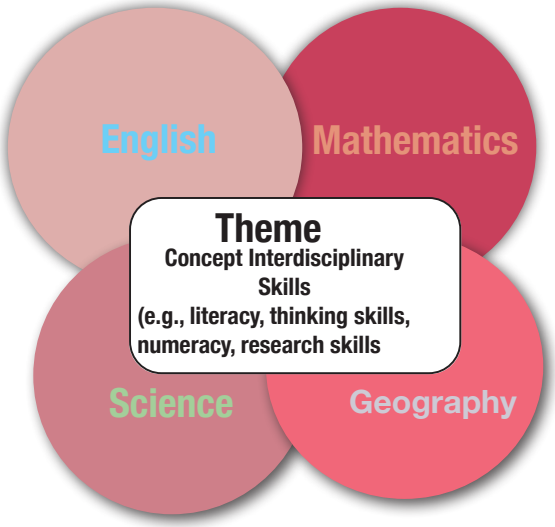
In this approach learning involves a theme or concept that will be taught right across all subject area of study by students. That is, content of a particular theme will be taught right across all subjects as shown in the diagram below. For instance, if the theme is global warming, subject areas create lessons or assessment as per their subjects around this theme. Social Science will address this issue, Science and all other subject likewise.



1. (ii) Interdisciplinary Approach

This approach addresses learning similarly to the multidisciplinary approach of integrated learning whereby learning takes place within the subject area. However, it is termed interdisciplinary in that the core curriculum of learning is interwoven into each subject under study by the students. For instance; in Social Science under the strand of geography students write essay on internal migration however, apart from addressing the issues of this topic, they are to apply the skill of writing text types in their essay such as argumentative essay, informative, explanatory, descriptive, expository and narrative essay while writing their essay. They must be able to capture the mechanics of English skills such as grammar, punctuation and so forth. Though these skills are studied under English they are considered as core skills that cut across all subjects under study. For example; if Science students were to write about human development in biology then the application of writing skills has to be captured by the students in their writing. It is not seen as an English skill but a standard essential skill all students must know and do regardless.

Therefore, essential knowledge, skills, values and attitudes comprising the core curriculum are interwoven and provide an essential and holistic framework for preparing all students for careers, higher education and citizenship in this learning.



2. Intradisciplinary Approach

This approach involves teachers integrate sub disciplines within a subject area. For instance, within the subject Social Science, the strands (disciplines) of geography, environment, history, political science and environment will all be captured studying a particular content for Social Science. For example, under global warming, students will study the geographical aspects of global warming, environmental aspect of global warming and likewise for history, political science and economics. Thus, children are well aware of the issues surrounding global warming and can address it confidently at each level of learning.

3. Trans disciplinary Approach

In this approach learning goes beyond the subject area of study. Learning is organized around students' questions and concerns. That is, where there is a need for change to improve lives, students develop their own curriculum to effect these need. The trans-disciplinary approach addresses real-life situations thus giving the opportunity to students to attain real life skills. This learning approach is more to do with Project–Based Learning also referred to as problem-based learning or place-based learning.

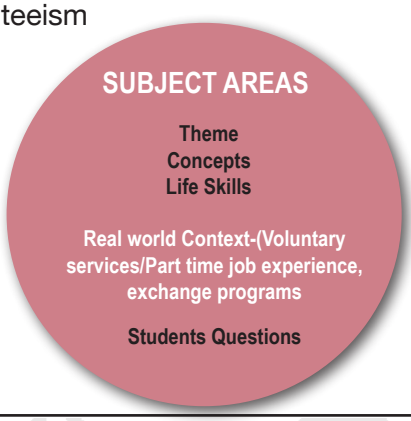
Below are the three steps to planning project based curriculum

1. Teachers and students select a topic of study based on student interests,
2. curriculum standards, and local resources.
3. The teacher finds out what the students already know and helps them generate questions to explore. The teacher also provides resources for students and
4. opportunities to work in the field
5. Students share their work with others in a culminating activity. Students display the results of their exploration and review and evaluate the project.

For instance; students may come up with slogans for school programs such as 'Our culture – clean city for a healthier PNG'. The main aim could be to curb betel nut chewing in public areas especially around bus stops and local markets. Here, students draw up their own instructions and criteria for assessment which is; they have to clean the nearest bus stop or local market once a week throughout the year. They also design and create posters to educate the general public as their program continues. They can also involve the town council and media to assist them especially to carry out awareness.

Studies have proven that Project based-programs have led to the following:

- Students go far beyond the minimum effort
- Make connections among different subject areas to answer open-ended questions
- Retain what they have learnt
- Apply learning to real-life problems
- Have fewer discipline problems
- Lower absenteeism



These integrated learning approaches will demand for teaches to be proactive in order to improve students learning and achievement. In order for PNG Standards-Based Curriculum to serve its purpose fully, these three approaches must be engaged for better learning for the children of PNG now an in the future

Essential Knowledge, Skills, Values and Attitude and Mathematical Thinking

Students' level of proficiency and progression towards the attainment of content standards will depend on their mastery and application of essential knowledge, skills, values, and attitudes in real life or related situations. Provided here are examples of different types of knowledge, processes, skills, values, and attitudes that all students are expected to learn and master as they progress through the grades. These are expanded and deepen in scope and the level of difficulty and complexity are increased to enable students to study in-depth the subject content as they progress from one grade to the next.

These knowledge, skills, values and attitudes have been integrated into the content standards and benchmarks. They will also be integrated into the performance standards. Teachers are expected to plan and teach essential knowledge, skills, values and attitudes in their lessons, and assess students' performance and proficiency, and progression towards the attainment of content standards.

Types of Knowledge

There are different types of knowledge. These include;	
• Public and private (privileged) knowledge • Specialised knowledge • Good and bad knowledge • Concepts, processes, ideas, skills, values, attitudes • Theory and practice • Fiction and non-fiction • Traditional, modern, and postmodern knowledge	• Subject and discipline-based knowledge • Lived experiences • Evidence and assumptions • Ethics and Morales • Belief systems • Facts and opinions • Wisdom • Research evidence and findings • Solutions to problems

Types of Processes

There are different types of processes. These include;	
• Problem-solving • Logical reasoning • Decision-making • Reflection	• Cyclic processes • Mapping (e.g. concept mapping) • Modelling • Simulating
Mathematics Inquiry processes include: • Gathering information • Analysing information • Evaluating information • Making judgements • Taking actions	

Mathematical Thinking Processes

The five Mathematical process skills that can help the students improve their mathematical thinking.

1. Mathematical Problem Solving

- Understand the meaning of the problem and look for entry points to its solution
- Analyse information (givens, constrains, relationships, goals)
- Make conjectures and plan a solution pathway
- Monitor and evaluate the progress and change course as necessary
- Check answers to problems and ask, “Does this make sense?

2. Mathematical Communication

- Use definitions and previously established causes/effects (results) in constructing arguments
- Make conjectures and use counter examples to build a logical progression of statements to explore and support their ideas
- Communicate and defend mathematical reasoning using objects, drawings, diagrams, actions
- Listen to or read the arguments of others
- Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments.

3. Mathematical Reasoning

- Make sense of quantities and relationships in problem situations
- Represent abstract situations symbolically and understand the meaning of quantities
- Create a coherent representation of the problem at hand
- Consider the units involved
- Flexibly use properties of operations.

4. Mathematical Connections

- Look for patterns or structure, recognizing that quantities can be represented in different ways
- Recognize the significance in concepts and models and use the patterns or structure for solving related problems
- View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems
- Notice repeated calculations and look for general methods and short cuts
- Continually evaluate the reasonableness of intermediate results (comparing estimates) while attending to details and make generalizations based on finding.

5. Mathematical Representation

- Apply prior knowledge to solve real world problems
- Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
- Make assumptions and approximations to make a problem simpler
- Check to see if an answer makes sense within the context of a situation and change a model when necessary.

Types of Skills

There are different types of skills. These include:

1. Cognitive (Thinking) Skills

Thinking skills can be categorized into **critical thinking** and **creative thinking** skills.

i. Critical Thinking Skills

A person who thinks critically always evaluates an idea in a systematic manner before accepting or rejecting it. Critical thinking skills include;	
• Attributing • Comparing and contrasting • Grouping and classifying • Sequencing • Prioritising • Analysing	• Detecting bias • Evaluating • Metacognition (Thinking about thinking) • Making informed conclusions.

ii. CreativeThinking Skills

A person who thinks creatively has a high level of imagination, able to generate original and innovative ideas, and able to modify ideas and products. Creative thinking skills include;	
• Generating ideas • Deconstruction and reconstruction • Relating • Making inferences • Predicting • Making generalisations • Visualizing	• Synthesising • Making hypothesis • Making analogies • Invention • Transformation • Modelling • Simulating

2. **Reasoning Skills** - Reason is a skill used in making a logical, just, and rational judgment.

3. **Decision-Making Skills** - Decision-making involves selection of the best solution from various alternatives based on specific criteria and evidence to achieve a specific aim.

4. **Problem Solving Skills** – These skills involve finding solutions to challenges or unfamiliar situations or unanticipated difficulties in a systematic manner.

5. Literacy Skills

A strong emphasis must be placed on various types of literacy, from financial to technological, from media to mathematical, from content to cultural. Literacy may be defined as the ability of an individual to use information to function in society, to achieve goals and to develop her or his knowledge and potential. Teachers emphasize certain aspects of literacy over others, depending on the nature of the content and skills they want students to learn.

The following literacy skills are intended to be exemplary rather than defi nitive	
• Listens, read, write, and speak with comprehension and clarity • Define and apply discipline-based conceptual vocabulary • Describe people, places, and events, and the connections between and among them • Arrange events in chronological sequence • Differentiate fact from opinion • Determine an author's purpose • Determine and analyse similarities and differences • Analyse cause and effect relationships • Explore complex patterns, interactions and relationships • Differentiate between and among various options	• Listens, read, write, and speak with comprehension and clarity • Define and apply discipline-based conceptual vocabulary • Describe people, places, and events, and the connections between and among them • Arrange events in chronological sequence • Differentiate fact from opinion • Determine an author's purpose • Determine and analyse similarities and differences • Analyse cause and effect relationships • Develop an ability to use and apply abstract principals • Explore and/or observe, identify, and analyse how individuals and/or societies relate to one another

6. **High Level Thinking Skills -** These skills include analysis, synthesis, and evaluation skills.

i. Analysis Skills – Analysis skills involve examining in detail and breaking information into parts by identifying motives or causes,underlying assumptions, hidden messages; making inferences and finding evidence to support generalisations, claims, and conclusions.

Keywords				
Analyse	Differences	Find	List	Similar to
Appraise	Discover	Focus	Motivate	Simplify
Arrange	Discriminate	Function	Omit	Take part in
Assumption	Discussion	Group	Order	Test for
Breakdown	Distinction	Highlight	Organize	Theme
Categorize	Distinguish	In-depth	Point out	
Cause & effect	Dissect	Inference	Research	
Choose	Divide	Inspect	See	
Classify	Establish	Isolate	Select	
Comparing	Examine	Investigate	Separate	

ii. Synthesis Skills – Synthesis skills involve changing or creating something new, compiling information together in a different way by combining elements in a new pattern proposing alternative solutions.

iii. Evaluation Skills – Evaluation skills involve justifying and presenting and defending opinions by making judgments about information, validity of ideas or quality of work based on set criteria.

Types of Values

Personal engagement and civic engagement strategies help young people to acquire and apply skills and dispositions that will prepare them to become competent and responsible citizens.

1. Personal Values (importance, worth, usefulness, etc.)

Core values	Sustaining values
• Sanctity of life • Truth • Aesthetics • Honesty • Human • Dignity • Rationality • Creativity • Courage • Liberty • Affectivity • Individuality	• Self-esteem • Self-reflection • Self-discipline • Self-cultivation • Principal morality • Self-determination • Openness • Independence • Simplicity • Integrity • Enterprise • Sensitivity • Modesty • Perseverance

2. Social Values

Core values	Sustaining values
• Equality • Kindness • Benevolence • Love • Freedom • Common good • Mutuality • Justice • Trust • Interdependence • Sustainability • Betterment of human kind • Empowerment	• Plurality • Due process of law • Democracy • Freedom and liberty • Common will • Patriotism • Tolerance • Gender equity and social inclusion • Equal opportunities • Culture and civilisation • Heritage • Human rights and responsibilities • Rationality • Sense of belonging • Solidarity • Peace and harmony • Safe and peaceful communities

Types of Attitudes

Attitudes - Ways of thinking and behaving, points of view	
• Optimistic • Participatory • Critical • Creative • Appreciative • Empathetic • Caring and concern • Positive • Confident • Cooperative	• Responsible • Adaptable to change • Open-minded • Diligent • With a desire to learn • With respect for self, life, equality and excellence, evidence, fair play, rule of law, different ways of life, beliefs and opinions, and the environment.

Teaching and Learning Strategies

Mathematics teaching emphasises and embraces the use of cognitive, reasoning, decision-making, problem solving and higher level thinking skills to teach to enhance students' understanding of inter-disciplinary concepts and issues in relation to environment, geography, history, politics and economic within PNG and globally. It aims to provide a meaningful pedagogical framework for teaching and learning essential and in demand knowledge, skills, values, and attitudes that are required for the preparation of students for careers, higher education and citizenship in the 21st Century.

Students must be prepared to gather and understand information, analyse issues critically, learn independently or collaboratively, organize and communicate information, draw and justify conclusions, create new knowledge, and act ethically. These teaching and learning strategies will help teachers to;

- familiarize themselves with different methods of teaching in the classroom
- develop an understanding of the role of a teacher for application of various methods in the classroom

Successful teachers always keep in view that teaching must "be dynamic, challenging and in accordance with the learner's comprehension. He/she does not depend on any single method for making his/her teaching interesting, inspirational and effective.

A detailed table of Teaching and Learning Strategies are outlined below:

STRATEGY	TEACHER	STUDENTS
CASE STUDY Used to extend students' understanding of real life issues.	Provide students with case studies related to the topic of the lesson and allow them to analyse and evaluate.	Study the case study and identify the problem addressed. They analyse the problem and suggest solutions supported by conceptual justifications and make presentations. This enriches the students' existing knowledge of the topic.
DEBATE A method used to increase students' interest, involvement and participation.	Provide the topic or question of debate on current issues affecting a bigger population, clearly outlining the expectations of the debate. Explain the steps involved in debating and set a criteria/ standard to be achieved.	Conduct researches to gather supporting evidence about the selected topic and summarising the points.
DISCUSSION The purpose of discussion is to educate students about the process of group thinking and collective decision.	The teacher opens a discussion on certain topic by asking essential questions. During the discussion, the teacher reinforces and emphasises on important points from students responses.	Students ponder over the question and answer by providing ideas, experiences and examples.

STRATEGY	TEACHER	STUDENTS
	Teacher guide the direction to motivate students to explore the topic in greater depth and the topic in more detail. Use how and why follow-up questions to guide the discussion toward the objective of helping students understand the subject and summarise main ideas.	
GAMES AND SIMULATIONS Encourages motivation and creates a spirit of competition and challenge to enhance learning.	Being creative and select appropriate games for the topic of the lesson. Give clear instructions and guidelines. The game selected must be fun and build a competitive spirit to score more than their peers to win small prices.	Go into groups and organize. Follow the instructions and play to win.
OBSERVATION Method used to allow students to work independently to discover why and how things happen as the way they are. It builds curiosity.	Give instructions and monitor every activity students do.	Students possess instinct of curiosity and are curious to see the things for themselves and particularly those things which exist around them. A thing observed and a fact discovered by the child for himself becomes a part of mental life of the child. It is certainly more valuable to him than the same fact or facts learnt from the teacher or a book. Students • Observe and ask essential questions • Record • Interpret.
PEER TEACHING & LEARNING (power point presentations, pair learning) Students teach each other using different ways to learn from each other. It encourages; team work, develops confidence, feel free to ask questions, improves communication skills and most importantly develop the spirit of inquiry.	Distribute topics to groups to research and teach others in the classroom. Go through the basics of how to present their peer teaching.	Go into their established working groups. Develop a plan for the topic. Each group member is allocated a task to work on. Research and collect information about the topic allocated to the group. Outline the important points from the research and present their findings in class.

STRATEGY	TEACHER	STUDENTS
PERFORMANCE-RELATED TASKS (dramatization, song/lyrics, wall magazines) Encourages creativity and take on the overarching ideas of the topic and are able to recall them at a later date	Students are given the opportunity to perform the using the main ideas of a topic. Provide the guidelines, expectations and the set criteria	Go into their established working groups. Being creative and create dramas, songs/lyrics or wall magazines in line with the topic.
PROJECT (individual/group) Helps students complete tasks individually or collectively	Teacher outline the steps and procedures of how to do and the criteria	Students are involved in investigations and finding solutions to problems to real life experiences. They carry out researches to analyse the causes and effects of problems to provide achievable solutions. Students carefully utilise the problem-solving approach to complete projects.
USE MEDIA & TECHNOLOGY to teach and generate engagement depending on the age of the students	Show a full movie, an animated one, a few episodes form documentaries, you tube movies and others depending on the lesson. Provide questions for students to answer before viewing	Viewing can provoke questions, debates, critical thinking, emotion and reaction. After viewing, students engage in critical thinking and debate

Strands, Units and Topics Suggested Lesson Titles

This section contains the overview of Mathematics content to be taught in Grade 12 General Mathematics. The table below outlines strands, units, topics with suggested lesson titles. Teachers will use this to develop their own termly and yearly programs.

Strand	Unit	Topic	Lesson Titles
Number, Operation and Computation	Financial Mathematics	Interest and Inflations	Interest
			Compound interest
			Inflation
			Appreciation
			Depreciation
		Consumer Credit	Loans
			Reduced Balance Loans
			Annuities Formula
			Credit /Debit Cards
			Hire-purchase agreements
		Investments	Types of Investments
			Investing in real estate
			Investing in Stock Market
		Insurance	Life Insurance
			Health Insurance
			Household Insurance
			Motor vehicle insurance
Geometry, Measurement and Transformation	Mensuration	Scale	Scale
			Area and scale
			Volumes and Scales
		Perimeter and area of polygons	Perimeter and area of triangles using Heron's Rule
			Perimeter and area of polygons
			Perimeter and area of regular polygons
		Volume and surface area	Application of volumes
			Application of surface areas
		Scales and dimensions	Similar figures and areas
			Similar solids, lengths, area and volume
			Total surface area and volume of truncated solids
			Modelling measurement problems
		Surveying	Estimating distances using pace length
			Offset surveys
			estimating area with irregular boundaries
			Surveying with obstacles

Strand	Unit	Topic	Lesson Titles
Patterns and Algebra	Application of Geometry	Polyhedra and Tessellations	Polyhedra
			Construction of Polyhedra
			Tessellation(tiling)
			Creating tessellations(complex)
	Functions and Graphs	Relations and functions	The sketches of functions and vertical line test
			The applications of the functions
		Domain and Range of a function	The main features of the functions including domain and range
			Calculate domain and range to functions
		Linear, quadratic, exponential functions and inequations	Linear, quadratic, exponential
			Linear, quadratic, exponential and inequality equations
		Sketches of hyperbolic, exponential, logarithmic functions and asymptotes	Hyperbolae of form $y = b/cx$
			Hyperbolae of form $y = (a/c-x) + b$
			Exponential curves of form $y = ax$
			Exponential curves of form $y = 1/ax$
			Logarithm functions ($y = \log ax$)
			Exponential and Logarithmic modelling
		Circles on Cartesian plane	Radius and centre of a unit circle
			General and standard form of the equation to circles
		Equations of Circles in standard forms	Conversion of the equation of circles in the standard for $(x-h)^2 + (y-k)^2 = r^2$
			Sketching circles in standard forms
Statistics and Probability	Bivariate Statistics	Bivariate data on scatter diagram	Scatter Diagrams
			Linear and nonlinear relationship
		Line Regression	Residuals
			Equation of Regression line
		Correlation	Types of correlation
			Interpret regression and correlation coefficients.
		Use of Technology in solving statistics problems	Using a Calculator
			Using an Excel Function

Grade 12
General Mathematics
Teaching Content

Strand 1: Number, Operations and Computation

Content Standard: Students will be able to represent numbers in various situations and forms, develop fluency in calculations through operations, use base ten as key for extending numbers and operations, and apply numbers in practical situations to develop number sense.

Unit	Benchmark	Topic	Lesson Title
Financial Mathematics	**12.1.1.1** Calculate compound interest, inflation, appreciation and depreciation.	Interest and Inflations	Interest
			Compound Interest
			Appreciation
			Depreciation
	12.1.1.2 Apply simple financial formulas to calculate personal loans, credit cards, hire-purchase, flat-rate interest loans, loan repayment schedules, service fees and charges, investments, stock markets, dividends, premium payments and returns.	Consumer Credit	Loans
			Reduced balance Loans
			Annuities Formula
			Credit/Debit Cards
			Hire-purchase agreements
	12.1.1.3 Explain types of investments and stock markets shares and calculate dividends, premium payments and returns.	Investments	Types of Investments
			Investing in real estate
	12.1.1.4 Identity and differentiate the types of insurance, policies and premium payments and calculate returns using financial formulas.	Insurance	Life Insurance
			Health Insurance
			House Insurance
			Motor Vehicle Insurance

Strand 1: Number, Operations and Computation

Unit : Financial Mathematics	Topic: Interest and Inflations

Content Standard	Students will be able to represent numbers in various situations and forms, develop fluency in calculations through operations, use base ten as key for extending numbers and operations, and apply numbers in practical situations to develop number sense.
Benchmark	**12.1.1.1** Calculate compound interest, inflation, appreciation and depreciation.

Learning Objective: By the end of the topic, students will be able to;
- calculate Compound interest or Appreciation, Depreciation, Inflation and Consumer Credit, and
- apply financial formulas in calculating interests.

Essential questions:

- What compound interest, inflation, depreciation and appreciation?
- How is compound interest calculated?

Key Concepts(ASK-MT)	
Attitudes/Values	Appreciate calculating different types of interest, depreciations and appreciations, inflation and use financial formulae in interest calculations.
Skills	Calculate simple and compound interests, appreciations and depreciations.
Knowledge	Calculating Interests and the use of financial formulas in calculating interests.
Mathematical Thinking	Think of how to modify formulas to make calculations easier and understandable.

Content Background

Interest

When people do have a lot of money for personal use or for business use, they can borrow money from a lender and they will pay this lender a fee for using their money. This fee is called **interest**

Alternatively, when people have more money than they need at the present time, they may **invest** this surplus money. By investing, the value of the amount invested increases overtime because the institution that they invest in pays for the use of that money. This payment is also called interest. Below are some situations where interest is likely to be charged

- Interest is charged by financial institutions such as banks when a loan is taken out.
- Interest is paid to an investor when he/she lends or invest money in a company or institution.
- Money left in bank accounts attracts a small amount of interest because the bank uses this money (for loans etc..)while it is in their care.

The sum of money invested (or borrowed) is called **principal.** The amount of interest paid depends on the amount of the principal. It also depends on the **term** (length of time) of investment (or loan) and on the **interest rate.** The interest rate is usually expressed as a percentage. Most calculations of interest involve percentage, so let's look at percentage increase.

Percentage increase if a quantity, δ, increases by r % then the increase is $\frac{r}{100}$ x δ The increased quantity is $\delta + \frac{r}{100}$ x $\delta = \delta = \delta(+\frac{r}{100})$

The multiplying factor for a percentage increase of $r\,\%$ is $(1+\frac{r}{100})$
To find the increased quantity we multiply by the multiplying factor

Simple interest

Simple interest is the interest calculated on the initial amount invested or borrowed (known as principal) for the entire term of the loan. A simple interest rate is sometimes referred to as **flat interest rate**. Below are common situations where simple interest is charged:

i. Savings account
ii. Fixed-term deposit account
iii. Government Bonds
iv. Company Debentures
v. Hire purchase agreement
vi. Bridging finance

Simple interest formula

When simple interest is calculated, a set percentage (the simple **interest rate**) of initial investment (the **principal**) is paid per time period. The interest rate is usually quoted as a particular percentage per annum (%p.a), which means percentage each year.

The formula for calculating simple interest is $I = \frac{PrT}{100}$ where P = principal
R = rate
T = time per annum

Total amount Accumulated

A kina, on a principal investment of P kina, for a T years at an interest rate of r % per annum, is given by:

$$A = P + 1$$
$$A = P + \frac{Prt}{100}$$

Taking P out as a common factor gives; $A = P\left(1 + \frac{rT}{100}\right)$

Compound interest

The calculation of compound interest is more complex because after each time period where interest is credited, these interest is added to the principal, creating a new principal, and so on. This creates a compounding effect on the interest- the investor is getting interest on the interest-hence the name compound interest.

We can use a formula to calculate the amount accumulated, A kina, when compound interest r % p.a. is applied to a principal amount of P Kina for a term of n years

If A_n is the amount in the account after n years then: $A_1 = P$

$$A_1 = P$$
$$A_1 = P + r\%\ of\ P$$
$$= P + \frac{r}{100}P$$
$$= P\left(1 + \frac{1}{100}\right)$$
$$= PR \quad \text{where} \quad R = \left(1 + \frac{1}{100}\right)$$

$$A_1 = PR + r\%\ of\ PR$$
$$= PR + \frac{r}{100}PR$$
$$= PR\left(1 + \frac{r}{100}\right)$$
$$= PR\ x\ R$$
$$= PR^2$$

Appreciation and Depreciation

Appreciation is the increase in value of an item or quantity over time. Common items that appreciate are property (houses, land) or items that are called 'collectables' such as gold, artwork, etcThe rate of increase in appreciation is similar to rate of increase in compound interest. For an item purchased for P Kina and appreciating at a rate of r % per year for n years

$$\text{Inflated Value after n years } = PR^n \; where \; R = 1 + \frac{r}{100}$$

Depreciation is the opposite of appreciation. It is the loss in value of a capital item over time. Basically there are three types of depreciations;

1. Flat rate depreciation.
2. Reducing-balance depreciation.
3. Unit-cost depreciation.

Inflation

Inflation is the general upward movement of the price of goods and services in a company. An increase in inflation is often caused by an increase in the supply of money. The annual rate of inflation can fluctuate greatly, ranging from nearly zero more than 20 %. Government like to ensure that the rate of inflation does not exceed more than 2-3% so that the spending power of people's money is maintained. If the inflation rate is high, prices will increase and people will not be able to purchase the same goods with the same amount of money.

Because inflation is usually quoted as a percentage rate per year, the calculation of inflated prices requires the same theory as growth with compound interest.
A formula to calculate this inflated value is the same as the formula for compound interest increase. If P is the original price of the item and r % is the inflation rate, then the inflated price after n years is

$$\text{Inflated Value after n years } = PR^n \; where \; R = 1 + \frac{r}{100}$$

Unit: Financial Mathematics	Topic: Consumer Credit

Benchmark	**12.1.1.2** Apply simple financial formulas to calculate personal loans, credit cards, hire-purchase, flat-rate interest loans, loan repayment schedules, service fees and charges.

Learning Objective: By the end of the topic, students will be able to;

- calculate consumer credit, and
- apply financial formulas in calculating interests for personal loan, credit cards payment and hire purchase.

Essential questions:

- How does a credit card works?
- What is a personal loan?
- What are the advantage and disadvantage of credit cards, debit cards and hire purchase agreement?

Key Concepts(ASK-MT)	
Attitudes/Values	Appreciate the importance of consumer credit services and their practical uses.
Skills	Calculate interest for loans, credits card payment and hire purchase and agreement.
Knowledge	Consumer credit services.
Mathematical Thinking	Think about how to modify formula to make calculations easier and understandable.

Content Background

Credit

Credit can be provided by a financial institution such as Bank Of South Pacific(BSP), Kina Bank, Westpac Bank..etc issuing a credit cards, or a retailer offering a store loyalty card such as CPL real reward card or Big Rooster loyalty card.
Financial institution offer credit card to customers that have stable income and a good credit history. Credit cards usually have a limit which is different for different individuals.
The credit card owner is sent an amount each month and the accumulated credit can be repaid within a specified time. There is a minimum amount payable and any amount that is not repaid will accumulate interest. It is easy to purchase with a credit card but it is also easier to overspend resulting in paying a large sum of interest.

interest is charged in various ways with credit cards

- Charged interest from the day it is borrowed
- Some cards have interest-free period while others do not but charge an annual fee for the use of the card.
- A minimum repayment amount is set each month.

Loyalty cards are organized in the same way as credit cards but are only used in that particular store that issues the card. Unlike credit cards, there are often incentives attached to loyalty cards but customers cannot access cash.

Debit cards

Debit cards are much more common than credit cards in PNG. There are different types of debit cards such as Kundu cards, Kina cards, Yumi cards and Access cards. In order to use a debit card you must sign to complete a transaction, or use EFPTOS and enter a PIN and the bank sends you regular statement detailing transactions and balance in your account. Debit cards allow you to use your own money which you have deposited into your account. There are no interests paid but the bank charges a very low fee compared to credit card.

Hire purchase Agreement

A hire purchase agreement usually requires the customer to pay a deposit then the balance owing plus the interest is paid in an agreed installment. The customer is hiring the item from the retailer until the final payment is made. If the customer defaults on any payments then the item will be repossessed by the retailer without any return of the payments. Interest paid on a hire purchase plan is calculated as simple interest per annum

Example

How much interest paid on a hire purchase contract that is used to pay off a laptop costing K1 500 and a deposit of K100. The contract is over 2 years at the rate of 16% p.a?

Solution

The balance owing after the deposit is paid = K1 500 –K100
= K 1 400

Interest to be paid in 2 years at 16% p.a. = K 1 400 X 16/100 X 2
= K448
The interest is K448

Personal Loan

A personal loan is money borrowed for the purchase of goods and services by the borrower.it can be obtained from a financial institution or a bank. To obtain such loans a borrower needs to meet the institutions requirements demonstrating that he/she will repay the loan. When a personal loan is give, the amount borrowed, the interest rate, the term of the loan and repayment amount is specified. The institution also charges a loan establishment fee and monthly fee.

Other types of loan include short term loan, eg bridging loan and long term loan eg reducing balance loan.

Unit : Financial Mathematics	Topic: Investment
Benchmark	**12.1.1.3** Explain types of investments and stock markets shares and calculate dividends, premimum payments and returns.

Learning Objective: By the end of the topic, students will be able to;

- identify different types of, investment and their policies, and
- calculating and compare interests for various investment.

Essential questions:

- What are the types of investments available in Papua New Guinea?
- Who is eligible to invest?

Key Concepts(ASK-MT)	
Attitudes/Values	Show confidents to identify the types of investment and appreciate the usefulness in real life.
Skills	Calculate interest for the different types of investment.
Knowledge	Types of Investment.
Mathematical Thinking	Think about how to calculate dividends of various types of investments.

Content Background

Investing in real estate

An investment property could be – a residential house

- A block of land that could be used for residential purpose later
- A business property such as a shop
- A farming land

Investing in real estate seems to be a certain method to increase wealth, but there are many hidden costs and pitfalls that can mean that this does not always happen. These costs include; initial cost of purchasing the real estate, duty amount on sale, ongoing costs, cost on selling the investment property and expected running costs.

Investing in stock market

In a way of expanding, businesses raise money by selling shares to the public. A share is a part of the ownership of a company. If you buy shares in a company then you are entitled to a share of the profit of the company; this is called dividend. A company selling shares is referred to as 'floating' the company and there are strict rules and regulations about the size and management of businesses that are floated. The rules are enforced to protect those buying shares because when the company sell shares it is under no obligations to repay the share price to the buyer. The process of floating a company, and buying and selling shares in the company, are all overseen by a body called the **stock exchange.** When a company is listed on the stock market than its share can be bought and sold at what is called the **market price**. The market price of the share depends on the demand for the share and this can fluctuate according to;

- Profitability of the company
- local demand for the product
- overseas demand for the product
- government legislation

There is a certain degree of risk associated with buying shares which is different for individual companies. The risk is high when there is a prospect of making a large profit but there is a large chance of losing all the invested money.
People buy shares in a company so that they can share in the profits of the company, either as a regular income from the dividends and/or from reselling the shares at a price greater than the buying price.

To buy a share on the stock market you need the expertise services of a **stockbroker.** Stockbrokers (share brokers) are independent financial advisors who are given the license to trade in shares. **Stockbrokers** charge a fee for their service and this is called brokerage. Brokerage is usually charged as a percentage of the value of a transaction but can also be a set amount.

Unit : Financial Mathematics	Topic: Insurance

Benchmark	**12.1.1.4** Identify and differentiate the types of insurance, polices and premium payments and calculate returns using financial formulas.

Learning Objective: By the end of the topic, students will be able to;

- identify different types of insurance and their policies, and
- apply financial formulas in calculating premium payments.

Essential questions:

- What is insurance and why is it important?
- What types of insurance are provided?

Key Concepts(ASK-MT)	
Attitudes/Values	Show interest in identifying the different types of insurances and their policies and value their importance.
Skills	Calculate premium payments and returns over a period of time using financial formulae.
Knowledge	Types of Insurances, policies and premium payments.
Mathematical Thinking	Think about how to calculate the premium payments using financial formulae.

Content Background

Insurance

Insurance is an arrangement by which a company or the state undertakes to provide a guarantee of compensation for specified loss, damage, illness, or death in the return for payment of a specified premium. A business that provides this service is called an Insurance company.

Types of insurance include:

i. Life insurance – term life insurance, whole life insurance
ii. Health insurance
iii. Household insurance - co-insurance, Liability
iv. Motor Vehicle insurance-compulsory third-party insurance, insurance cover for loss and damage of vehicle

An example of a Brokerage on the value of the transaction taken from *'Save Buk Gr.12 General Maths, pg. 147'* is shown below:

Brokerage on the value of the transaction

Value of transaction(K)	Brokerage
5000 or below	2.5%(minimum of K65)
5001 – 20 000	2.0%
20 001 – 50 000	1.5%
50 000 and above	1.0%

There are various figures and ratios that help buyers of shares to assess the suitability of companies that sell shares:

- The dividend per share (DPS). $Dividend\ per\ share = \frac{total\ dividend\ paid}{number\ of\ shares}$
  (The amount is usually quoted in cents per share)
- The dividend yield which allows comparison of income generated by different investments.

  $Dividend\ yield = \frac{DPS}{share\ price}$ expressed as a percentage (%).
- The earnings per share (EPS)

  $Earning\ per\ share = \frac{DPS}{share\ price}$ expressed as cents
- The price - earnings ratio (PE ratio) shows how many years it will take for the purchase of shares to be returned by earnings.

  $price - earning = \frac{share\ price}{EPS}$

Strand 2: Geometry, Measurement and Transformation

Content Standard: Students will be able to comprehend the meaning and significant of geometry, measurements and spatial relationship including units and system of measurement and develop and use techniques, tools, and formulas for measuring the properties of objects and relationships among the properties and use transformations and symmetry to analyze mathematical situations.

Unit	Benchmark	Topic	Lesson Title
Mensuration	**12.2.2.1** Calculate actual area and dimensions of the field given the ratios.	Scale	Scale
			Area and scale
			Volumes and Scales
	12.2.2.2 Apply the Pythagorean Theorem, sine ratio and Heron's formula to solve problems involving triangles.	Perimeter and area of polygons	Perimeter and area of triangles using Heron's Rule
			Perimeter and area of polygons
			Perimeter and area of regular polygons
	12.2.2.3 Calculate area and total internal angles of polygons by identifying the triangular shapes.	Volume and surface are	Application of volumes
			Application of surface areas
	12.2.2.4 Use formulas, including appropriate units of measure, to determine the surface area and volumes pyramids, cylinders, spheres, and truncated solids.	Scales & Dimensions	Similar figures and areas
			Similar solids, lengths, area and volume
			Calculating surface area and volume of truncated solid
			Modelling measurement problems
	12.2.2.5 Apply methods of measurements in surveying on level ground without obstacle and around obstacle	Surveying	Estimating distances using pace length
			Offset surveys
			estimating area with irregular boundaries
			Surveying with obstacles
Application of Geometry	**12.2.2.6** Identify and construct shapes and solids from regular polygons and prisms to form tessellation patterns.	Polyhedra	Polyhedra
			Construction of Polyhedra
		Tessellation	Tessellation (tiling)
			Creating complex tessellations

Strand 2: Geometry, Measurement and Transformation

Unit : Mensuration	Topic: Scales

Content Standard 2	Students will be able to comprehend the meaning and significant of geometry, measurements and spatial relationship including units and system of measurement and develop and use techniques, tools, and formulas for measuring the properties of objects and relationships among the properties and use transformations and symmetry to analyse mathematical situations.
Benchmark	**12.2.2.1** Calculate actual area and dimensions of the field given the ratios.

Learning Objective: By the end of the topic, students will be able to;

- identify scale factors used in the shapes and solids,
- calculate actual area of shapes using a given scale factor, and
- find the volumes of solids with a scale factor of the model.

Essential questions:

- What is scale factor?
- How can scale factor be used when calculating area of shapes and volume of solids?

Key Concepts(ASK-MT)	
Attitudes/Values	Express confidence in using scale factors to either enlarge or reduce shapes and solids
Skills	Calculate area and volume of solids and shapes modified by assigned scales.
Knowledge	Identify scale factors and its relationship pattern to area and volume.
Mathematical Thinking	Think about how to relate models to authentic situations from the given scale.

Content Background

Scales

To find a scale factor between two similar figures, find two corresponding sides and write the ratio of the two sides. If you begin with the smaller figure, your scale factor will be less than one. If you begin with the larger figure, your scale factor will be greater than one. When looking at scales we look at what happens to the area of shapes and the volume of solids when lengths in polygons and solids are enlarged by a particular scale factor. The examples below will explain further.

Area

Example 1.
Consider a rectangle 5 cm by 2 cm
The area of the rectangle is 5 cm x 2 cm = 10 cm^2, but happens if the rectangle is enlarged?
Lets start by multiplying the rectangle by scale factor 2. The rectangle is now 10 cm by 4 cm and its area is 40 cm^2

The lengths were multiplied by 2, but the area has been multiplied by a scale factor of 4. Now we will multiply the lengths in the original rectangle by the scale factor 3. The rectangle is now 15 cm by 6 cm and its area is 90 cm^2. The lengths were multiplied by 3, but the area has been multiplied by a scale factor of 9.

Finally, we will try multiplying the lengths by scale factor 5. The rectangle is now 25 cm by 10 cm and its area is 250 cm^2. The lengths were multiplied by 5, but the area has been multiplied by a scale factor of 25.

Students should see a pattern as stated below.

When all the lengths are multiplied by k, the areas are multiplied by k^2.

Volume

When all the lengths are multiplied by k, the areas are multiplied by k^2.

Example 2.
Consider a Cuboid with side of length 3 cm, 4 cm and 5 cm.
The volume of the cuboid is 3 cm x 4 cm x 5 cm = 60 cm^3, but what happens if the cuboid is enlarged. We will start by multiplying the length by scale factor 2. The cuboid is now 6 cm by 8cm by 10 cm and the volume is 480 cm^3.

The lengths were multiplied by 2, but the volume has been multiplied by a scale factor of 8.
Now we will multiply the lengths in the original cuboid by the scale factor 3. The prism is now 9 cm by 12 cm by 15 cm and its volume is 1620 cm^3. The lengths were multiplied by 3, but the volume has been multiplied by a scale factor of 27.

Finally, we will try multiplying the lengths by scale factor 10. The rectangle is now 30 cm by 40 cm by 50 cm and its volume is 60 000 cm^3. The lengths were multiplied by 10, but the area has been multiplied by a scale factor of 1 000.

Students should see a pattern again.

When all the lengths are multiplied by k, the areas are multiplied by k^2.

If the lengths in a shape or solid are all multiplied by a scale factor of k, then the areas will be multiplied by a scale factor of k^2 and the volumes will be multiplied by a scale factor of k^3

Unit : Mensuration	Topic: Triangle Formula

Benchmark	**12.2.2.2** Apply the Pythagorean Theorem, sine ratio and Heron's formula to solve problems involving triangles.

Learning Objective: By the end of the topic, students will be able to;

- solve triangle problems using the cosine rule and the sine rule, and
- find areas of triangles using Herons formula.

Essential questions:

- How non-right angle triangles' can be calculated?
- What are the rules used to calculate the area of non-right angled triangle?

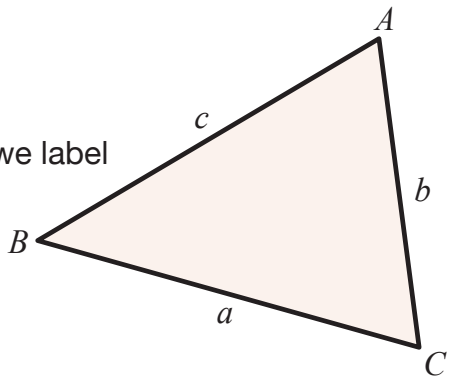
Key Concepts(ASK-MT)	
Attitudes/Values	Appreciate knowledge on sine rule, cosine rule and Heron's formula.
Skills	Calculate area of triangles using the sine rule, cosine rule and Heron's formula appropriately.
Knowledge	calculating area of right-angled and non-right-angled triangles.
Mathematical Thinking	Think about how to use the appropriate application of rules involved in calculating area of triangles.

Content Background

A common mathematical problem is to find the angles or lengths of the sides of a triangle when some, but not all of these quantities are known. It is also useful to be able to calculate the area of a triangle from some of these information. In this topic we will illustrate several formulae for doing this.

Consider a triangle as shown in the figure below,

A triangle with six pieces of information: angles at A, B, and C; sides a, b and c. There are six pieces of information available: angles at A, B and C, and the sides a, b and c. The angle at A is usually written A, and so on. Notice that we label the sides according to the following convention:



side b is opposite the angle B
side c is opposite the angle C
side a is opposite the angle A

The Sine Rule

The sine rule is used to find unknown side lengths and angles in a triangle where;

(i) Two angles and one side are known.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin c}$$

Two parts of this formula are used to find the length of another side

(ii) Two side lengths and a non-included angle are known

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin c}{c}$$

Two parts of this formula are used to find the size of another side

We can use the cosine Rule when three sides of the triangle are given. When given three sides, we can find unknown angles from the following formulae:

$$Cos\ A = \frac{b^2 + C^2 - a^2}{2bc}$$
$$Cos\ B = \frac{c^2 + a^2 - b^2}{2ca}$$
$$Cos\ A = \frac{a^2 + b^2 - c^2}{2ab}$$

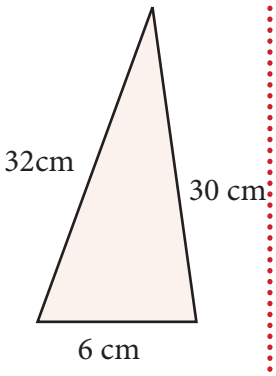
The cosine Rule given above can be rearranged into the following forms to find unknown sides

Herons Formula

Herons Formula allows us to find the area of a triangle when only the lengths of the three sides are given. His formula states:

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

Where a, b and c are the lengths of the sides and s is the semiperimeter of the triangle. The semiperimeter, s, of the triangle with sides a, b, and c, is $s = \frac{a+b+c}{2}$



Example:

Find the area of the triangle on the right.

Solutions

Before we can use Heron's formula, we will have to calculate the semiperimeter first. Extracting the values a = 32 cm, b = 30 cm and c =6 cm.

$$s = \frac{a+b+c}{2}$$
$$= \frac{32+30+6}{2}$$
$$= \frac{68}{2}$$
$$S = 34cm$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{34(34-32(34-30)(34-6)}$$
$$= \sqrt{34(2)(4)(28)}$$
$$= \sqrt{7616}$$
$$\approx 87.2696...$$
$$\approx 87.27cm$$
$$A \approx 87.27cm^2 \text{ correct to 2 d.p}$$
$$\text{Hence, the area of the triangle is } 87.27\text{cm}^3$$

Unit : Mensuration	Topic: Perimeter and area of polygons

Benchmark	**12.2.2.3** Calculate area and total internal angles of polygons by identifying the triangular shapes.

Learning Objective: By the end of the topic, students will be able to;

- divide the polygon into triangles,
- calculate area and internal angles of a polygon,and
- know the properties of regular polygon.

Essential questions:

- How do we find the total internal angler and area of polygons?
- How can we divide a polygon into triangles?

Key Concepts(ASK-MT)	
Attitudes/Values	Show confidence in finding the perimeter and area of polygon.
Skills	Calculate total internal angle and area of polygon.
Knowledge	Perimeter and area of polygons.
Mathematical Thinking	Think about how to divide regular polygons into different triangles to calculate total internal angle and area.

Content Background

Area of polygons

For us to find the area of any polygon we must first look at a triangle. There 3 different types of formulas for calculating the area of a triangle and the one that is used depends on given measurements of the triangle.

1. If the measurement for the base and its perpendicular height is given, then the area of the triangle is:

$$\text{Area of triangle} = \frac{1}{2}\text{x base x height}$$

$$\text{Area} = \frac{bh}{2}$$

2. If the measurement for the 3 sides is given, then the area of the triangle is

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

Where a,b and c are the sides of the triangle
s is the semi-perimeter calculated using the following formula

$$s = \frac{1}{2}(a+b+c)$$

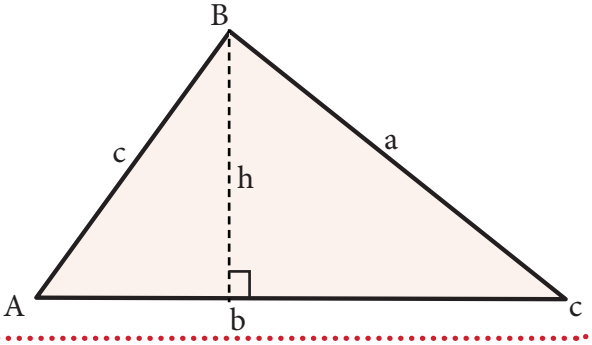
The above formula is called the Herons Formula. Also discussed in the previous topic

3. If the measurement for the 2 sides and the size of angle are given are given, then the area of the triangle is

$$\text{Area of triangle} = \frac{1}{2}\text{x } base \text{ x } height$$

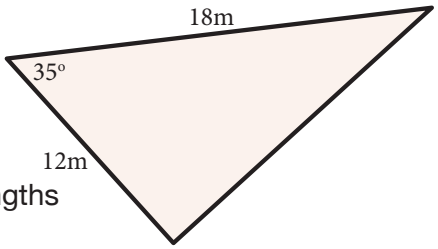
$$= \frac{1}{2}\text{x } base \text{ x } c \sin A$$

$$\text{Area of triangle} = \frac{1}{2}bc \sin A$$



This formula is used when the length of two sides, b and c, and the included angle size, A, is known.

Example
Find the area of the triangle on the right.



Solution
Collecting the necessary information from the diagram, two side lengths

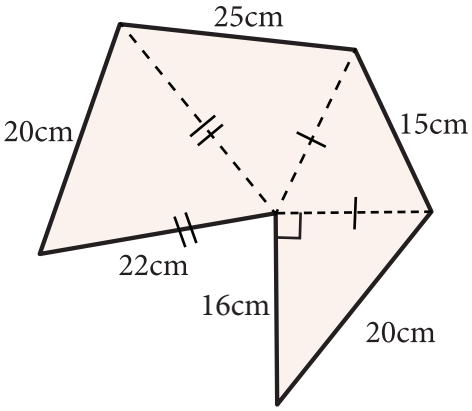
a =12 m,b = 18 m and the size of the included angle c =35^0 so we can use the formula

$$\text{Area of triangle} = \frac{1}{2}bc\sin A$$
$$= \frac{1}{2}\text{x}12m\text{ x }18m\text{ x }\sin 35^0$$
$$= 61.9462.....m^2$$
$$= 61.95m^2., \text{correct to 2 d.p}$$

Then the area of a polygon can be found by dividing the polygon into triangles, first finding the area of the triangles and then summing these areas.

An n-sided polygon can be divided into (n-2) triangles

Example
Find the area of the polygon on the right.



Solution
From the diagram there is a right-angled triangle an equilateral triangle,a scalene triangle and an isosceles triangle with all sides known.

1. Area of a right-angled triangle $= \frac{1}{2}\text{x }base\text{ x }height$
$= \frac{1}{2}\text{x }16cm\text{ x }15cm$
$= 120cm^2$

For these 3 triangles, 3 sides are known so Herons formula will be used

2. The semi-perimeter of an equilateral triangle $s = \frac{1}{2}(a+b+c)$
$= \frac{1}{2}(15cm+15cm+15cm)$
$= 22.5cm$

Area of an equilateral triangle$= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{22.5(22.5-15)(22.5-15)22.5-15)}$
$= \sqrt{9492.1875}$
$= 97.43cm^2 \text{correct to } 2d.p$

3. The semi-perimeter of a scalene triangle $s = \frac{1}{2}(a+b+c)$
$= \frac{1}{2}(25cm+22cm+15cm)$
$= 25.5cm$

Area of a scalene triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{25.5(25.5-25)(25.5-22)(25.5-15)}$$
$$= \sqrt{468.5625}$$
$$= 21.65 cm^2 \text{ correct to 2 d.p}$$

Area of an isosceles triangle

Area of the polygon = 120 cm^2 + 97.43cm^2 + 21.65 cm^2 + 1 296.45 cm^2
=1 535.53 cm^2

The area of the polygon is 1 535.53 cm^2

Total Internal Angles of Polygons

The same principle for calculating the area of polygons by dividing them into triangles can be used to find the interior angle sum of any polygon. If an angle is unknown then use one of the many formulas for calculating the angle of a triangle covered in the previous topic.
For exterior angles of regular polygons, just extend one of the sides and form the angle with the adjacent side to the extended side. The total exterior angle of a polygon is 360^0.

Unit : Mensuration	Topic: Volume and surface area
Benchmark	**12.2.2.4** Use formulas, including appropriate units of measure, to determine the surface area and volumes of pyramids, cylinders, spheres, and truncated solids.

Learning Objective: By the end of the topic, students will be able to;
- effectively calculate volume, and
- use different formulas of various types of polygons to calculate surface area.

Essential questions:
- What is volume?
- Is the surface area the same as area of polygons
- What are truncated solids?

Key Concepts(ASK-MT)	
Attitudes/Values	Show confidence in applying formulae for the different types of polygons.
Skills	Different types of formulas to calculate surface area.
Knowledge	Volume and surface area.
Mathematical Thinking	Think about how to calculate volume and surface areas.

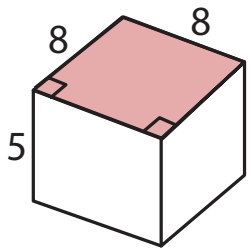
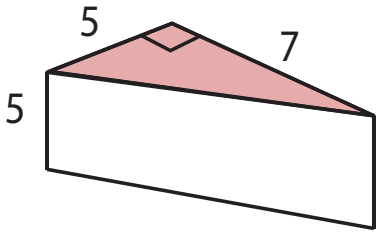
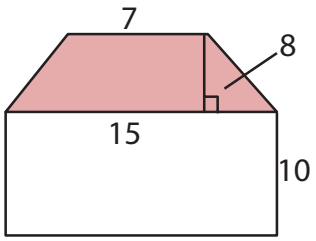
Content Background

Volume of a prism

Volume is a three–dimensional concept. It measures the amount of interior space of a three dimensional figure based on a cubic unit, that is, the number of 1 by 1 by 1 cubes that will fit inside a figure. The volume of a prism is the area of either base (B) multiplied by the height (h) of the prism.

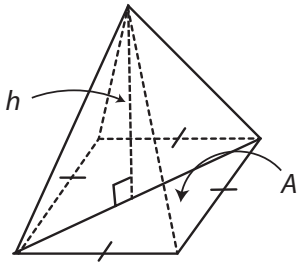
V = (Area of base) · (height) or V = Bh

Example
Find the volume of the prisms below.

Square Prism	Triangular Prism	Trapezoidal Prism
		
The base is a square with area (base) 8.8=64 cm² $\text{Volume} = Bh$ $= 64(5)$ $= 320cm^3$	The base is a right triangle with area $\frac{1}{2}(5)(7) = 17.5\text{units}^2$ $\text{Volume} = Bh$ $= 17.5(9)$ $= 157.5\text{units}^3$	The base is a trapezoid with area of $\frac{1}{2}(7+15).8 = 88\text{units}^2$ $\text{Volume} = Bh$ $= 88(10)$ $= 880 \text{ units}$

Volumes: Pyramids and prisms

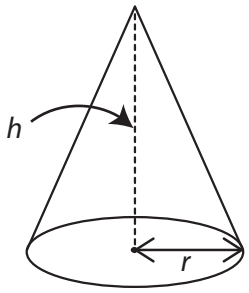
It can be shown that the volume of a square pyramid is one third of the volume of the corresponding right prism with the same height and base.



Volume of a pyramid $= \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height measured from the base.

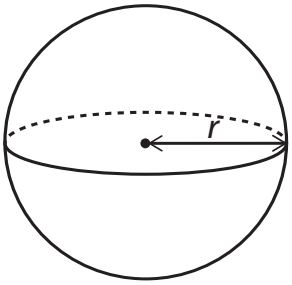
This formula holds for pyramids with a polygonal base with area A.

The cross-sections of a cone (or sphere) are circles but the radii of the cross-sections differ. The volume of a cone is one third of the volume of the corresponding cylinder with the same height and radius.



Volume of a cone $= \frac{1}{3}\pi r^2 h$ where r is the radius of the base and h is the height.

Finally, the volume of a sphere is given by Volume of a sphere $= \frac{4}{3}\pi r^3$, where r is the radius of the sphere.



Surface Area

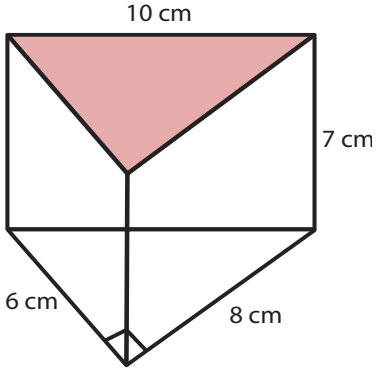
In the same way that we 'cut open' a prism to find the surface area, we can 'cut open' a cylinder of radius r and height h to show that the area of the curved surface is $2\pi rh$. Adding in the two circular ends, we arrive at the formula $A = 2\pi rh + 2\pi r^2$ for the total surface area of a cylinder. The surface area formula for a cone is $A = \pi r^2 + nrl$, where r is the radius and l is the slant height. Finally, the surface area of a sphere is given by $A = 4\pi r^2$, where r is the radius of the sphere.

Surface Area of a Prism

The surface area of a prism is the sum of the areas of all of the faces, including the bases. Surface area is expressed in square units.

Example

Find the surface area of the triangular prism at right.



Step 1: Area of the 2 bases: $2\left[\frac{1}{2}(6cm)(7cm\right] = 48cm^2$

Step 2: Area of the 3 lateral faces

Area of face 1: $(6\ cm)\ (7cm) = 42\ cm^2$

Area of face 1: $(8\ cm)(7cm) = 56\ cm^2$

Area of face 1: $(10\ cm)(7cm) = 70\ cm^2$

Step 3: Surface area of prism = sum of bases and lateral faces:

SA = $48\ cm^2 + 42\ cm^{2+}\ 56\ cm^2 + 70\ cm^2 = 216\ cm^2$

Unit : Mensuration	Topic: Surveying
Benchmark	**12.2.2.5** Apply methods of measurements in surveying on level ground without obstacle and around obstacle.

Learning Objective: By the end of the topic, students will be able to;

- determine pace length by measuring 100, using tape measure,
- estimate distances and areas using pace length,
- define survey lines, offset and field book, and
- interpret and calculate areas and perimeter using sketch or scale diagram.

Essential questions:

- What is surveying?
- What are the different methods of surveying?

Key Concepts(ASK-MT)	
Attitudes/Values	Appreciate surveying and value the use of different methods of measurement of irregular boundaries.
Skills	Measuring and calculating perimeter or area of irregular boundaries on level ground without and aroud obstacles.
Knowledge	Surveying
Mathematical Thinking	Think about how to measure irregular boundaries and adapt the importance of surveying.

Content Background

Surveying

Surveying or land surveying is the technique, profession, art and science of determining the terrestrial or 3-dimensional position of points and the distances and angles between them. A land surveying professional is called a surveyor.

Surveyors survey properties for various reasons, including calculating the area of the land enclosed by the boundaries.

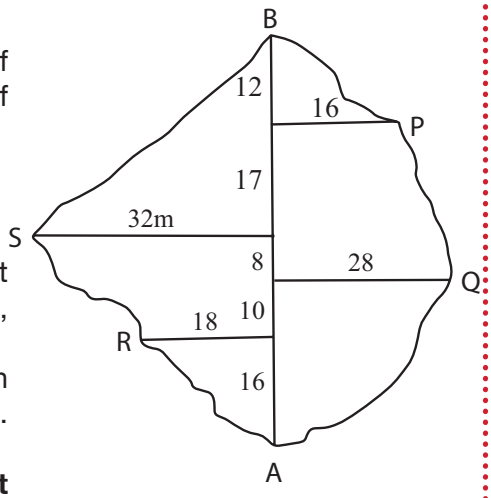
The units used to measure the land areas are as follows:

$$1 \text{ square metre } (m^2) = 1\ m\ x\ 1\ m = 1m^2$$
$$1 \text{ hectare } (ha) = 100\ m \text{ x } 100\ m = 1000m^2$$
$$1 \text{ square kilometer} = 1000\ m \text{ x } 1000\ m = 1000\,000\ m^2$$

Which unit to use depends on the size of the property? For instance, a building block of land is measured in square metres. There are number of ways calculating area, depending on whether it is calculated from a map or from measurements obtained in survey. Here we will look at ways of calculating areas from survey notes (also called field notes or a filed book)

Offset surveys

Offset surveys are used by surveyors to measure irregular block of land. The measurements recorded are used to make estimates of the perimeter or area of the block.

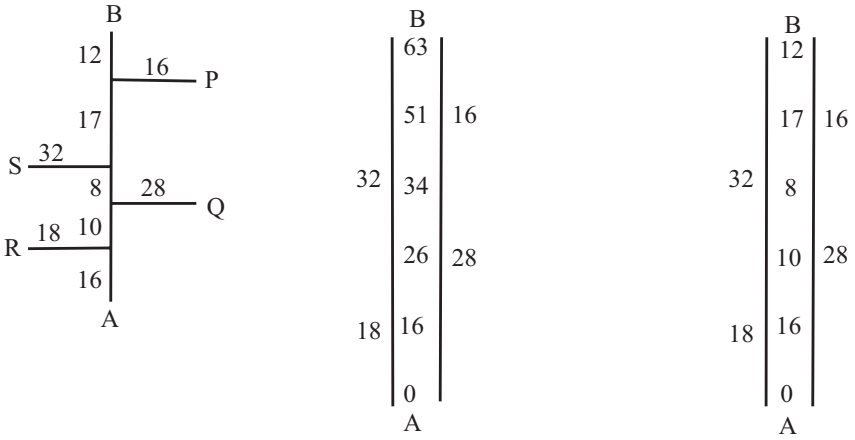


To carry out an offset survey of an irregular block of land:

1. The length of the straight line joining the two points furthest apart is measured and recorded. This line, AB in the diagram, is called the base line. Its direction is also recorded.
2. Measurements, along AB and perpendicular to AB, are then recorded for various features (P,Q,R and S) of the block of land.

The measurements perpendicular to the base line are called **offset distances**

A surveyor would record these measurements as field notes in one of the following ways.
A surveyor would record these measurements as field notes in one of the following ways.



The measurements along the baseline AB are from point A

Areas with irregular boundaries

Some areas that need to be measured have irregular or curved edges. In the field, irregular boundaries are sketched by using a survey line and a number of equally spaced offsets from the

survey line to the irregular boundaries. Once the survey line has been drawn, the area of irregular shape can readily be calculated. This can be calculated by treating individually each of the regions formed by the offset.

Definition and Technical Terms

Simply stating, *surveying* involves the measurement of distances and angles. The distance may be horizontal or vertical in direction. Vertical distances are also called *elevations.* Similarly, the angles may be measured in horizontal and vertical plane. Horizontal angles are used to express the directions of land boundaries and other lines. There are two fundamental purposes for measuring distances and angles.

- The first is to determine the relative positions of existing points or objects on or near the surface of the earth.
- The second is to layout or mark the desired positions of new points or objects, which are to be pl

Importance of Surveying

Surveying is one of the world's oldest and most important arts because, as noted previously, from the earliest times it has been necessary to mark boundaries and divide land. Surveying has now become indispensable to our modern way of life. The results of today's surveys are being used to:

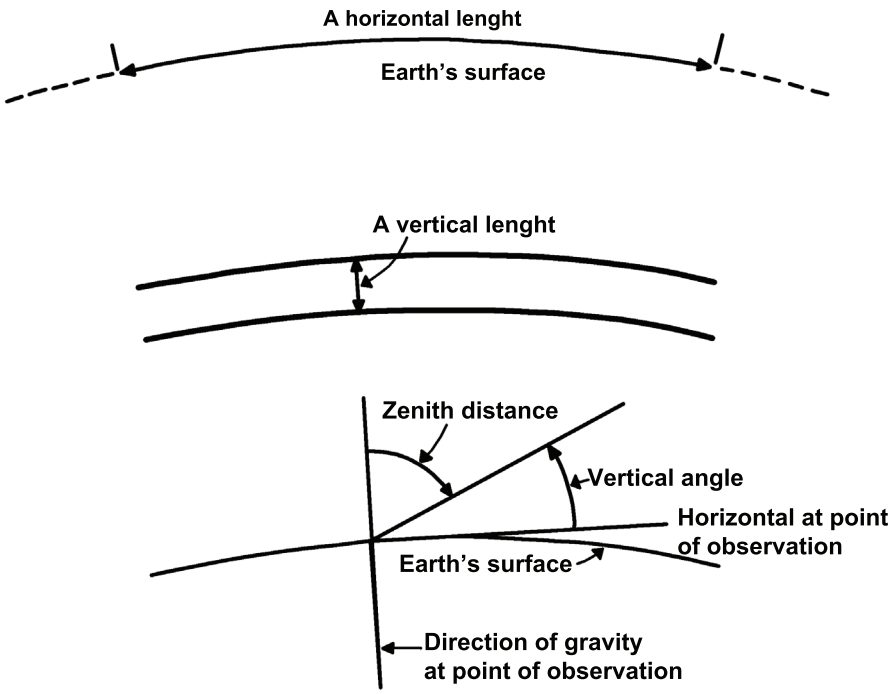
1. Map the earth above and below sea level,
2. Prepare navigational carts for use in the air, on land and at sea,
3. Establish property boundaries of private and public lands,
4. Develop data banks of land-use and natural resources information which aid in managing our environment,
5. Determine facts on the size, shape, gravity and magnetic fields of the earth and
6. Prepare charts of our moon and planets.

Application of Surveying

Surveying plays an essential role in the planning, design, layout, and construction of our physical environment and infrastructure (all the constructed facilities and systems which human communities use to function and thrive productivity). It is also the link between design and construction. Roads, bridges, buildings, water supply, sewerage, drainage systems, and many other essential public work projects could never have been built without surveying technology.

Measuring Distances and Angles

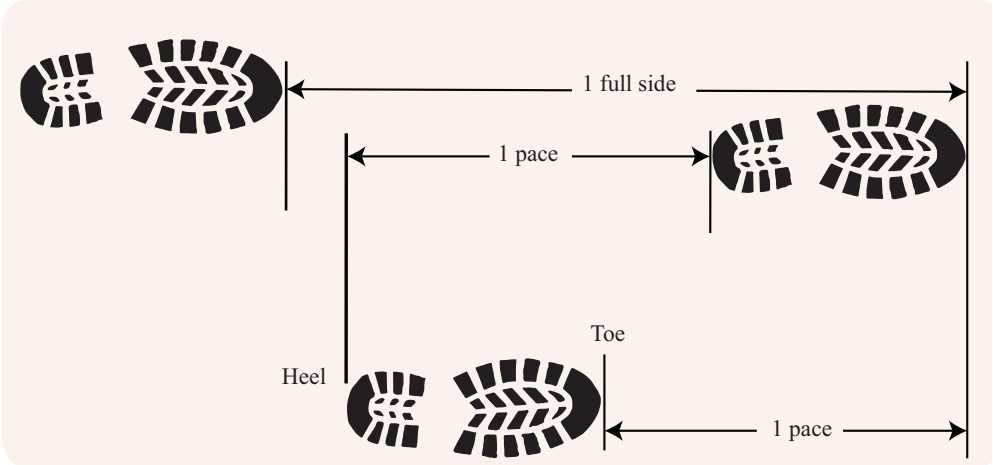
Horizontal distance is measured along a level surface. At every point along that length, the line tangent to the level surface is horizontal. It can be measured by tape or Electronic Distance Measurement (EDM). A true horizontal distance is actually curved, like the surface of the earth. A vertical distance is measured along the direction of gravity and is equivalent to a difference in height between two points. When the height is measured with reference to a given level surface, like mean sea level, it is called an elevation. An instrument called level, which is used to observe the rod at different points, can measure elevation. The relative vertical position of several points separated by long distances can be determined by a continuous series of level rod observations. This procedure is called leveling.



Rough Distance Measurement

In certain surveying applications, only a rough approximation of distance is necessary; a method called pacing, or the use of a simple measuring wheels, may be sufficient in these instances , e.g. locating topographic features during the preliminary reconnaissance of a building site, searching for the property corners etc. In this method, distances can be measured with an accuracy of about 1:100 by pacing. While providing only a crude measurement of distances, pacing has the significance advantage of requiring no equipment. It is a skill every surveyor should have. Pacing simply involves counting steps or paces while walking naturally along the line to be measured.

$$Distance = Unit\ Pace \times Number\ of\ Paces$$



Pacing provides a simple yet useful way to make distance measurement.
Depending on the skill and care applied, a pace distance can be determined with a relative accuracy of between 1:50 and 1:200.

Using the Measuring Wheel

A simple measuring wheel mounted on a rod can be used to determine distances, by pushing the rod and rolling the wheel along the line to be measured. An attached device called an odometer serves to count the number of turns of the wheels. From the known circumference of the wheel and the number of revolutions, distances for reconnaissance can be determined with relative accuracy of about 1:200. This device is particularly useful for rough measurement of distance along curved lines.

$$Distance = Odometer\ Reading \times Circumference\ of\ the\ Wheel(\pi D)$$

Where, D is the diameter of the measuring wheel.

Taping Equipment's and Methods

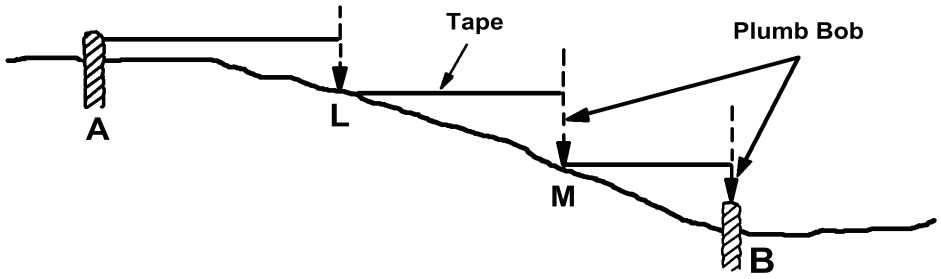
Measuring horizontal distances with a tape is simple in theory, but in actual practice, it is not as easy as it appears at first glance. It takes skill and experience for a surveyor to be able to tape a distance with a relative accuracy between 1:3000 and 1:5000, which is generally acceptable range for most preliminary surveys.

Steel Tapes

Modern steel tapes are available in variety of lengths and cross sections; among the most commonly used are the 100*ft* tape and the 30-m tape, which $\frac{1}{4}$ are in and 6 mm wide, respectively. Both lighter as well as heavier duty tapes are also available.

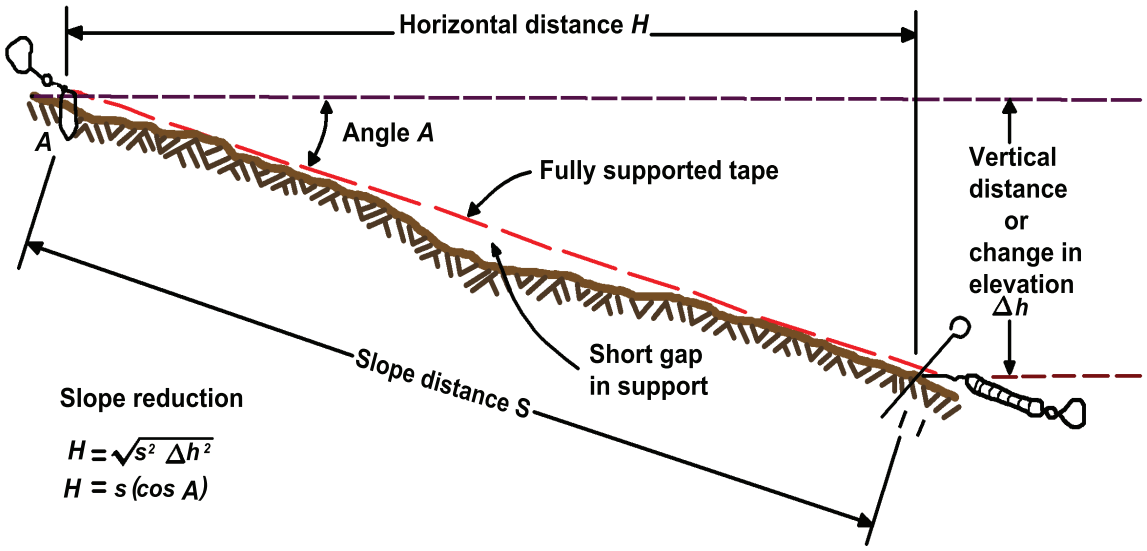
Horizontal Measurement on Sloping

In taping on uneven or sloping ground, it is standard practice to hold the tape horizontal and use a plumb bob at one or both ends. It is difficult to keep the plumb line steady for height above the chest. Wind exaggerates the problem and may make accurate work impossible. When a 100 m length cannot be held horizontally without plumbing from above shoulder level, shorter distances are measured and accumulated to total a full tape length. This procedure, called breaking tape.



Breaking tape

In measuring the distance between two points on a steep slope, rather than break tape every few meters, it may be desirable to tape along the slope and compute the horizontal component. This requires measurement also of either the angle of inclination A or the difference in elevation Δh. Breaking tape is more time consuming and generally less accurate due to the accumulation of random errors from making tape ends and keeping the tape level and aligned for many short sections.



Unit : Application of Geometry	Topic: Polyhedra and Tessellations

Benchmark	**12.2.2.6** Identify and construct shapes and solids from regular polygons and prisms to form tessellation patterns.

Learning Objective: By the end of the topic, students will be able to;

- identify and construct Polyhedra, and
- identify and create simple and complex tessellations.

Essential questions:

- What are polyhedrons and tessellations?
- How can we construct polyhedrons to form tessellations?

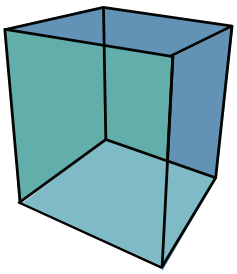
Key Concepts(ASK-MT)	
Attitudes/Values	Enjoy and appreciate identifying and drawing polyhedrons and being enthusiastic about creating complex tessellations.
Skills	Construct polyhedral and create simple or complex tessellations.
Knowledge	Identify and extract polyhedral shapes from solids and construct different polyhedrae tessellations.
Mathematical Thinking	Think about the usefulness of the construction of polyhedrons and logical organization of tessellations in real life.

Content Background

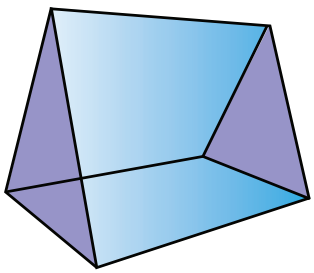
Polyhedra

A **polyhedron** is a solid with flat faces. From Greek poly meaning "many" and –hedron meaning 'faces'. Each face is a polygon (a flat shape with straight sides). There are many polyhedral that can be constructed that combine regular and irregular polygons. Each of this solids can be deconstructed to show the underlying flat faces of the solid which is called the net of the solid.

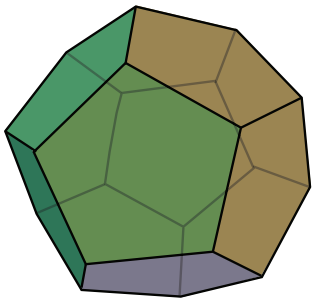
Examples of Polyhedra



Cube
Its faces are all squares



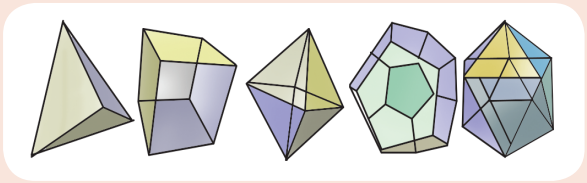
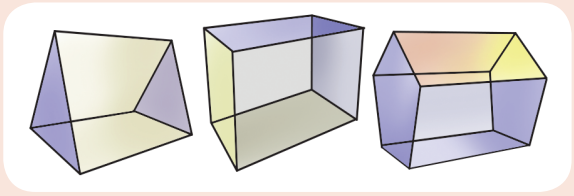
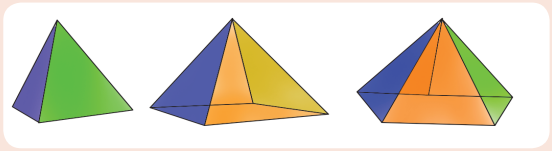
Triangular Prism
Its faces are triangles and rectangles



Dodecahedron
What faces does it have?

So no **curved surfaces**: cones, spheres and cylinders are not polyhedrons.

Common Polyhedra

	Platonic Solids
	Prisms
	Pyramids

Note: The plural of Polyhedron is Polyhedra or Polyhedrons

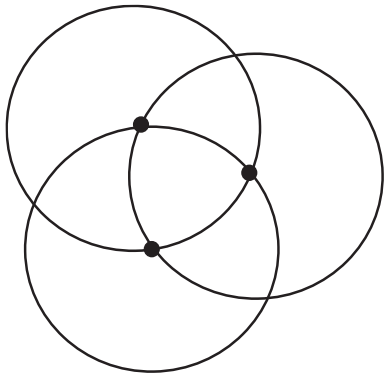
Constructing polyhedral requires careful attention to measuring lengths and angles. It is advisable to draw a net that minimizes the number of edges to be joined. If there are many repeated shapes, using templates would be the best option. Adding tabs to glue the structure together also makes it easier to construct.

Example

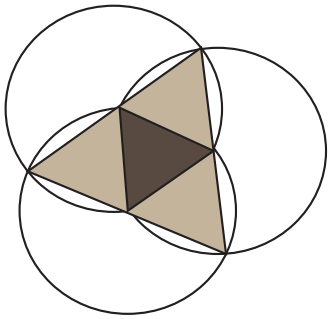
Construct a platonic solid.

From the illustration above, there five platonic solids. We will construct a tetrahedron since it is the first figure listed under the platonic solids. A tetrahedron has four faces that are equilateral triangles; four vertices and three faces meet at every vertex. Therefore; to construct a tetrahedron

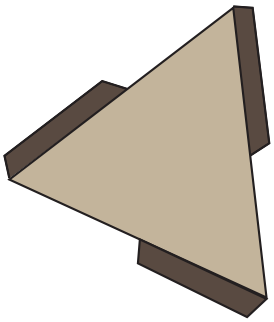
1. Using a compass, construct 3 intersecting circles of the same size radius so that the circumference of each of them goes through the centres of the other two.



2. Join the points of intersection on the circumference to form a large equilateral triangle.
   Join the points of intersection of the centre to form the smaller equilateral triangle.



3. Add some tabs for gluing your tetrahedron together, crease along the lines and glue together to form the tetrahedron. The best effect is achieved if the tabs are glued on the inside of the polyhedra

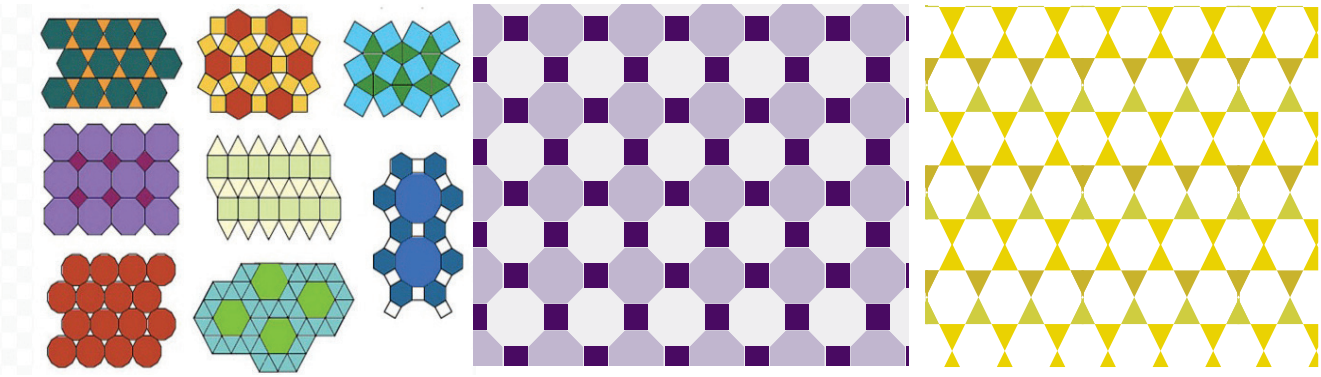


Net for the tetrahedron. Darker regions are tabs for gluing

Tessellations

A tessellation of a flat surface is the tiling of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. In mathematics, tessellations can be generalized to higher dimensions and a variety of geometries. A periodic tiling has a repeating pattern. Some special kinds include regular tilings with regular polygonal tiles all of the same shape, and semi-regular tilings with regular tiles of more than one shape and with every corner identically arranged.

Tessellations also known as tilings are a collection of polygons that fill the plane with no overlaps or gaps.



There are regular tessellations that tessellates with just one polygon and semi-regular tessellations that use two or more regular polygons. Tessellations are a good way to introduce students to the beauty of mathematics which can be very artistic and interesting to view and study.

Creating Complex Tessellation

In order to create any tessellation, one has to know that shapes should be symmetrical

Strand 3: Patterns and Algebra

Content Standard: Students will be able to interpret various types of patterns and functional relationships, use symbolic forms to represent, model, and analyze mathematical situations and collect, organize, and represent data to answer questions.

Unit	Benchmark	Topic	Lesson Title
Functions and Graphs	**12.3.3.1** Define and describe a function using the vertical line test.	Relations and Functions	The sketches of the functions and the vertical line test
			The applications of the functions
	12.3.3.2 Recognize and explain different functions and their features and calculate their range and domain.	Domain and Range of a function	The main features of the functions including Domain and Range
			Calculate domain and range to functions
	12.3.3.3 Define absolute value of a number and graph absolute value functions.	Absolute Value Functions	Absolute Value
			Graphing Absolute value function
			Absolute value equations
	12.3.3.4 Solve problems involving linear, quadratic and exponential and inequality equations involving absolute values individually or simultaneously using algebraic or graphs.	Linear, Quadratic and Exponential equations and Inequalities	Problems on Linear and Quadratic equations and Inequalities
			Problems on Exponential equations and Inequalities
		Absolute Value Function	Introductions of absolute value functions
			Graphs of absolute value functions
	12.3.3.5 Derive and sketch graphs of hyperbolic, exponential, logarithm functions and discuss their asymptotes and applications.	Sketches of hyperbolic, exponential, Logarithmic functions and Asymptotes	Sketching Hyperbolic functions
			Sketching exponential functions
			Sketching Logarithmic functions
			Definition and application of asymptotes
	12.3.3.6 Derive and sketch graphs of circles.	Circles on Cartesian Plane	Radius and centre of a unit circle
			Deriving General form of circle equations from the graph
			Sketching the graphs of circles
	12.3.3.7 Convert and write equations of circles in standard form $(x-h)^2+(y-k)^2=r^2$.	Equation of Circles in Standard Form	Conversion of the equations of circles in Standard form: $(x - h)^2 + (y - k)^2 = r^2$
			Sketch circles in standard form

Strand 3: Patterns and Algebra

Unit: Functions and Graphs | **Topic: Relations and functions**

Content Standard 3	Students will be able to interpret various types of patterns and functional relationships, use symbolic forms to represent, model, and analyse mathematical situations and collect, organize, and represent data to answer questions.
Benchmark	**12.3.3.1** Define and describe a function using the vertical line test.

Learning Objective: By the end of the topic, students will be able to;

- identify and define functions and its features,
- examine the behaviour of function displayed on a graph, and
- demonstrate and explain the vertical line test.

Essential questions:

- What is a function?
- What are the features to a function?
- How can the behaviours to a function be described when displayed on a graph?

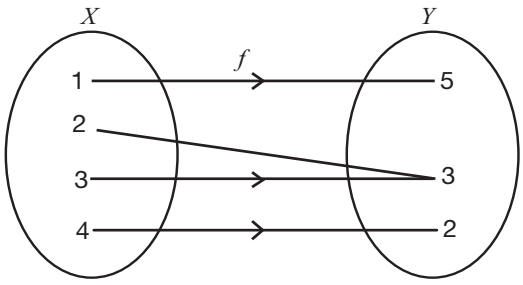
Key Concepts(ASK-MT)	
Attitudes/Values	Discuss confidently the relationship of elements within a functions displayed on a diagram.
Skills	Analysis and solve Problem involving functions.
Knowledge	Functions and vertical line test.
Mathematical Thinking	Think about how to explain relations and functions and vertical line test using diagrams or graphs.

Content Background

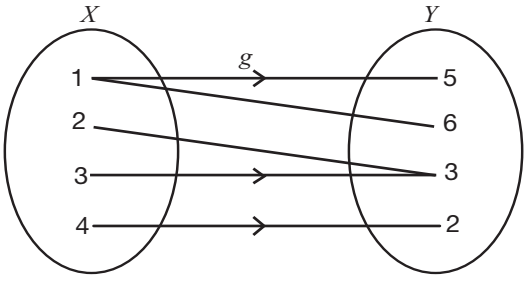
What is a Function?

Definition of a function: A function f from a set of elements X to a set of elements Y is a rule that assigns to each element x in X exactly one element y in Y

One way to demonstrate the meaning of this definition is by using arrow diagrams.



$f: X \rightarrow Y$ is a function. Every element in X has associated with it exactly one element of Y.



g: $X \rightarrow Y$ is not a function. The element 1 in set X is assigned two elements, 5 and 6 in set Y.

A function can also be described as a set of ordered pairs (x,y) such that for any x-value in the set, there is only one y-value. This means that there cannot be any repeated x values with different y-values.

The examples above can be described by the following sets of ordered pairs.

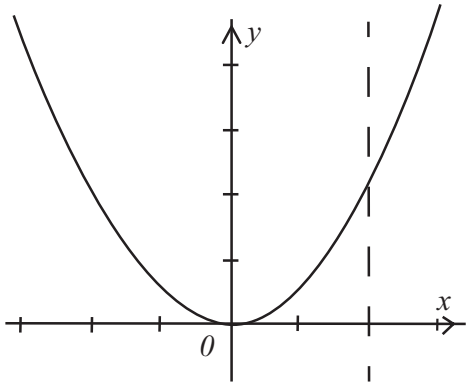
$F = \{(1,5),(3,3),(2,3),(4,2)\}$ is a function. $G = \{(1,5),(4,2),(2,3),(3,3),(1,6)\}$ is not a function.

The definition we have given is a general one. While in the examples we have used numbers as elements of X and Y, there is no reason why this must be so. However, in these notes we will only consider functions where X and Y are subsets of the real numbers.

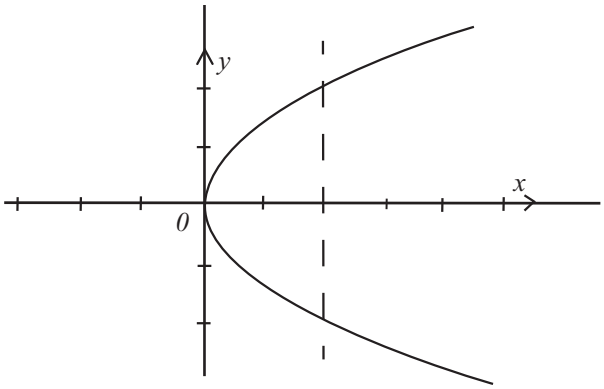
In this setting, we often describe a function using the rule, $y = f(x)$, and create a graph of that function by plotting the ordered pairs $(x,f(x))$ on the Cartesian plane. This graphical representation allows us to use a test to decide whether or not we have the graph of a function: The Vertical Line Test.

The Vertical Line Test

The Vertical Line Test states that if it is not possible to draw a vertical line through a graph so that it cuts the graph in more than one point, then the graph is a function.



This is the graph of a function. All possible vertical lines will cut this graph only once.



This is not the graph of a function. The vertical line we have drawn cuts the graph twice.

Unit: Functions and Graphs	Topic: Domain and Range of a Function

Benchmark **12.3.3.2** Recognize and explain different functions and their features and calculate their range and domain.

Learning Objective: By the end of the topic, students will be able to;

- identify and discuss the domain and range of a function, and
- determine and interpret the domain and range of a given function.

Essential questions:

- What is a domain of a function?
- What is a range of a function?
- What are the significant features of a function?

Key Concepts(ASK-MT)	
Attitudes/Values	Share ideas on identifying the domain and range of a function.
Skills	Analysing and describe the domain and range of a function.
Knowledge	Definition of domain and range of a function.
Mathematical Thinking	Think about how to determine the domain and range of a given function.

Content Background

Domain of a Function

For a function $f: X \to Y$ the domain of f is the set X.

This also corresponds to the set of x-values when we describe a function as a set of ordered pairs (x, y). If only the rule $y = f(x)$ is given, then the domain is taken to be the set of all real x for which the function is defined. For example, $y = \sqrt{x}$ has domain; all real $x \geq 0$. This is sometimes referred to as the natural domain of the function.

Range of a Function

For a function $f: X \to Y$ the range of f is the set of y-values such that $y = f(x)$ for some x in X. This corresponds to the set of y-values when we describe a function as a set of ordered pairs (x,y). The function $y = \sqrt{x}$ has range; all real $y \geq 0$.

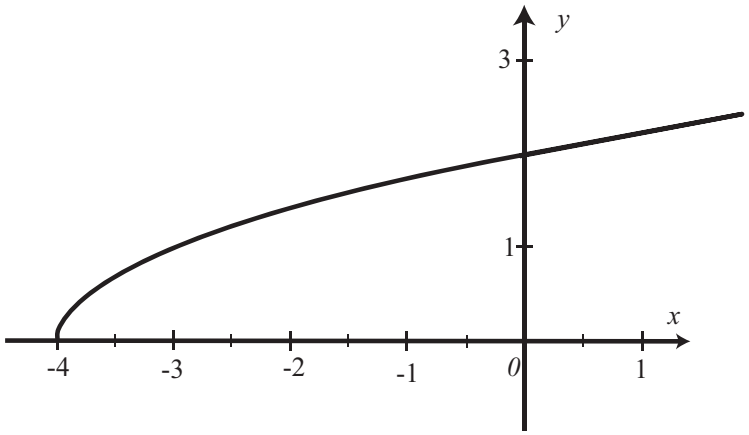
Example 1

a. State the domain and range of $y = \sqrt{x+4}$
b. Sketch, showing significant features, the graph of $y = \sqrt{x+4}$

Solution

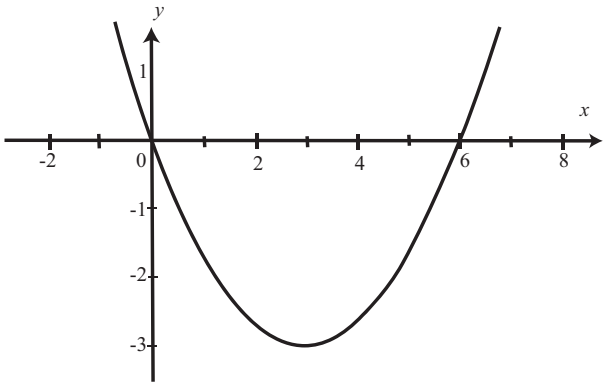
a. The domain of $y = \sqrt{x+4}$ is all real $x \geq -4$. We know that square root functions are only defined for positive numbers so we require that $x + 4 \geq 0$, *i.e.* $x \geq -4$. We also know that the square root functions are always positive so the range of $y = \sqrt{x+4}$ is all real $y \geq 0$.

b. The graph of $y = \sqrt{x+4}$



Example 2

(i) State the equation of the parabola sketched below, which has vertex (3,-3).
(ii) Find the domain and range of this function.



Solution

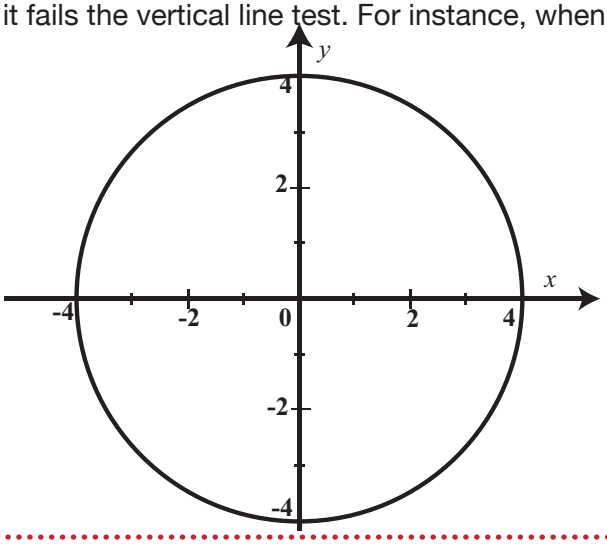
(i) The equation of the parabola is $y = \frac{x^2 - 6x}{3}$
(ii) The domain of this parabola is all real x. The range is all real $y \geq -3$

Example 3

Sketch $x^2+y^2=16$ and explain why it is not the graph of a function.

Solution

$x^2+y^2=16$ is not a function as it fails the vertical line test. For instance, when x = 0 $y = \pm 4$.



Unit: Functions and Graphs	Topic: Absolute Value Functions
Benchmark	**12.3.3.3** Define absolute value of a number and graph absolute value functions.

Learning Objective: By the end of the topic, students will be able to;

- identify absolute value functions and its features, and
- solve problems involving absolute functions and inequalities.

Essential questions:

- What is an Absolute Value Function?
- What is significant about Absolute Value Functions?
- How is an Absolute Value Function solved?

Key Concepts(ASK-MT)	
Attitudes/Values	Show confident to explain and discuss features of an absolute value function.
Skills	Analysis and solve Problem involving absolute functions and inequalities.
Knowledge	Solve problems with absolute values and inequalities through graphs or simultaneously.
Mathematical Thinking	Think about how to obtain solutions to problems involving absolute values and inequalities.

Content Background

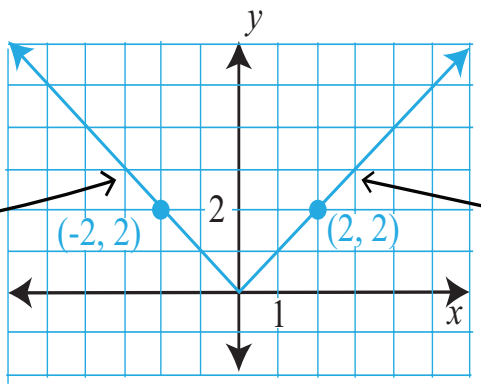
1. The Absolute Value

The absolute value of a number is its distance from 0 on the number line. Since distance is non-negative, the absolute value of a number is always non-negative. The symbol $|x|$ is used to represent he absolute value of a number x.

The absolute value of x is defined by: $|x| = \begin{cases} x, & if\ x > 0 \\ 0, & if\ x = 0 \\ -x & ,x < 0 \end{cases}$

The graph of this piece wise function consists of two rays, is V-shaped, and opens up. The corner point of the graph, called the vertex occurs at the origin.

To the left of x = 0 the graph is given by the line y = -x



To the left of x = 0 the graph is given by the line y = x

Notice that the graph of $y = |x|$ is symmetric in the y-axis because for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.

2. GRAPHING ABSOLUTE VALUE FUNCTIONS

The graph of $y = a|x - h| + k$ has the following characteristics.

- The graph has vertex (h, k) and is symmetric in the line $x = h$.
- The graph is V-shaped. It opens up if $a > 0$ and down if $a < 0$.
- The graph is wider than the graph of $y = |x|$ if $|a| < 1$.
- The graph is narrower than the graph of $y = |x|$ if $|a| > 1$.

To graph an absolute value function you may find it helpful to plot the vertex and one other point. Use symmetry to plot a third point and then complete the graph.

Example: Graph $y = -|x + 2| + 3$.

Solution
To graph $y = -|x + 2| + 3$, plot the vertex at (-2, 3). Then plot another point on the graph, such as (-3, 2). Use symmetry to plot a third point, (-1, 2). Connect these three points with a V-shaped graph. Note that $a = -1 < 0$ and $|a| = 1$, so the graph opens down and is the same width as the graph of $y = |x|$.

3. ABSOLUTE VALUE EQUATIONS

1. Isolate the absolute value.
2. Identify what the isolated absolute value is set equal to..

- If the absolute value is set equal to zero, remove absolute value symbols & solve the equation to get one solution.
- If the absolute value is set equal to a negative number, there is no solution.
- If the absolute value is set equal to a positive number, set the argument (expression within the absolute value) equal to the number and set it equal to the opposite of the number, using an 'or' statement in between the two equations. Then solve each equation separately to get two solutions.

Example

a. $|3x + 12| + 7 = 7$
$|3x + 12| = 0$

Solution
Because this equals 0, there is ONE solution.
$3x + 12 = 0$
$x = -4$

b. $|3x - 7| + 7 = 2$
$|3x - 7| = -5$

Because this equals a negative number, there is NO solution.

c. $|3x - 7| + 7 = 9$
$|3x - 7| = 2$

Because this equals A positive number, there are TWO solutions

$3x - 7 = 2$
$3x = 9$
$x = 3$

$3x - 7 = -2$
$3x = 5$
$x = \frac{5}{3}$

4. SOLVING ABSOLUTE VALUE INEQUALITIES

When absolute value inequalities are written to describe a set of values, like the inequality $y|x - 5| \leq 4$, it is sometimes desirable to express this set of values without the absolute value, either using inequalities, or using interval notation.

Example: Solve $|x - 5| \leq 4$

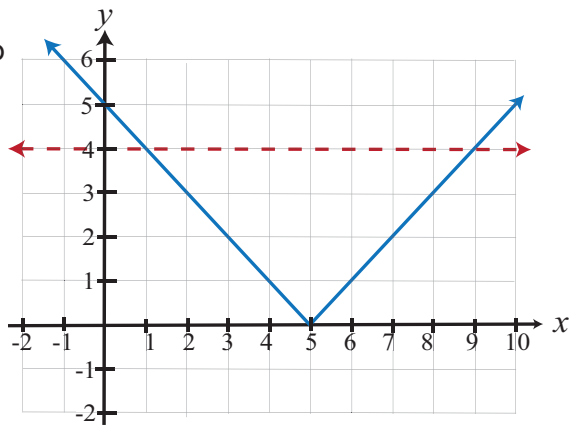
Solution: We will need to know first where the corresponding equality is true. In this case, we first will find where $|x - 5| \leq 4$. We do this because the absolute value is a nice friendly function with no breaks, so the only way the function values can switch from being less than 4 to being greater than 4 is by passing through where the values equal 4.

$|x - 5| \leq 4$.

$x - 5 = 4$ or $x - 5 = -4$

$x = 9$ $x = 1$

To use a graph, we can sketch the function $f(x) = |x - 5|$. To help us see where the outputs are 4, the line $g(x) = 4$ could also be sketched.



On the graph, we can see that indeed the output values of the absolute value are equal to 4 at $x = 1$ and $x = 9$. Based on the shape of the graph, we can determine the absolute value is less than or equal to 4 between these two points, when $1 \leq x \leq 9$. In interval notation, this would be the interval [1,9].

Example : Given the function $f(x) = -\frac{1}{2}|4x - 5| + 3$, determine for what x values the function values are negative.

Solution

We are trying to determine where $f(x) < 0$, which is when 4. We begin by isolating the absolute value:

$-\frac{1}{2}|4x - 5| < -3$ ..... when we multiply both sides by -2, it reverses the inequality

$\Rightarrow |4x - 5| > 6$

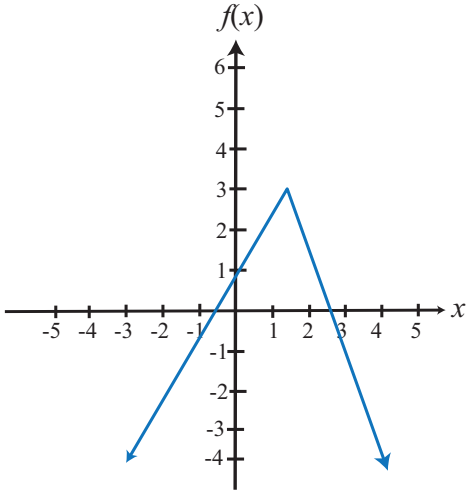
Next we solve for the equality $|4x - 5| > 6$

$4x - 5 = 6$ or $4x - 5 = -6$

$x = \frac{11}{4}$ $x = -\frac{1}{4}$

We can sketch a graph of the function to determine on which intervals the original function values are negative. Notice that it is not even really important exactly what the graph looks like, as long as we know that it crosses the horizontal axis at $x = \frac{11}{4}$ and $x = -\frac{1}{4}$, and that the graph has been reflected vertically. From the graph of the function, we can see the function values are negative to the left of the first horizontal intercept at $x = -\frac{1}{4}$ and negative to the right of the second intercept at $x = \frac{11}{4}$ This gives us the solution to the inequality: $x < -\frac{1}{4}$ *or* $x > \frac{11}{4}$

In interval notation, this would be $\left(-\infty, -\frac{1}{4}\right) \cup \left(\frac{11}{4}, \infty\right)$



Unit: Functions and Graphs	Topic: Linear, Quadratic and Exponential equations and Inequalities

Benchmark	**12.3.3.3** Solve problems involving linear, quadratic, and exponential and inequality equation involving absolute values, individually or simultaneously using algebra or graphs.

Learning Objective: By the end of the topic, students will be able to;

- describe linear, quadratic and exponential equations, and
- examine the inequalities that involve linear, quadratic and exponential equations.

Essential questions:

- How can you differentiate between a linear, quadratic and exponential equation?
- How can inequalities involving linear, quadratic and exponential equations be solved?

Key Concepts(ASK-MT)	
Attitudes/Values	Clearly explain or discuss features of an absolute value function.
Skills	Analysis and solve Problem involving absolute functions and inequalities.
Knowledge	Solve problems with absolute values and inequalities through graphs or simultaneously.
Mathematical Thinking	Think about how to obtain solutions to problems involving absolute values and inequalities.

Content Background

Graphing and solving quadratic inequalities

Four types of quadratic inequalities in two variables.

$$y<ax^2+bx+c\,,\quad y>ax^2+bx+c\,,\quad y\le ax^2+bx+c,\quad y\ge ax^2+bx+c$$

The graph of any such inequality consists of all solutions (x,y) of the inequality. The steps used to graph a quadratic inequality are very much like those used to graph a linear inequality.

To graph one of the four types of quadratic inequalities shown above, follow these steps:

(i) Draw the parabola with equation $y = ax^2 + bx + c$. Make the parabola dashed for inequalities with < or > and solid for inequalities with ≤ or ≥.
(ii) Choose a point (x,y) inside the parabola and check whether the point is a solution of the inequality.
(iii) If the point from Step 2 is a solution, shade the region inside the parabola. If it is not a solution, shade the region outside the parabola.

Example

Graph $y > x^2 - 2x - 3$

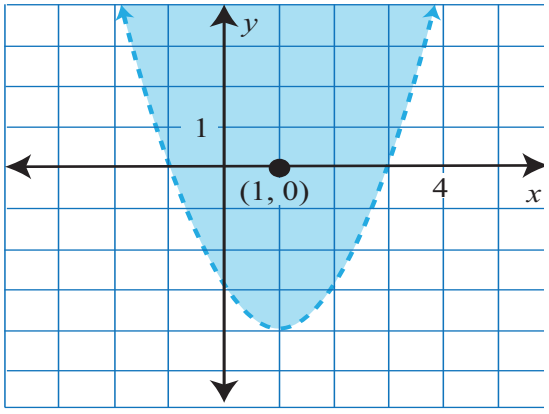
Solution: Follow the above three steps

(i) Graph $y = x^2 - 2x - 3$.
Since the inequality symbol is >, make the parabola dashed.

(ii) Test a point inside the parabola, such as (1,0).

$$y > x^2 - 2x - 3 \Leftrightarrow 0 > 1^2 - 2(1) - 3 \Leftrightarrow 0 > -4$$

So, (1,0) is a solution of the inequality.
Shade the region inside the parabola.



Graphing a system of quadratic inequalities is similar to graphing a system of linear inequalities. First graph each inequality in the system. Then identify the region in the coordinate plane common to all the graphs. This region is called the graph of the system.

Example: Graph the system of quadratic inequalities.

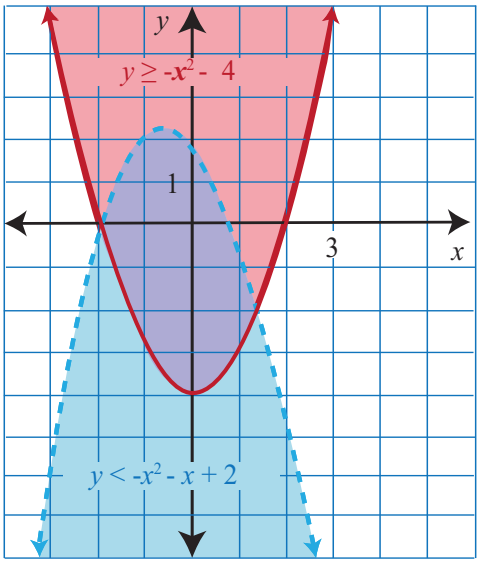
$y \geq x^2 - 4$ ..... Inequality 1
$y < -x^2 - x + 2$ .....Inequality 2

Solution

Graph the inequality $y \geq x^2 - 4$. The graph is the red region inside and including the parabola $y = x^2 - 4$.

Graph the inequality $y < -x^2 - x + 2$. The graph is the blue region inside (but not including) the parabola $y = -x^2 - x + 2$.

Identify the purple region where the two graphs overlap. This region is the graph of the system.



Unit: Functions and Graphs	Topic: Sketches of Hyperbolic, exponential, logarithmic functions and asymptotes

Benchmark	**12.3.3.4** Derive and sketch graphs of hyperbolic, exponential, logarithmic functions and discuss their asymptotes and applications.

Learning Objective: By the end of the topic, students will be able to;

- recognise and discuss hyperbolic, exponential and logarithmic functions and their features,
- sketch the graph to the hyperbolic, exponential and logarithmic functions, and
- explain the meaning to asymptotes.

Essential questions:

- What are the significances of exponential, hyperbolic and logarithmic functions?
- How are these functions solved and sketched?
- What is the definition to assymptotoes?

Key Concepts(ASK-MT)	
Attitudes/Values	Appreciate and enjoy discussing the features of hyperbolic, exponentials and logarithmic functions and their applications.
Skills	Sketching the graphs of hyperbolic, exponential and logarithmic functions.
Knowledge	Features of hyperbolic, exponential, logarithmic functions and the assymptotes.
Mathematical Thinking	Think about how to describe the features of hyperbolic, exponential, logarithmic functions and the assymptotes.

Content Background

1. Exponential Function

Exponential functions are one of the most important functions in mathematics. Exponential functions have many scientific applications, such as population growth and radioactive decay. Exponential function are also used in finance, so if you have a credit card, bank account, car loan, or home loan it is important to understand exponential functions and how they work.

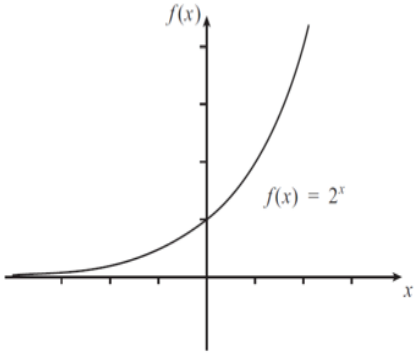
Exponential functions are function where the variable x is in the exponent. Some examples of exponential functions are $f(x)=2^x, f(x)=5^{x-2}$ *or* $f(x)=9^{2x+1}$. In each of the three examples the variable x is in the exponent, which makes each of the examples exponential functions.

An Exponential Function is a function of the form $f(x)=b^x$ *or* $y=b^x$ where b is called the "base" and b is a positive real number other than 1(b > 0 and b ≠ 1). The domain of an exponential function is all real numbers, that is, x can be any real number.

2. Graphing Exponential Functions

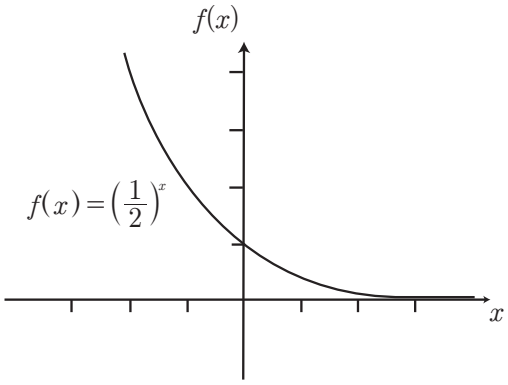
To begin graphing exponential functions we will start with two examples. We will graph the exponential functions $f(x)=2^x$ by making a table of values and plotting the points.

When creating a table of values, start with the numbers x =-3,–2,–1,0,1,2 and 3 because it is important to have different types of numbers, some negative, some positive, and zero.



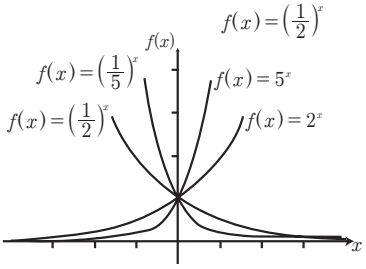
x	-3	-2	-1	0	1	2	3
$f(x)=2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Notice that as the x-values get smaller, x = –1, –2, etc. the graph of the function gets closer and closer to the x-axis, but never touches the x-axis. This means that there is a horizontal asymptote at the x-axis or y = 0. A horizontal asymptote is a horizontal line that the graph gets closer and closer to.



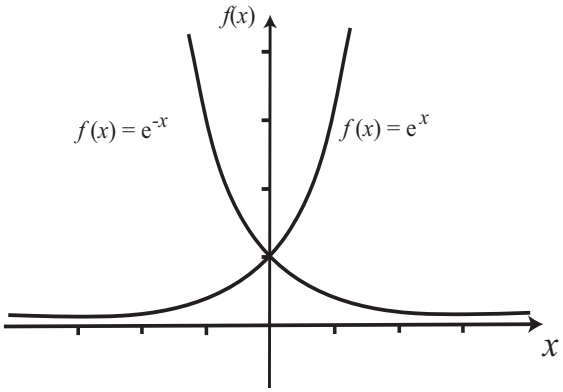
The example above demonstrates the general shape for graphs of functions of the form $f(x) = b^x$ when $b > 1$.
What happens if 0 < b < 1? To examine this case, take another numerical example. Suppose that $b = \frac{1}{2}$ *i.e.* $f(x) = \left(\frac{1}{2}\right)^x$

x	-3	-2	-1	0	1	2	3
$f(x) = (\frac{1}{2})^x$	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$



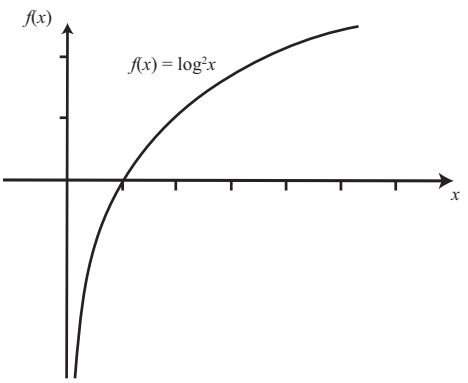
Notice that these properties are the same as when b > 1. One interesting thing that you might have spotted is that $f(x) = \left(\frac{1}{2}\right)^x = 2^{-2}$ is a reflection of $f(x) = 2^x$ in the $f(x)$ axis and that $f(x) = \left(\frac{1}{5}\right)^x = 5^{-x}$ is a reflection of $f(x) = 5^x$ in the $f(x)$ axis.
In general, $f(x) = \left(\frac{1}{b}\right)^x = b^{-x}$ is a reflection of $f(x)=b^x$ in the $f(x)$ axis.

A particularly important example of an exponential function arises when $b = e$. You might recall that the number e is approximately equal to 2.718. The function $f(x) = e^x$ is often called 'the' exponential function. Since $e > 1$ and $1/e < 1$, we can sketch the graphs of the exponential functions $f(x) = e^x$ and $f(x) = e^{-x} = \left(\frac{1}{e}\right)^x$.



2. Logarithm Functions

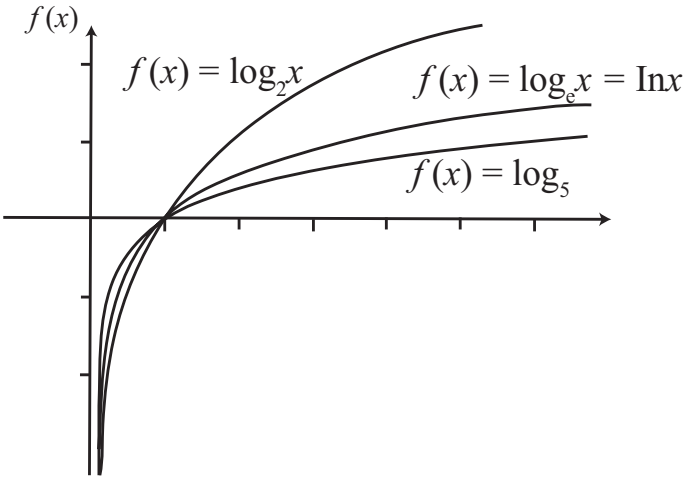
We shall now look at logarithm functions. These are functions of the form $f(x) = \log_a x$ where a > 0.



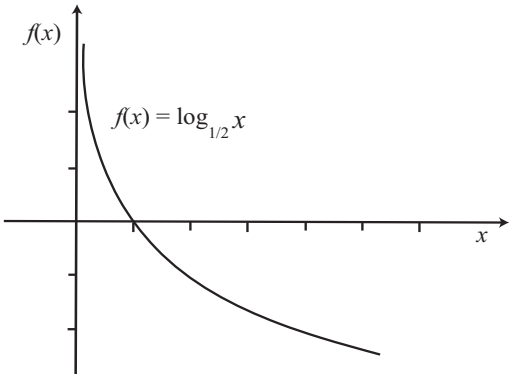
Let examine the case $f(x) = \log_2 x$ means $2^{f(x)} = x$. An important point to note here is that, regardless of the argument, $2^{f(x)} > 0$. So we shall consider only positive arguments.

x	$\frac{1}{2}$	$\frac{1}{2}$	1	2	4
$f(x) = \log_2 x$	-2	-1	0	1	2

This example demonstrates the general shape for graphs of functions of the form $f(x) = \log_2 x$ when a > 1.
What is the effect of varying a? We can see by looking at sketches of a few graphs of similar functions. For the special case where $a = e$, we often write ln x instead of loge x.



What happens if 0 < a < 1? To examine this case, take another numerical example. Suppose that a =1/2 .Then f(x) = $\log_{1/2} x$ means $(1/2)^{f(x)} = x$.

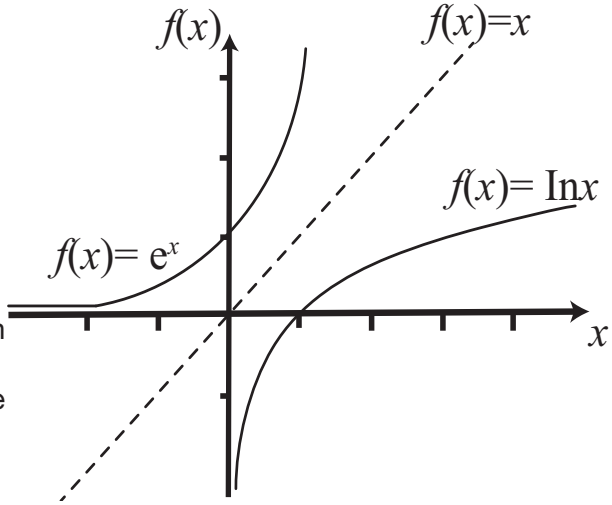


This example demonstrates the general shape for graphs of functions of the form $f(x) = \log_{1/2} x$ when 0 < a < 1.

3. The Relationship between Exponential Functions and Logarithm Functions

We can see the relationship between the exponential function $f(x) = e^x$ and the logarithm function $f(x) = \ln x$ by looking at their graphs.

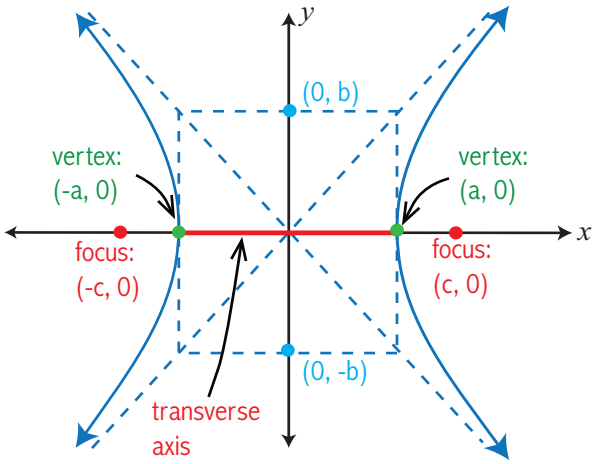
You can see straight away that the logarithm function is a reflection of the exponential function in the line represented by $f(x) = x$. In other words, the axes have been swapped: x becomes $f(x)$, and $f(x)$ becomes x.



Key Point: The exponential function $f(x) = e^x$ is the inverse of the logarithm function $f(x) = \ln x$

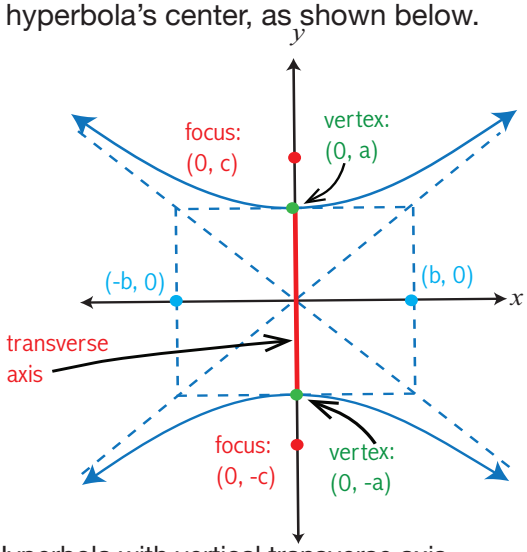
Hyperbola

A Hyperbola is the set of all points P such that the difference of the distances from P to two fixed points, called the foci, is constant. The line through the foci intersects the hyperbola at two points, the vertices. The line segment joining the vertices is the transverse axis and its midpoint is the centre of the hyperbola. A hyperbola has two branches and two asymptotes. The asymptotes contain the diagonals of a rectangle centered at the hyperbola's center, as shown below.



Hyperbola with horizontal transverse axis

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$



Hyperbola with vertical transverse axis

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

Characteristics of a Hyperbola (Center At Origin)

The standard form of the equation of a hyperbola with center at (0, 0) is as follows.

EQUATION	*TRANSVERSE AXIS*	*ASYMPTOTES*	*VERTICES*
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	Horizontal	$y=\pm\frac{b}{a}x$	$(\pm a,0)$
$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$	Vertical	$y=\pm\frac{a}{b}x$	$(0\pm a)$
The foci of the hyperbola lie on the transverse axis, c units from the center where $c^2=a^2+b^2$			

Graphing an Equation of a Hyperbola

Example: Draw the hyperbola given by $4x^2 - 9y^2 = 36$

Solution

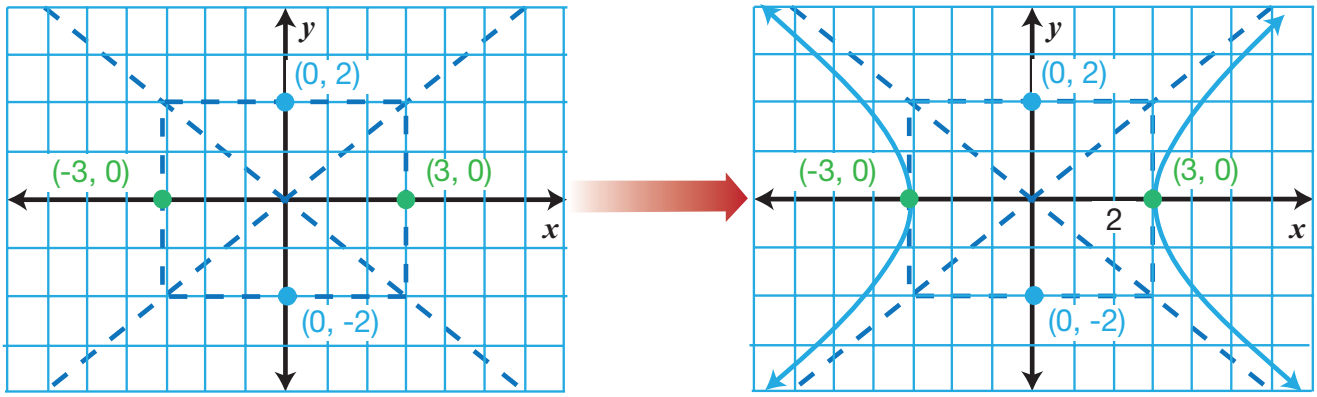
First rewrite the equation in standard form.

$4x^2 - 9y^2 = 36$ .....Write original equation

$\frac{4x^2}{36} - \frac{9y^2}{36} = \frac{36}{36}$ .....Divide each side by 36

$\frac{x^2}{9} - \frac{y^2}{4} = 1$ ..... simplifying

Note from the equation that $a^2 = 9$ and $b^2 = 4$, so $a = 3$ and $b = 2$. Because the x^2-term is positive, the transverse axis is horizontal and the vertices are at (-3,0) and (3,0). To draw the hyperbola, first draw a rectangle that is centered at the origin, $2a = 6$ units wide and $2b = 4$ units high. Then show the asymptotes by drawing the lines that pass through opposite corners of the rectangle. Finally, draw the hyperbola.



Unit: Functions and Graphs	Topic: Circles on Cartesian Plane

Benchmark **12.3.3.5** Derive and sketch graphs of circles.

Learning Objective: By the end of the topic, students will be able to;

- identify and discuss the features of a circle, and
- sketch circles on cartesian planes.

Essential questions:

- What are the main features to a circle?
- What is a unit circle?
- What is the general equation to a circle?

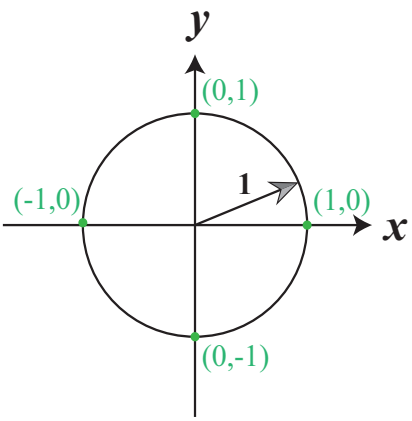
Key Concepts(ASK-MT)	
Attitudes/Values	Confidently and carefully sketch circles onto the cartesian planes and discuss the features.
Skills	Sketching circles onto the cartesian planes.
Knowledge	Sketches of circles and their equations.
Mathematical Thinking	Think about how to sketch circles onto cartesian planes given the equations.

Content Background

A circle is the set of points in a plane that are a fixed distance, called the radius, from a fixed point, and called the center. Because all of the points on a circle are the same distance from the center of the circle, you can use the Distance Formula to find the equation of a circle.

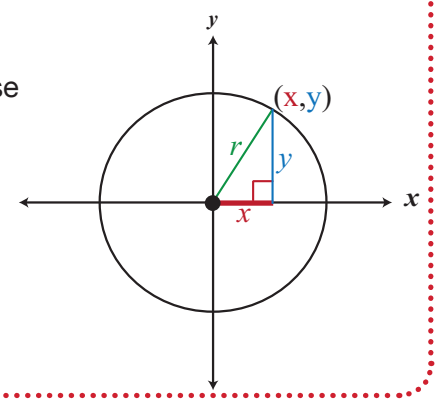
1. What is a 'Unit Circle?'

A "unit" circle has a radius of 1. In other words, the distance from the center of the circle to any part of the edge (circumference) is always 1. The unit of measurement doesn't really matter, because the most important thing about the unit circle is that it makes many equations and calculations much simpler.



It also serves as a useful basis for looking at the definitions of angles. Imagine that the center of the circle sits at the center of a coordinate system with an x-axis running horizontal and a y-axis running vertically. The circle crosses the x-axis at $x = 1$, $y = 0$. Scientists and mathematicians define the angle from that point moving in a counter-clockwise direction. So the point $x = 1$, $y = 0$ on the circle is at an angle of 0°.

2. Circles with Centre the Origin



In the circle below, let point (x,y) represent any point on the circle whose centre is at the origin. Let r represent the radius of the circle.
In the right triangle

r = length of the hypotenuse
x = length of leg
y = length of a leg

By Pythagorean Theorem, you can write

$x^2+y^2=r^2$

This is an equation of a circle with centre at the origin.

In general: Two quantities are needed to find the equation of a circle: (1) Centre and (2) Radius.
If the centre is is (0,0), the equation of the circle will be in the form $x^2 + y^2 = r^2$.

Example:
Find the equation of the following circles, each of the centre (0, 0).

(i) Which has the radius $\sqrt{13}$ **(ii)** which contains the point (4,-1)

Solution
Centre (0,0) is of the form $x^2 + y^2 = r^2$

$$x^2 + y^2 = (\sqrt{13})^2$$
$$x^2 + y^2 = 13$$

Thus the equation of the circle is $x^2 + y^2 = 13$

Centre (0,0) is of the form $x^2+y^2=r^2$

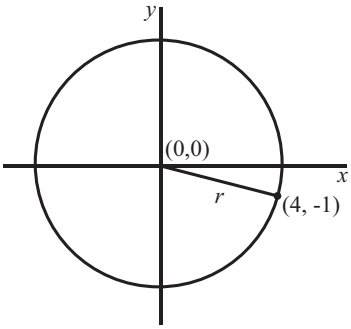
$$(4)^2 + (-1)^2 = r^2$$
$$16 + 1 = r^2$$
$$17 = r^2$$

Thus the equation of the circle is $x^2 + y^2 = 17$

Alternatively, the radius is the distance from (0,0) to (4,-1).
Using the distance formula, the radius

$$= \sqrt{(4-0)^2 - (-1-0)^2}$$
$$\sqrt{16+1}$$
$$\sqrt{17}$$

Thus the equation of the circle is $x^2 + y^2 = 17$ $\quad (\sqrt{17})^2 = 17)$



Unit: Functions and Graphs	Topic: Equation of Circles in Standard Form

Benchmark	**12.3.3.6** Covert and write equation of circles in standard form $(x-h)^2+(x-k)^2=r^2$

Learning Objective: By the end of the topic, students will be able to;

- state the conditions under which the general equation of second degree in two variables represents a circle, and
- derive and find the centre and radius of a circle whose equation is given in general form.

Essential questions:

- Can we find a mathematical expression for a given circle?
- How can you sketch a circle onto the cartesian plane?
- How does the equation of a circle look like?

Key Concepts(ASK-MT)	
Attitudes/Values	Confidently express equations to circles in the standard forms.
Skills	Converting equations of circles to the standard forms.
Knowledge	Standard form for equations to circles.
Mathematical Thinking	Think about how to derive equations of circles given certain points.

Content Background

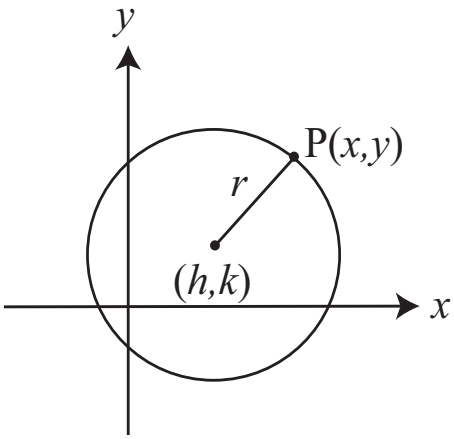
1. Circle

A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point in the same plane remains constant. The fixed point is called the centre of the circle and the constant distance is called the radius of the circle.

An equation of a circle is an equation in x and y which is satisfied by the coordinates of a point if and only if the point is on the circle.

2. Standard Form of the Equation of a Circle

Let $P(x,y)$ be a point in a plane which moves so that it is always a constant distance, called the radius r, from the fixed point (h,k), called the centre of the circle. Then by distance formula



$(x-h)^2+(y-k)^2=r^2$ ..... (1)

Equation (1) is called standard form of the circle, with centre (h,k) and radius r.

If the centre is at the origin (0, 0), equation (1) reduces to

$(x)^2+(y)^2=r^2$ ..... (2)

Also if the centre is at the origin (0,0) and radius is 1 (one), then the equation (1) reduces to the unit circle i.e;

$(x)^2+(y)^2=1$

Note that any equation equivalent to equation (1) is also an equation of the circle. We may reduce the equation (1) to the form.

$x^2+y^2-2ky+(h^2+k^2-r^2)=0$ ..... (3)

we observe that

(i) The equation (3) is second degree in x and y.
(ii) The coefficients of x^2 and y^2 are equal.
(iii) There is no product term xy.

Example: (i) Find the equation of the circle with centre at (–2, 3) and radius 6.
Solution: From the standard form

$$(x-h)^2+(y-k)^2=r^2 \text{ ,here } (h,k)=(-2,3), r=6$$
$$(x+2)^2+(y-3)^2=36$$

(i) State the coordinates of the centre and give the radius for the circle

$$x^2+y^2+4x-6y-3=0$$
$$x^2+4x+y^2-6y=3$$
$$(x^2+4x+4)+(y^2-6y+9)=3+4+9$$
$$(x+2)^2+(y-3)^2=16$$

complete the squares write as standard equation therefore, the centre is at (-2,3) and the radius is 4

Strand 4: Statistics and Probability

Content Standard: Students will be able to investigate how to interpret data using methods of exploratory data analysis, develop and evaluate inferences, predictions and arguments that are based on data and understand and apply basic notions of chance and probability.

Unit	Benchmark	Topic	Lesson Title
Bivariate Statistics	**12.4.4.1** Plot scatter diagrams, draw line of best fit and recognize linear and non-linear relationship.	Bivariate data on scatter diagrams.	Scatter Diagrams
			Linear and non-linear Relationship
	12.4.4.2 Calculate regression and correlation coefficients, write equation of the regression line, and interpret regression and correlation coefficients in the context of the problem.	Linear Regression	Residual
			Equation of Regression Line
		Correlation	Types of Correlation
			Regression and Correlation Coefficient
	12.4.4.3 Use appropriate technology to aid concept development as a tool for problem solving.	Use of Technology in problem solving	Using a Calculator
			Using an Excel Function

Unit : Bivariate Statistics | **Topic : Bivariate data on scatter diagrams**

Benchmark	**12.4.4.1** Plot scatter diagrams draw line of best fit and recognize linear and non-linear relationship.

Learning Objective: By the end of the topic, students will be able to;

- define bivariate data and understand what it means, and
- plot scatter diagram and recognize relationship of bivariate data.

Essential questions:

- What is bivariate data?
- What are some examples of bivariate data?

Key Concepts(ASK-MT)	
Attitudes/Values	Appreciate the concept of bivariate data and be able to describe and explain the relationships between the variables.
Skills	Describing the relationships between variables given bivariate data.
Knowledge	Bivariate data and sketching the data.
Mathematical Thinking	Think about how to analyse bivariate data and explain the results displayed on a graph.

Content Background

1. Bivariate Data

We often want to know if there is a relationship between the items in two different data sets. Examples of these are:

- Is there a relationship between children's ages and their heights?
- Is there a relationship between people's height and weights?
- Is there a relationship between student's marks in an English examination and their marks in mathematics examinations?

In each of the above, two pieces of information are to be collected from each person in the investigation and the two data sets are to be compared. When two pieces of information are collected from each subject in an investigation, then we are concerned with **bivariate data.**

A **scatter graph** or **scatter plot** is a type of display that uses coordinates to display values for two variables for a set of data. That data is displayed as a collection of points, each having the value of one of one variable determining the position of the horizontal coordinates and the value of the other variable determining the position of the vertical coordinate.

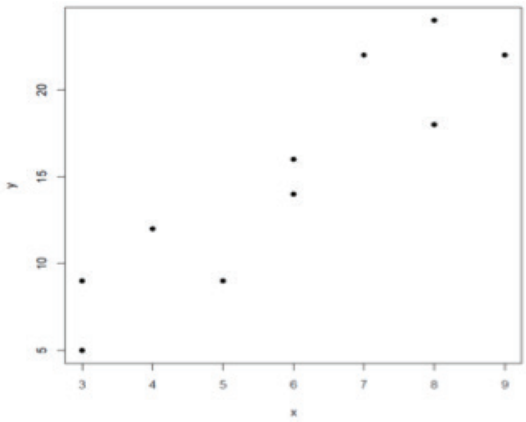
For example, a researcher wishes to investigate whether there is a relationship between the age and the blood pressure of people 50 years or older. To study this, an experiment is conducted using a random sample of ten patients and the following observations are recorded:

Dosage (x)	3	3	4	5	6	6	7	8	8	9
Duration of relief (y)	9	5	12	9	14	16	22	18	24	22

Note: There are two variables in the problem and they are labeled by x and y for our convenience. The common sense suggests that the variable y depends on x and x can be independently selected and controlled by the researcher. However, the variable y cannot be controlled and is dependent on x.

Statisticians are often interested to see whether there is a linear relationship (or linear association) between the two variables x and y: Such observations as collected as pairs on x and y (or $(x;y)$) are called bivariate data.

For the above bivariate data, '$x_1 = 3$' corresponds to '$y_1 = 9$' and there are $n = 10$ pairs. In general, there are n pairs of such bivariate data given by $(x_1,y_1),(x_2,y_2)....(x_n,y_n)$. A **scatter plot** of the above bivariate data is given below.



This scatter plot shows that the points seem to cluster around a straight line. It tells us that there is a possible (approximate) linear relationship between x and y.

Then we want to further investigate: *Is the linear relationship between the variables* $\boldsymbol{x}$ *and* $\boldsymbol{y}$ *clear and significant?*

We need to find a measure to investigate the strength of a possible linear relationship between two variables x and y: This measure is known as the correlation coefficient (or Pearson's correlation coefficient) between x and y.

Unit: Bivariate Statistics	Topic: Linear Regression

Benchmark	**12.4.4.2** Calculate regression and correlation coefficients, write equation of the regression line, and interpret regression and correlation coefficients in the context of the problem.

Learning Objective: By the end of the topic, students will be able to;

- understand the need for regression lines in determining the degree to which two variables of interest are correlated,
- find the equation of the line of best fit $(y = mx + c)$ from a data set of two variables of interest, and
- extrapolate and interpolate from the regression line equation, $(y = mx + c)$ and make predictions.

Essential questions:

- How can you define 'regression'?
- What causes regression?
- What is a regression line?

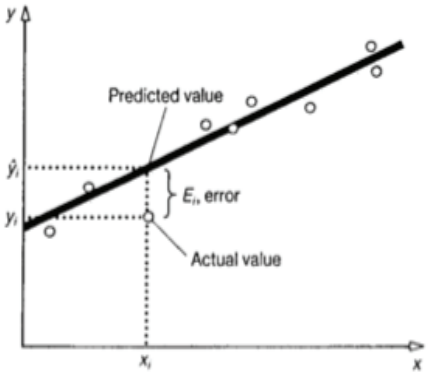
Key Concepts(ASK-MT)	
Attitudes/Values	Confidently express the relations between two quantities by way of the equation of the line of best fit.
Skills	Identify the relations between two quantities by way of the equation of the line of best fit.
Knowledge	Linear Regression.
Mathematical Thinking	Think about how to draw a line of best fit and equation of a regression line.

Content Background

Linear Regression

1.Residuals

After verifying that the linear correlation between two variables is significant, next we determine the equation of the line ***(line of best fit)*** that can be used to predict the value of y for a given value of x.



Each data point E_i (error) represents the difference between the observed y-value and the predicted y-value for a given x-value on the line. These differences are called residuals.

2. Regression Line – Least Square method

A regression line, also called a *line* of best fit, is the line for which the sum of the squares of the residuals is a minimum. The equation of a regression line for an independent variable x and a dependent y is $(y = mx + b)$ where $\hat{y}$ is the predicted y – value for the given x – value. The slope m and y- intercept b are given by

$$m = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} \text{ and } b = \bar{y} - m\bar{x} = \frac{\sum y}{n} - m\frac{\sum x}{n}$$

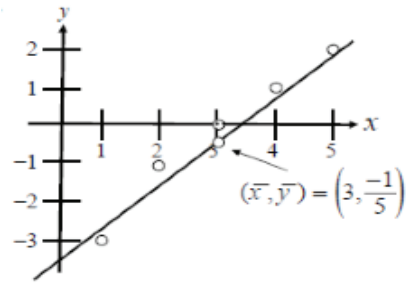
where $\bar{y}$ is the mean of the y- values and $\bar{y}$ is the mean of the x-values. The regression line always passes through $(\bar{x},\bar{y})$.

Example 1: Find the equation of the regression line

x	y	xy	x^2	y^2
1	-3	-3	1	9
2	-1	-2	4	1
3	0	0	9	0
4	1	4	16	1
5	2	10	25	4
$\sum x = 15$	$\sum y = -1$	$\sum xy = 9$	$\sum x^2 = 55$	$\sum y^2 = 15$

$$m = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} = \frac{5(9)-(15)(-1)}{5(55)-(15)^2} = \frac{60}{50} = 1.2$$

$$b = \hat{y} - m\hat{x} = \frac{-1}{5} - (1.2)\frac{15}{5} = -3.8$$



The regression line is $\hat{y} = 1.2x - 3.8$

Example 2: The following data represents the number of hours 12 different students watched television during the weekend and the scores of each student who took a test the following Monday.

a.) Find the equation of the regression line.

b.) Use the equation to find the expected test score for a student who watches 9 hours of TV.

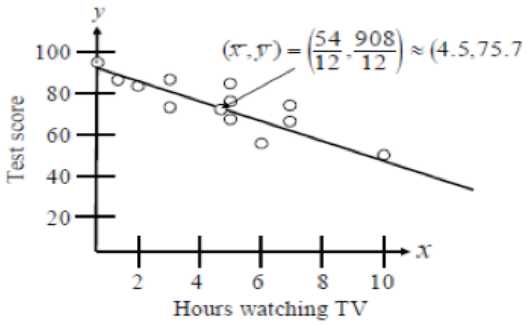
Hours, x	0	1	2	3	3	5	5	6	7	7	10
Test score, y	96	85	82	74	95	68	76	58	65	75	50
xy	0	85	164	222	285	340	380	348	455	525	500
x^2	0	1	4	9	9	25	25	36	49	49	100
y^2	9216	7225	6724	5476	9025	4624	5776	3364	4225	5625	2500

$$\sum x = 54,\ \sum y = 908,\ \ \sum xy = 3724, \sum x^2 = 332, \sum y^2 = 7083$$

$$m = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} = \frac{12(3724)-(54)(908)}{12(332)-(54)^2} \approx -4.067$$

$$b = y - m\hat{x} = \frac{908}{12} - (-4.067)\frac{54}{12} \approx 93.97$$

$$\hat{y} = -4.07x + 93.97$$



Unit: Bivariate Statistics	Topic : Correlation
Benchmark	**12.4.4.3** Calculate regression and correlation coefficients, write equation of the regression line, and interpret regression and correlation coefficients in the context of the problem.

Learning Objective: By the end of the topic, students will be able to;

- understand what a correlation coefficient is and interpret it, and
- calculate Correlation Coefficient (r) from a set of data.

Essential questions:

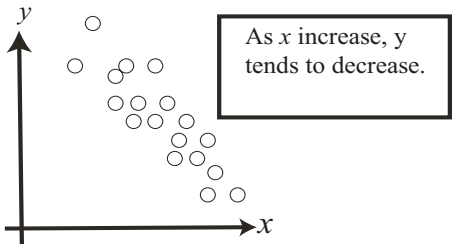
- What is correlation?
- What is the significance of studying correlation?
- What is the meaning to the values of r?
- What is q correlation?

Key Concepts(ASK-MT)	
Attitudes/Values	Express confidence in talking about correlations between quantities and explain the relationship between the variables.
Skills	Identify correlations between quantities and explain the relationship between the variables.
Knowledge	Correlation
Mathematical Thinking	Think about how to identify correlations between quantities and explain the relationship between the variables.

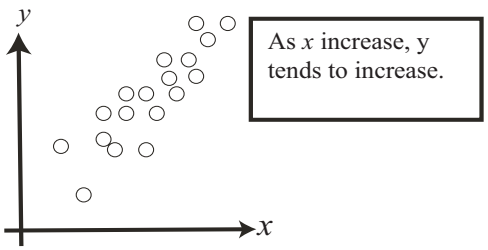
Content Background

1. Types of Correlation

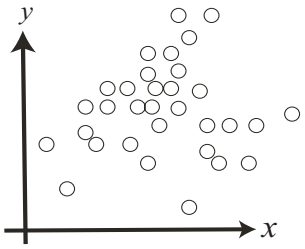
Once the intercept and slope have been estimated using least squares, various indices are studied to determine the reliability of these estimates. One of the most popular of these reliability indices is the correlation coefficient. The correlation coefficient, or simply the correlation, is an index that ranges from -1 to 1. When the value is near zero, there is no linear relationship. As the correlation gets closer to plus or minus one, the relationship is stronger. A value of one (or negative one) indicates a perfect linear relationship between two variables. Actually, the strict interpretation of the correlation is different from that given in the last paragraph. The correlation is a parameter of the bivariate normal distribution. This distribution is used to describe the association between two variables. This association does not include a cause and effect statement. That is, the variables are not labeled as dependent and independent. One does not depend on the other. Rather, they are considered as two random variables that seem to vary together. The important point is that in linear regression, y is assumed to be a random variable and x is assumed to be a fixed variable. In correlation analysis, both y and x are assumed to be random variables.



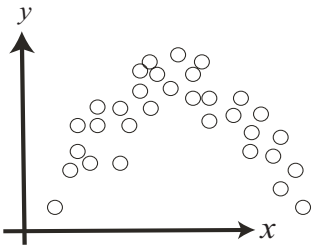
Negative Linear Correlation



Positive Linear Correlation



No Correlation



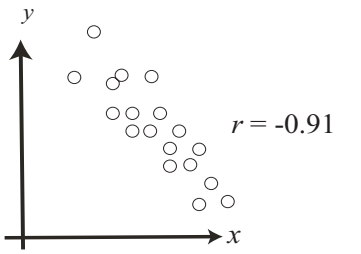
Nonlinear Correlation

Correlation Coefficient

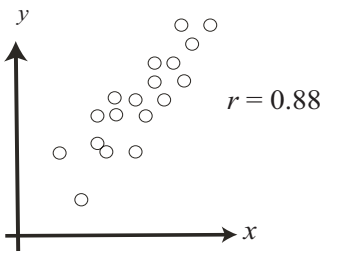
The correlation coefficient is a measure of the strength and the direction of a linear relationship between two variables. The symbol r represents the sample correlation coefficient. The formula for r is

$$r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

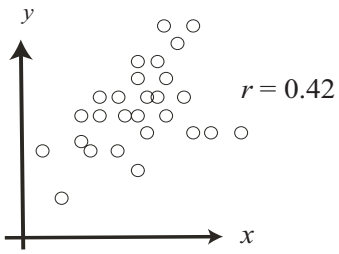
The range of the correlation coefficient r is –1 to 1. If x and y have a strong positive linear correlation, r is close to 1. If x and y have a strong negative linear correlation, r is close to –1. If there is no linear correlation or a weak linear correlation, r is close to 0. r is a measure of the linear (straight-line) association between two variables.



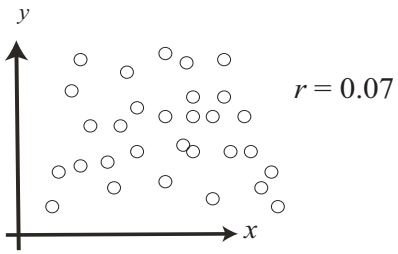
Strong negative correlation



Strong positive correlation



Weak positive correlation



Nonlinear Correlation

Example 1: Calculate the correlation coefficient r for the following data.

x	y	xy	x^2	y^2
1	-3	-3	1	9
2	-1	-2	4	1
3	0	0	9	0
4	1	4	16	1
5	2	10	25	4
$\sum x = 15$	$\sum y = -1$	$\sum xy = 9$	$\sum x^2 = 55$	$\sum y^2 = 15$

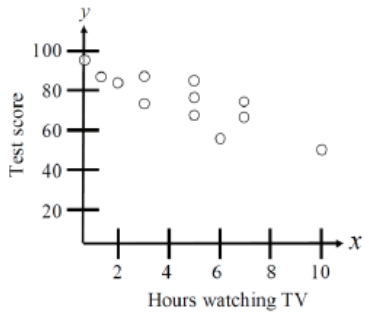
$$r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}} = \frac{5(9) - (15)(-1)}{\sqrt{5(55) - 15^2}\sqrt{5(15) - (-1)^2}}$$

$$= \frac{60}{\sqrt{50}\sqrt{74}} \approx 0.986$$

There is a strong positive linear correlation between x and y.

Example 2: The following data represents the number of hours 12 different students watched television during the weekend and the scores of each student who took a test the following Monday.
a) Display the scatter plot.
b) Calculate the correlation coefficient r.

Hours (x)	0	1	2	3	3	5	5	5	6	7	7	10
Test score (y)	96	85	82	74	95	68	76	84	58	65	75	50



Hours (x)	0	1	2	3	3	5	5	5	6	7	7	10
Test score (y)	96	85	82	74	95	68	76	84	58	65	75	50
xy	0	85	164	222	285	340	380	420	348	455	525	500
x^2	0	1	4	9	9	25	25	25	36	49	49	100
y^2	9216	7225	6724	5476	9025	4624	5776	7056	3364	4225	5625	2500

$\sum x = 54$, $\sum y = 908$, $\sum xy = 3724$, $\sum x^2 = 332$, $\sum y^2 = 70836$

$$r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}} = \frac{12(3724) - (54)(908)}{\sqrt{12(332) - 54^2}\sqrt{12(70836) - (908)^2}} \approx -0.831$$

There is a strong negative linear correlation. As the number of hours spent watching TV increases, the test scores tend to decrease.

Unit: Bivariate Statistics	Topic : Use of technology in problem solving
Benchmark	**12.4.4.3** Use appropriate technology to aid concept development as a tool for problem solving.

Learning Objective: By the end of the topic, students will be able to;

- use appropriate technologies to do basic statistical calculations such as mean, variance, standard deviation, correlation coefficient, covariance etc.

Essential questions:

- What statistical application or software is used in calculating the mean, variance, standard deviation, correlation coefficient, covariance etc either from the smartphone or laptop?
- How efficient and effective is the statistical application or software?

Key Concepts(ASK-MT)	
Attitudes/Values	Be appreciative to apply technology to calculate statistical problems.
Skills	Apply technology to calculate statistical problems.
Knowledge	Using technology to calculate the basic statistical calculations.
Mathematical Thinking	Think about how to use technology to evaluate satistical calculations.

Content Background

This topic is for students to make use of available technology to aid the concept development statistical calculation such as calculation of linear regression and correlation co-efficient of a set of bivariate numerical data. Calculators and MS Excel are examples of technologies which can be used to aid concept development.

Examples

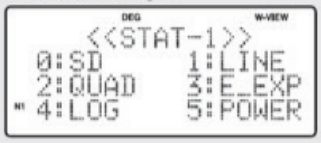
1.Using the Calculator

We can use a calculator to find equation of a line of linear regression. When a series of bivariate data has been entered correctly, then the calculator can be used to find the values of a and b, to give the equation of the line of regression from which we can make estimates of the variable y, if we know a value for the other variable x.

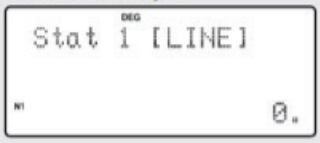
Set the calculator in Statistics mode.

MODE key

Numeric key 1



Numeric key 1



The calculator is now set for bivariate data entry; each pair p, q being input as p [(x, y) key] q [DATA key]. For example, if the bivariate pair 0, 2 were the first item of a set of data, they would appear on the calculator as:

For the following table, enter the bivariate data shown. The calculator screens are shown for the whole process

p	0	1	2	3	4
q	2	3	5	4	6

Having entered the data, the values of the **a** and **b** in the equation of the line of regression of y on x, $y = \mathbf{a} + \mathbf{b}\,x$, can be found using the calculator. The ALPHA key identifies **a** and **b** as extra functions on the bracket keys

ALPHA **a** =

ALPHA **b** =

The line of regression of y on x is therefore $y = 2.2 + 0.9\,x$

Estimates of the value of y can be found by substituting given values of x in the equation.

2. Using an Excel function

We can calculate correlation coefficient for the following data using an excel function.

x	1.0	2.3	3.1	4.8	5.6	6.3
y	2.6	2.8	3.1	4.7	5.1	5.3

The Correlation Coefficient is calculated according to the following formula:

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\left[n\sum x^2 - \left(\sum x\right)^2\right]\left[n\sum y^2 - \left(\sum y\right)^2\right]}}$$

It may appear that the above equations are quite complicated, however upon inspection; we see that their components are nothing more than simple algebraic manipulations of the raw data. We can expand our spread sheet to include these components and use inbuilt function to do our calculations.

A11 = =COUNT(B3:B8)

	A	B	C	D	E	F	G	H	I	J
1										
2		x	y	xy	x^2	y^2				
3		1.0	2.6	2.6	1.0	6.8				
4		2.3	2.8	6.44	5.3	7.8				
5		3.1	3.1	9.61	9.6	9.6				
6		4.8	4.7	22.56	23.0	22.09				
7		5.6	5.1	28.56	31.4	26.0				
8		6.3	5.3	33.39	39.7	28.1				
9										
10	n	Σ x	Σ y	Σ (xy)	Σ (x^2)	Σ (y^2)				
11	6	23.1	23.6	103.16	110.0	100.4				
12										
13		$(\Sigma x)^2$	$(\Sigma y)^2$							
14		533.61	556.96							
15										
16	slope, m =	0.5842		=(A11*D11-B11*C11)/(A11*E11-B14)						
17	y-int, b =	1.6842		=(C11-C16*B11)/A11						
18	r =	0.9741		=(A11*D11-B11*C11)/SQRT((A11*E11-B14)*(A11*F11-C14))						

Standards-Based Lesson Planning

What are Standards-Based Lessons?

In a Standards-Based Lesson, the most important or key distinction is that, a student is expected to meet a defined standard for proficiency. When planning a lesson, the teacher ensures that the content and the methods of teaching the content enable students to learn both the skills and the concepts defined in the standard for that grade level and to demonstrate evidence of their learning.

Planning lessons that are built on standards and creating aligned assessments that measure student progress towards standards is the first step teacher must take to help their students reach success. A lesson plan is a step-by-step guide that provides a structure for an essential learning.

When panning a standards-based lesson, teacher instructions are very crucial for your lessons. How teachers instruct the students is what really points out an innovative teacher to an ordinary teacher. Teacher must engage and prepare motivating instructional activities that will provide the students with opportunities to demonstrate the benchmarks. For instance, teacher should at least identify 3-5 teaching strategies in a lesson; teacher lectures, ask questions, put students into groups for discussion and role play what was discussed.

Why is Standards-Based Lesson Planning Important?

There are many important benefits of having a clear and organized set of lesson plans. Good planning allows for more effective teaching and learning. The lesson plan is a guide and map for organizing the materials and the teacher for the purpose of helping the students achieve the standards. Lesson plans also provide a record that allows good, reflective teachers to go back, analyse their own teaching (what went well, what didn't), and then improve on it in the future. Standards-based lesson planning is vital because the content standards and benchmarks must be comparable, rigorous, measurable and of course evidence based and be applicable in real life that we expect students to achieve. Therefore, teachers must plan effective lessons to teach students to meet these standards. As schools implement new standards, there will be much more evidence that teachers will use to support student learning to help them reach the highest levels of cognitive complexity. That is, students will be developing high-level cognitive skills.

Components of a Standards-Based Lesson Plan

An effective lesson plan has three basic components;

- aims and objectives of the course;
- teaching and learning activities;
- assessments to check student understanding of the topic.

Effective teaching demonstrates deep subject knowledge, including key concepts, current and relevant research, methodologies, tools and techniques, and meaningful applications.

Planning for under-achievers NORMA

Who are underachieving students?

Under achievers are students who fail or do not perform as expected. Underachievement may be caused by emotions (low self-esteem) and the environment (cultural influences, unsupportive family)

How can we help underachievement?

Underachievement varies between students. Not all students are in the same category of underachievement.

Given below a suggested strategies teachers may adopt to assist underachievers in the classroom.

• Examine the Problem Individually
It is important that underachieving students are addressed individually by focusing on the student's strengths.

• Create a Teacher-Parent Collaboration
Teachers and parents need to work together and pool their information and experience regarding the child. Teachers and parents begin by asking questions such as;

- In what areas has the child shown exceptional ability?
- What are the child's preferred learning styles?
- What insights do parents and teachers have about the child's strengths and problem areas?

• Help student to plan every activity in the classroom
• Help students set realistic expectations
• Encourage and promote the student's interests and passions.
• Help children set short and long-term academic goals
• Talk with them about possible goals.
• Ensure that all students are challenged (but not frustrated) by classroom activities
• Always reinforce students

Example of Standards-Based Lesson Planning

The following sample lesson can help teachers to plan effective lessons. Teachers are encouraged to study the layout of the different components of these lessons and follow this design in their preparation and teaching of each lesson. Planning a good lesson helps the teacher in maintaining a standard teaching pattern which should not deviate students learning of the concept from the topic.

Sample Lesson Plan

Strand 1: Number, Operation and Computation

Content Standard 4: Students will be able to represent numbers in various situations and forms, develop fluency in calculations through operations, use base ten as key for extending numbers and operations, and apply numbers in practical situations to develop number sense.

Unit: Financial Mathematics

Benchmark: 12.1.1.1 Calculate compound interest, inflation, appreciation and depreciation.

Topic: Interests and Inflations

Lesson Title: Depreciation

Objective: By the end of the lesson, students should be able to;

- understand the concept of depreciation;
- comprehend methods of calculating depreciation charges;
- calculate depreciation.

Materials: Handouts, Worksheets, Blackboard/Whiteboard, Chalk/Marker

ASK-MT	
Attitudes/Values	Appreciate calculating depreciations values of an item or quantity over time.
Skills	Define and calculate depreciation values of an item or quantity over time.
Knowledge	Depreciation.
Mathematical Thinking	Think of how to define and calculate depreciation values of an item or quantity.

Lesson Procedure

TEACHER ACTIVITIES TEACHER ACTIVITIES	STUDENT ACTIVITIES
INTRODUCTION	**5 minutes**
• Review the concept of Appreciation by asking short response questions. • An example of a Toyota car is used to help students grasp the concept of depreciation including the following terms: „ - Cost. „ - Estimated useful life. „ - Expected scrap value.	• Listen carefully and use their previous knowledge on appreciation to answer teachers questions • Listen and comprehend difference between appreciation and depreciation using the scenario provided by the teacher. • Think about the concept of depreciation using the terms; cost, estimated useful life and expected scape value.
BODY	**20 minutes**
Modeling	
• Ask Students to form groups of four or five to set up their own business. The business can be a: Restaurant, Game Centre, Tutorial Centre, Karaoke or Supermarket. • Invited students to provide verbal feedback. • There is no right or wrong answer. Teacher may verify and comment on practically and reasonableness of answers.	• Discuss the name of the company, nature of the business, the capital required, and most importantly, list 5 essential fixed assets they must acquire. They record discussion outcomes. • Provide feedback on their company and name 5 essential assets they must acquire.
Guided Practice	
• Ask students students how to estimate useful life, calculate residual value, depreciable amount and discuss the depreciation methods. • Provide an example using the Toyota car to estimate useful life, calculate residual value, depreciable amount and discuss the depreciation methods.	• Think about how to estimate useful life, calculate residual value, depreciable amount and discuss the depreciation methods. • Observe attentively on the example provided by teacher to estimate useful life, calculate residual value, depreciable amount and discuss the depreciation methods.
Independent Practice	
• Ask students to remain in the same group to determine the estimated useful life, residual value and depreciable amounts of their fixed assets; and identify the possible depreciation methods to apply.	• Discuss to determine the estimated useful life, residual value and depreciable amounts of their fixed assets; and identify the possible depreciation methods to apply.
CONCLUSION	**15 minutes**
• Allow students to present their solutions on the board for class discussions. • Make corrections were necessary and summarize suggested methods and explains that there are various methods of allocating depreciable amounts. • Teacher concludes lesson and highlights the key concepts.	• Present their solutions on the board and explain their answers • Make necessary corrections if any through consolidating the key points of the lesson highlighted by the teacher.

Assessment/ lesson Evaluation

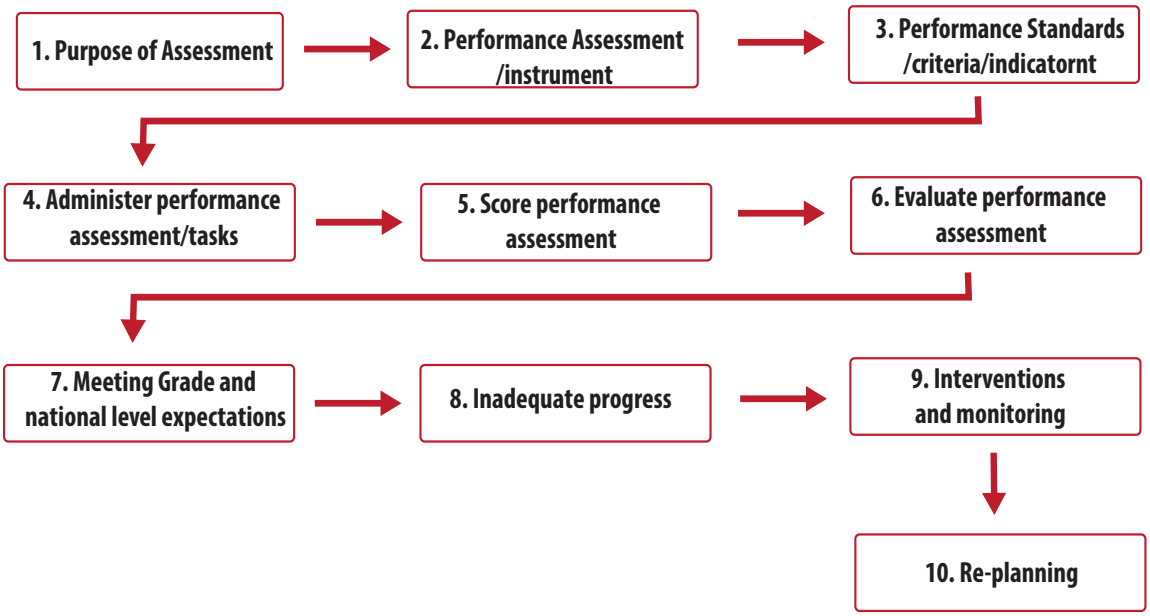
Students can:

- understand the concept of depreciation;
- comprehend methods of calculating depreciation charges;
- calculate depreciation.

Assessment, Monitoring and Reporting

What is Standards-Based Assessment (SBA)?

Standards-Based Assessment is an on-going and a systematic process of **assessing, evaluating, reporting** and **monitoring** students' performance and progression towards meeting grade and national level expectations. It is the measurement of students' proficiency on a learning objective or a specific component of a content standard and progression towards the attainment of a benchmark and content standard.



Purpose of Standards-Based Assessment

Standards-Based Assessment (SBA) serves different purposes. These include instruction and learning purposes. The primary purpose of SBA is to improve student learning so that all students can attain the expected level of proficiency or quality of learning.

Enabling purposes of SBA is to:

- measure students' proficiency on well-defined content standards, benchmarks and learning objectives
- ascertain students' attainment or progress towards the attainment of specific component of a content standard
- ascertain what each student knows and can do and what each student needs to learn to reach the expected level of proficiency
- enable teachers to make informed decisions and plans about how and what they would do to assist weak students to make adequate progress towards meeting the expected level of proficiency
- enable students to know what they can do and help them to develop and implement strategies to improve their learning and proficiency level
- communicate to parents, guardians, and relevant stakeholders the performance and progress towards the attainment of content standards or its components
- compare students' performances and the performances of other students

Principles of Standards-Based Assessment

The principle of SBA is for assessment to be;

- emphasing on tasks that should encourage deeper learning
- be an integral component of a course, unit or topic and not something to add on afterward
- a good assessment requires clarity of purpose, goals, standards and criteria
- of practices that should use a range of measures allowing students to demonstrate what they know and can do
- based on an understanding of how students learn
- of practices that promote deeper understanding of learning processes by developing students' capacity for self-assessment
- improving performance that involves feedback and reflection
- on-going rather than episodic
- given the required attention to outcomes and processes
- be closely aligned and linked to learning objectives, benchmarks and content standards

Standards-Based Assessment Types

In standards-Based Assessment, there are three broad assessments types.

1. Formative Assessment

Formative assessment includes 'assessment *for* and *as'* and is conducted during the teaching and learning of activities of a topic.

Purposes of assessment for Learning

- On-going assessment that allows teachers to monitor students on a day-to-day basis.
- Provide continuous feedback and evidence to the teachers that should enable them to identify gaps and issues with their teaching, and improve their classroom teaching practice.
- Helps students to continuously evaluate, reflect on, and improve their learning

Purposes of assessment as Learning

- Occurs when students reflect on and monitor their progress to inform their future learning goals
- Helps students to continuously evaluate, reflect, and improve their own learning
- Helps students to understand the purpose of their learning and clarify learning goals

2. Summative Assessment

Summative assessment focuses on 'assessment *of learning'* and is conducted after or at the conclusion of teaching and learning of activities or a topic.

Purposes of assessment of Learning

- Help teachers to determine what each student has achieved and how much progress he/she has made towards meeting national and grade-level expectations
- Help teachers to determine what each student has achieved at the end of a learning sequence or a unit.
- Enable teachers to ascertain each student's development against the unit or topic objectives and to set future directions for learning.
- Help students to evaluate, reflect on, and prepare for next stage of learning

3. Authentic Assessment

- Is performed in a real life context that approximates as much as possible, the use of a skill or concept in the real world.
- Is based on the development of a meaningful product, performance or process
- Students develop and demonstrate the application of their knowledge, skills, values and attitudes in real life situations which promote and support the development of deeper levels of understanding.

Authentic assessment refers to assessment that:

- Looks at students actively engaged in completing a task that represents the achievement of a learning objective or standard
- Takes place in real life situations
- Asks students to apply their knowledge, skills, values and attitudes in real life situations
- Students are given the criteria against which they are being assessed

Performance Assessment

Performance assessment is a form of testing that requires students to perform a task rather than select an answer from a ready-made list. For example, a student may be asked to explain historical events, generate scientific hypotheses, solve math problems, converse in a foreign language, or conduct research on an assigned topic. Teachers, then judge the quality of the student's work based on an agreed-upon set of criteria. It is an assessment which requires students to demonstrate that they have mastered specific skills and competencies by performing or producing something.

Types of performance assessment;

i. Products

This refers to concrete tangible items that students create through either the visual, written or auditory media such as;

- Creating a health/physical activity poster
- Video a class game or performance and write a broadcast commentary
- Write a speech to be given at a school council meeting advocating for increased time for health and physical education in the curriculum
- Write the skill cues for a series of skill photo's
- Create a brochure to be handed out to parents during education week
- Develop an interview for a favourite sportsperson
- Write a review of a dance performance
- Essays
- Projects

ii. Process Focused Tasks

It shows the thinking processes and learning strategies students use as they work such as;

- Survival scenarios
- Problem solving initiative/adventure/ activities
- Decision making such as scenario's related to health issues
- Event tasks such as creating a game, choreographing a dance/gymnastics routine, creating an obstacle course
- Game play analysis
- Peer assessment of skills or performances
- Self-assessment activities
- Goal setting, deciding a strategy and monitoring progress towards achievement

iii. Portfolio

This refers to a collection of student work and additional information gathered over a period of time that demonstrates learning progress.

iv. Performances

It deals with observable affective or psycho-motor behaviours put into action such as;

- Skills check during game play
- Role plays
- Officiating a game
- Debates
- Performing dance/gymnastics routines
- Teaching a skill/game/dance to peers

Performance Standards

Performance Standards are concrete statements of how well students must learn what is set out in the content standards, often called the "be able to do" of "what students should know and be able to do." Performance standards are the indicators of quality that specify how competent a students' demonstration or performance must be. They include explanations of how well students must demonstrate the content, explaining how good is good enough.

Performance standards:

- measure students' performance and proficiency (using performance indicators) in the use of a specific knowledge, skill, value, or attitude in real life or related situations
- provide the basis (performance indicators) for evaluating, reporting and monitoring students' level of proficiency in use of a specific knowledge, skills, value, or attitude
- are used to plan for individual instruction to help students not yet meeting expectations (desired level of mastery and proficiency) to make adequate progress towards the full attainment of benchmarks and content standards
- are used as the basis for measuring students' progress towards meeting grade-level benchmarks and content standards

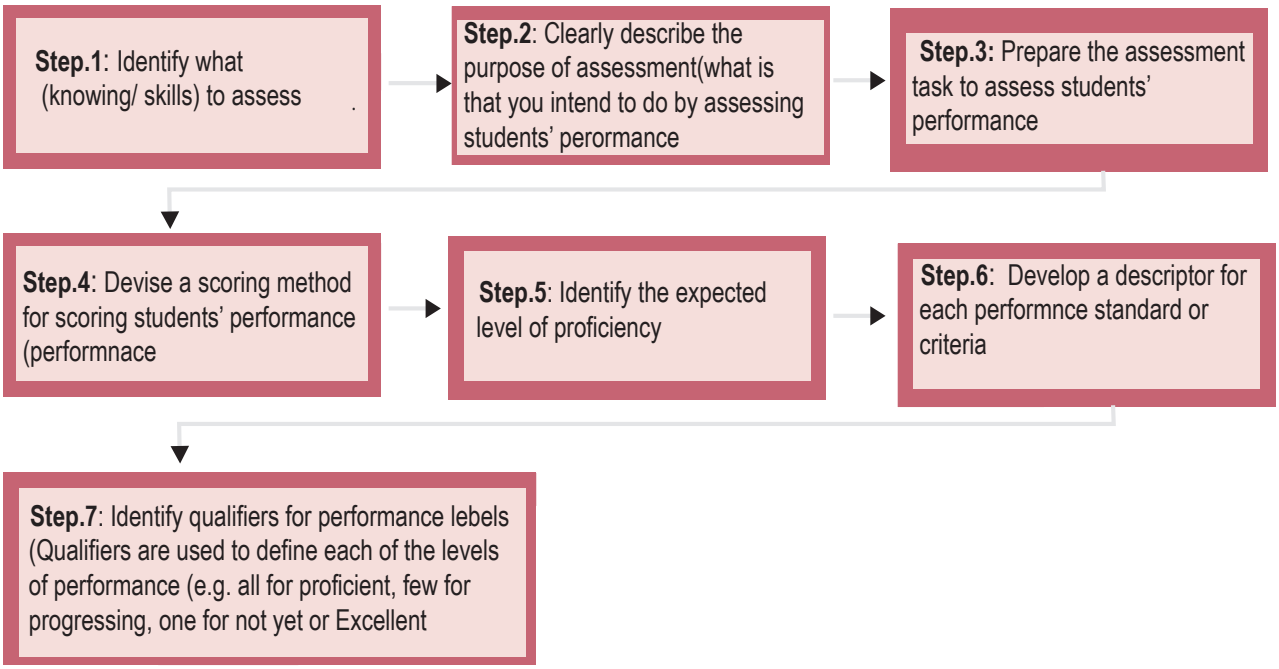
Assessment Strategies

It is important for teachers to know that, assessment is administered in different ways. Assessment does not mean a test only. There are many different ways to find out about student's strengths and weaknesses. Relying on only one method of assessing will not reflect student's achievement.

Provided in the appendices is a list of suggested strategies you can use to assess student's performances. These strategies are applicable in all the standards-based assessment types.

How to Develop effective SBA tools

Teachers are required to use the steps outlined below when planning assessment. These steps will guide you to develop effective assessments to improve student's learning as well as evaluating their progress towards meeting national and grade –level expectations.



Sample of Assessment Types

Formative - Sample 1

STRAND 1: Number, Operation and Computation

Content Standard 1 : Students will be able to represent numbers in various situations and forms, develop fluency in calculations through operations, use base ten as key for extending numbers and operations, and apply numbers in practical situations to develop number sense.

Unit: Financial Mathematics

Benchmark: 12.1.1.1 Calculate compound interest, inflation, appreciation and depreciation.

Topic: Interests and Inflations

Lesson Title: Depreciation

Objective: By the end of the lesson, students should be able to;

- understand the concept of depreciation;
- comprehend methods of calculating depreciation charges;
- calculate depreciation.

What is to be assessed? - (ASK-MT)

ASK-MT	
Attitudes/Values	Appreciate calculating depreciations values of an item or quantity over time.
Skills	Define and calculate depreciation values of an item or quantity over time.
Knowledge	Depreciation.
Mathematical Thinking	Think of how to define and calculate depreciation values of an item or quantity.

Purpose of the assessment
To measure students' proficiency on the achievement of the benchmark and learning objectives.

Expected level of proficiency
- understand the concept of depreciation;
- comprehend methods of calculating depreciation charges;
- calculate depreciation.

Performance Task
Students will be given an activity for them to work in group or individually to determine the estimated useful life, residual value and depreciable amounts of their fixed assets using different depreciation methods.

Assessment Strategy
This activity will be assessed in one lesson to determine how well students can determine the estimated useful life, residual value and depreciable amounts of their fixed assets using different depreciation methods.

Assessment Scoring
Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess the child's learning through the assessment tool for a lesson exercise.

Performance standards/ Criteria	Exemplar (20 points)	Proficient (13-19 points)	Developing (6-12 Points)	Beginning (2-5 points)	Score __/20 Marks
Problem Solving	No errors when solving problems	Few errors when solving problems	Numerous errors when solving problems	Little or no understanding of the problem is evidenced	
Math Content (Depreciation)	Demonstrate a clear knowledge of depreciation	Demonstrate a general knowledge of depreciation	Demonstrate a limited knowledge of depreciation	Demonstrate a little or no of depreciation	
Use of different Depreciation Methods	All depreciation methods are used correctly	Few depreciation method used correctly	Only 1 depreciation, method used.	Depreciation method used but not correctly	
Teacher Comments:					

Summative - Sample 2

STRAND 1 : Number, Operation and Computation

Content Standard 1: Students will be able to represent numbers in various situations and forms, develop fluency in calculations through operations, use base ten as key for extending numbers and operations, and apply numbers in practical situations to develop number sense.

Unit: Financial Mathematics

Benchmark: 12.1.1.1 – 12.1.1.4 (Refer to Benchmarks in Unit: Financial Mathematics of Strand 1)

Topics: (Refer to Topics in Unit: Financial Mathematics of Strand 1)

Lesson Titles: (Refer to lessons in Unit: Financial Mathematics of Strand 1)

Lesson Objectives: (Refer to lessons objectives in Unit: Financial Mathematics of Strand 1)

What to be assessed? Key Concepts (ASK-MT)

ASK-MT	
Attitudes/Values	Show confidence in applying mathematics knowledge and skills in solving financial problems.
Skills	Solve problems on financial mathematics
Knowledge	Financial Mathematics
Mathematical Thinking	Think of how the mathematical processes involved in solving financial problems.

Purpose of the assessment
To measure students' proficiency on the achievement of the benchmarks and learning objectives in this unit. (This assessment is to be conducted after teaching the unit)

Expected level of proficiency

All students are expected to:

- Calculate compound interest, inflation, appreciation and depreciation.
- Calculate personal loans, credit cards, hire-purchase, flat-rate interest loans, loan repayment schedules, services fees and charges.
- Calculate dividends, premium payments and return.
- Differentiate types of insurance, policies and premium payments and calculate returns using financial formulas.

Performance Task

Students will do an assignment out of 20 marks. You can use other assessment tools (Test, assignment, projects etc...) to assess students' proficiency on these benchmarks.

Task: Students are to:
- Do an assignment out of 20 marks on Financial Mathematics covering benchmarks 12.1.1.1 to 12.1.1.4.
- Show all calculation processes to attain full marks.

Assessment Strategies
An assignment will be used to measure students' proficiency.

Assessment Scoring
Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess child's learning as an assessment tool of an assignment.

Performance standards/ Criteria	Exemplar (20 points)	Proficient (13-19 points)	Developing (6-12 Points)	Beginning (2-5 points)	Score __/20 Marks
Effort	Maximum effort was put forth to complete the assignment.	Good effort was put forth to complete the assignment.	Minimum effort was made to complete the assignment.	Little or no effort to do the assignment.	
Problem Solving	No errors when solving problems.	Few errors when solving problems.	Numerous errors when solving problems.	Little or no understanding of the problem is evidenced.	
Math Content (Financial mathematics)	Demonstrate a clear knowledge of financial mathematics.	Demonstrate a general knowledge of financial mathematics.	Demonstrate a limited knowledge of financial mathematics.	Demonstrate a little or no of financial mathematics.	
Calculations Process.	All calculations are very clear, organized and neatly completed with no inaccuracies.	All calculations are very clear, organized and neatly completed with 1-6 inaccuracies.	Most calculations are very clear, organized and neatly completed with 6-8 inaccuracies.	All calculations are unclear and not organized.	
Teacher Comments:					

Authentic Assessment- Sample 3

Strand 1: Number, Operation and Computation

Content Standard 1: Students will be able to represent numbers in various situations and forms, develop fluency in calculations through operations, use base ten as key for extending numbers and operations, and apply numbers in practical situations to develop number sense.

Unit: Financial Mathematics

Benchmark: 12.1.1.1 – 12.1.1.4 (Refer to Benchmarks in Unit: Financial Mathematics of Strand 1)

Topics: (Refer to Topics in Unit: Financial Mathematics of Strand 1)

Lesson Titles: (Refer to lessons in Unit: Financial Mathematics of Strand 1)

Lesson Objectives: (Refer to lessons objectives in Unit: Financial Mathematics of Strand 1)

What to be assessed? Key Concepts (ASK-MT)

ASK-MT	
Attitudes/Values	Show confidence in applying mathematics knowledge and skills in solving authentic financial problems.
Skills	Solve authentic problems on financial mathematics.
Knowledge	Financial Mathematics.
Mathematical Thinking	Think of how the mathematical processes involved in solving authentic financial problems.

Purpose of the assessment

To measure students proficiency on the achievement of the benchmarks and learning objectives in this unit. This assessment is to be conducted after teaching this unit.

Expected level of proficiency

All students are expected to;

- Calculate compound interest, inflation, appreciation and depreciation.
- Calculate personal loans, credit cards, hire-purchase, flat-rate interest loans, loan repayment schedules, services fees and charges.
- Calculate dividends, premium payments and return.
- Differentiate types of insurance, policies and premium payments and calculate returns using financial formulas.

Performance Task

Students will do a project out of 20 marks. You can use other assessment tools (Test, assignment, projects etc...) to assess students' proficiency on these benchmarks.

Students will be given two (2) weeks to complete this project.

Task
Students in groups of 5 to 6 students visit of the following business organizations such as an Insurance company, a Financial Institution or a Furniture Store.

Task Details:

(i) Describe the type business organization they visited and its key business activity.

(ii) Provide an example of how their business is conducted. For example, students visiting Furniture Store should detail description of how Hire Purchase is facilitated for customers, and provide an example of a customer who wishes to buy furniture at the store. Students who visit Financial Institution should also provide an example of a customer who wishes to take a loan or deposited a certain amount of money. Similarly, students who visit Insurance Company should provide detail description of a customer who wishes insure themselves or their properties.

(iii) Show all calculation processes involved in task (ii) above.

(iv) Present your findings in class.

Assessment Strategies

A project will be used to measure students' proficiency.

Assessment Scoring

Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess child's learning as an assessment tool of the project.

Performance standards/ Criteria	Exemplar	Proficient	Developing	Beginning	Score
	(20 points)	(13-19 points)	(6-12 Points)	(2-5 points)	__/20 Marks
Effort	Maximum effort was put forth to complete the project.	Good effort was put forth to complete the project	Minimum effort was made to complete the project.	Little or no effort to do the project.	
Mathematics knowledge and Understanding and Calculation Process	The students consistently use appropriate financial formulas in their calculations with no inaccuracies.	The students generally use appropriate financial formulas in their calculations with 1-6 inaccuracies.	The students some sometimes uses appropriate financial formulas in their calculations with 6-8 inaccuracies.	The students attempts to apply methods, financial formulas in their calculations.	
Level of Difficulty	Topic explores beyond material covered in class	Topic has been covered in class and has been successfully extended.	Topic has been covered in class and their has been an attempt to extend the topic	Topic has been covered in class and not extended.	
Presentation	The presentation is easy to follow and is obvious that considerable effort has been made	The presentation is generally easy to follow and is obvious that some effort has been made	Problems with the presentation make it difficult to follow.	Little effort appears to have been put into the presentation	
Teacher Comments:					

STEAM Assessment

Sample 4: Integrated Strands in relation to the project from integrated subjects

Unit: (Integrated Units from all Subjects in this project)

Content Standard: (Integrated Content Standard from all Subjects in project)

Benchmark: (Integrated Benchmarks from all Subjects in this project)

Topic: (Integrated Topics from all Subjects in this project)

Lesson topic: (Integrated Topics from all Subjects in concern)

Instructional Objective (s): Students will be able to;

- Create a STEAM project "building a prototype model of a catapult launching system" to enhance their understand of this concept

ASK-MT	
Values/Attitudes	Appreciate the beauty of the application of mathematics during the designing process of the project.
Skills	Calculating size and space Time management and efficiency, Linear measurement and scaling techniques, Calculating mechanical advantage
Knowledge	Size and space Time management and efficiency, Linear measurement and scaling techniques
Mathematical Thinking	Think about how to integrate and apply the mathematical knowledge in the project

What is to be assessed? - (KSAVs)

Integrated subjects concepts used designing the projects.

Purpose of the assessment

To measure students proficiency on the achievement of the benchmarks and learning objectives for integrated subjects in the project. (STEAM Project)

Expected level of proficiency

All students are expected to;

- "Build a prototype model of a catapult launching system" through integrating concepts learned in other subjects.

Performance Task

Student will carry out a project worth 30 marks that should contribute to the School Learning Improvement Program (SLIP). This project will assess students proficiency on the mentioned benchmarks. In order for this assessment type to attain its intended purpose the following must be done carefully;

Task: Students will be given a month to complete this project.

(1) all grade 12 Mathematics teachers discuss the STEAM project with their HOD
(2) the Mathematics HOD brings this project to the attention of the Head Teacher hence it will involve the learning of all grade 12 classes in the school.
(3) once approved by the Head Teacher, the Mathematics HOD now convenes a meeting with all other subject HOD to integrate this project into their learning. HOD for Mathematics will have developed criteria already and will discuss around that.
4) the HOD for other subjects meet with their respective subject teachers to gauge their views and write up criteria's with reference to the theme of the project, "STEM Design and Engineering Challenge" bringing out the essence of their subjects in this project.
(5) the Head Teacher then convenes a meeting with all teachers as they are now aware of the project. HOD for respective subjects give feedback from their meetings. Issues concerning this project must be ironed out and all subjects now carry out this assessment, starting with Mathematics.

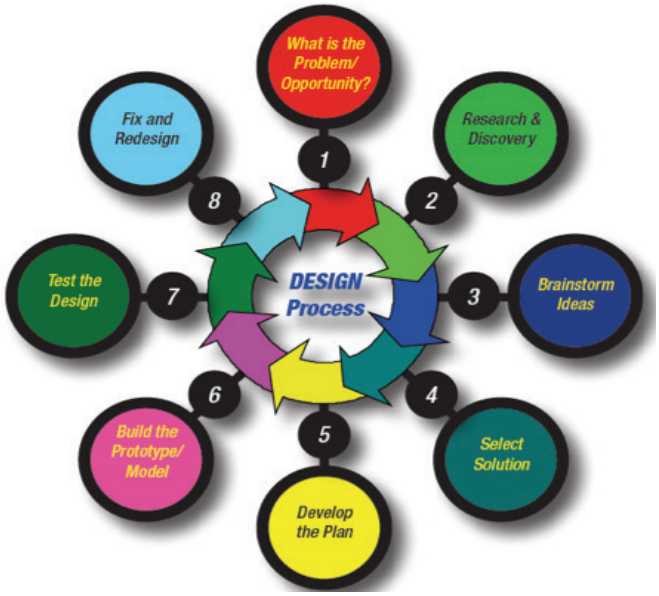
The grade 12 Mathematics teacher will now do the following;

(i) Group the students into groups of 6 to design (drawing and manual) a tangible technology that will enhance the notion of "building a prototype model of a catapult launching system"
(ii) The teacher then assesses their designs and the best designs now compete with the other best designs from other grade 12 classes.

(iii) All the best designers now create models of their designs with assistance from their class members. At this stage the other subjects now carry forward this assessed projects theme, 'building a prototype model of a catapult launching system" however in the context of their subjects. STEAM is an integrated approach of teaching. All subjects must incorporate the theme put forward by Mathematics. They develop criteria that should address this theme. For instance; Technology and Industrial Arts (TIA) will develop criteria that will engage the students to construct the models. Science teachers will develop criteria to test students' knowledge of the Science process of Engineering Design thinking when they create the models around the theme of "prototype model of a catapult launching system". The English subject teachers will set criteria and guidelines for students on how to write reports so they write to tell others what they have learned and experienced. They must also be given guidelines to writing report. Students get to write report of how they designed this technology. The Mathematics teacher will provide criteria for the students in terms of the measurements, angles and operations used to work out the size and shape of the technology.

Task: Students will be given 6 weeks to complete this project. They are to;

- Design and build a prototype model of a catapult launching system that is easy to use and easy to transport.
- Follow the Design Process to prepare their prototype model in time.
- Write and prepare a short presentation to explain the catapult that was built and the process of building it.



Design Specification:

The catapult should be designed to launch a golf ball at least fifteen feet, to a 18cm x 18cm target.

- The catapult should include a system for determining range, reliability, and accuracy.
- The catapult should be mobile, yet stable. Outriggers or other support systems need to be included to maintain stability when the launcher is used.
- The catapult should be no larger than 30cm long x 30 cm deep x 90cm tall.
- The catapult should feature a locking pin or trigger that activates the catapult to launch.
- Your team should prepare to deliver a presentation about the merits of your catapult model and design.

Assessment Strategy

Design Project will be used to measure student's proficiency.

The students will be reinforced in the following STEAM concepts.

Science

- Applications of simple machines, including wheels and axles, levers, and pulleys
- Balance and equilibrium
- Energy transformations, such as rotary motion to linear motion
- Mechanical advantage

Technology and Engineering

- Prototyping and modelling
- Invention and innovation
- Structural integrity/strength
- Brainstorming and problem solving
- Trial and error engineering concepts

ARTS

- Perspective drawing (3D)
- Critical Thinking Process
- Applying the Principles of Graphic design
  - Balance
  - proximity
  - Repetition
  - colour
  - negative/positive space
  - Applying creative process

Math

- Calculating size and space
- Time management and efficiency
- Linear measurement and scaling techniques
- Calculating mechanical advantage

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Appendices

APPENDIX 1: BLOOM'S TAXONOMY

LEVEL OF UNDERSTANDING	KEY VERBS
CREATING Can the student create a new product or point of view?	Construct, design, and develop, generate, hypothesize, invent, plan, produce, compose, create, make, perform, plan, produce, assemble, formulate,
EVALUATING Can the student justify a stand or decision?	Appraise, argue, assess, choose, conclude, critique, decide, defend, evaluate, judge, justify, predict, prioritize, provoke, rank, rate, select, support, monitor,
ANALYSING Can the student distinguish between the different parts?	Analysing, characterize, classify, compare, contrast, debate, criticise, deconstruct, deduce, differentiate, discriminate, distinguish, examine, organize, outline, relate, research, separate, experiment, question, test,
APPLYING Can the student use the information in a new way	Apply, change, choose, compute, dramatize, implement, interview, prepare, produce, role play, select, show, transfer, use, demonstrate, illustrate, interpret, operate, sketch, solve, write,
UNDERSTANDING Can the student comprehend ideas or concepts?	Classify, compare, exemplify, conclude, demonstrate, discuss, explain, identify, illustrate, interpret, paraphrase, predict, report, translate, describe, classify,
REMEMBERING Can the student recall or remember the information?	Define, describe, draw, find, identify, label, list, match, name, quote, recall, recite, tell, write, duplicate, memorise, recall, repeat, reproduce, state,

APPENDIX 2: 21ST CENTURY SKILLS

WAYS OF THINKING	**Creativity and innovation** • Think creatively • Work creatively with others • Implement innovations **Critical thinking, problem solving and decision making** • Reason effectively and evaluate evidence • Solve problems • Articulate findings **Learning to learn and meta-cognition** • Self-motivation • Positive appreciation of learning • Adaptability and flexibility
WAYS OF WORKING	**Communication** • Competency in written and oral language • Open minded and preparedness to listen • Sensitivity to cultural differences **Collaboration and teamwork** • Interact effectively with others • Work effectively in diverse teams • Prioritise, plan and manage projects
TOOLS FOR WORKING	**Information literacy** • Access and evaluate information • Use and manage information • Apply technology effectively **ICT literacy** • Open to new ideas, information, tools and ways of thinking • Use ICT accurately, creatively, ethically and legally • Be aware of cultural and social differences • Apply technology appropriately and effectively
LIVING IN THE WORLD	**Citizenship – global and local** • Awareness and understanding of rights and responsibilities as a global citizen • Preparedness to participate in community activities • Respect the values and privacy of others **Personal and social responsibility** • Communicate constructively in different social situations • Understand different viewpoints and perspectives **Life and career** • Adapt to change • Manage goals and time • Be a self-directed learner • Interact effectively with others

APPENDIX 3: TEACHING AND LEARNING STRATEGIES

STRATEGY	TEACHER	STUDENTS
CASE STUDY **Used to extend students' understanding of real life issues**	Provide students with case studies related to the topic of the lesson and allow them to analyse and evaluate.	Study the case study and identify the problem addressed. They analyse the problem and suggest solutions supported by conceptual justifications and make presentations. This enriches the students' existing knowledge of the topic.
DEBATE **A method used to increase students' interest, involvement and participation**	Provide the topic or question of debate on current issues affecting a bigger population, clearly outlining the expectations of the debate. Explain the steps involved in debating and set a criteria/standard to be achieved.	Conduct researches to gather supporting evidence about the selected topic and summarising the points. They are engaged in collaborative learning by delegating and sharing tasks to group members. When debating, they improve their communication skills.
DISCUSSION **The purpose of discussion is to educate students about the process of group thinking and collective decision.**	The teacher opens a discussion on certain topic by asking essential questions. During the discussion, the teacher reinforces and emphasises on important points from students responses. Teacher guide the direction to motivate students to explore the topic in greater depth and the topic in more detail. Use how and why follow-up questions to guide the discussion toward the objective of helping students understand the subject and summarise main ideas.	Students ponder over the question and answer by providing ideas, experiences and examples. Students participate in the discussion by exchanging ideas with others.
GAMES AND SIMULATIONS **Encourages motivation and creates a spirit of competition and challenge to enhance learning**	Being creative and select appropriate games for the topic of the lesson. Give clear instructions and guidelines. The game selected must be fun and build a competitive spirit to score more than their peers to win small prices.	Go into groups and organize. Follow the instructions and play to win
OBSERVATION **Method used to allow students to work independently to discover why and how things happen as the way they are. It builds curiosity.**	Give instructions and monitor every activity students do	Students possess instinct of curiosity and are curious to see the things for themselves and particularly those things which exist around them. A thing observed and a fact discovered by the child for himself becomes a part of mental life of the child. It is certainly more valuable to him than the same fact or facts learnt from the teacher or a book. Students Observe and ask essential questions Record Interpret

PEER TEACHING & LEARNING *(power point presentations, pair learning)* Students teach each other using different ways to learn from each other. It encourages; team work, develops confidence, feel free to ask questions, improves communication skills and most importantly develop the spirit of inquiry.	Distribute topics to groups to research and teach others in the classroom. Go through the basics of how to present their peer teaching.	Go into their established working groups. Develop a plan for the topic. Each group member is allocated a task to work on. Research and collect information about the topic allocated to the group. Outline the important points from the research and present their findings in class.
PERFORMANCE-RELATED TASKS (dramatization, song/ lyrics, wall magazines) Encourages creativity and take on the overarching ideas of the topic and are able to recall them at a later date	Students are given the opportunity to perform the using the main ideas of a topic. Provide the guidelines, expectations and the set criteria	Go into their established working groups. Being creative and create dramas, songs/ lyrics or wall magazines in line with the topic.
PROJECT (individual/group) Helps students complete tasks individually or collectively	Teacher outline the steps and procedures of how to do and the criteria	Students are involved in investigations and finding solutions to problems to real life experiences. They carry out researches to analyse the causes and effects of problems to provide achievable solutions. Students carefully utilise the problem-solving approach to complete projects.
USE MEDIA & TECHNOLOGY to teach and generate engagement depending on the age of the students	Show a full movie, an animated one, a few episodes form documentaries, you tube movies and others depending on the lesson. Provide questions for students to answer before viewing	Viewing can provoke questions, debates, critical thinking, emotion and reaction. After viewing, students engage in critical thinking and debate

APPENDIX 4 : ASSESSMENT STRATEGIES

STRATEGY	DESCRIPTION
ANALOGIES	Students create an analogy between something they are familiar with and the new information they have learned. When asking students to explain the analogy, it will show the depth of their understanding of a topic.
CLASSROOM PRESENTATIONS	A classroom presentation is an assessment strategy that requires students to verbalize their knowledge, select and present samples of finished work, and organize their thoughts about a topic in order to present a summary of their learning. It may provide the basis for assessment upon completion of a student's project or essay.
CONFERENCES	A conference is a formal or informal meeting between the teacher and a student for the purpose of exchanging information or sharing ideas. A conference might be held to explore the student's thinking and suggest next steps; assess the student's level of understanding of a particular concept or procedure; and review, clarify, and extend what the student has already completed
DISCUSSIONS	Having a class discussion on a unit of study provides teachers with valuable information about what the students know about the subject. Focus the discussions on higher level thinking skills and allow students to reflect their learning before the discussion commences.
ESSAYS	An essay is a writing sample in which a student constructs a response to a question, topic, or brief statement, and supplies supporting details or arguments. The essay allows the teacher to assess the student's understanding and/or ability to analyse and synthesize information.
EXHIBITIONS/ DEMONSTRATIONS	An exhibition/demonstration is a performance in a public setting, during which a student explains and applies a process, procedure, etc., in concrete ways to show individual achievement of specific skills and knowledge.
INTERVIEWS	An interview is a face-to-face conversation in which teacher and student use inquiry to share their knowledge and understanding of a topic or problem, and can be used by the teacher to explore the student's thinking; assess the student's level of understanding of a concept or procedure and gather information, obtain clarification, determine positions, and probe for motivations.
LEARNING LOGS	A learning log is an ongoing, visible record kept by a student and recording what he or she is doing or thinking while working on a particular task or assignment. It can be used to assess student progress and growth over time.
OBSERVATION	Observation is a process of systematically viewing and recording students while they work, for the purpose of making programming and instruction decisions. Observation can take place at any time and in any setting. It provides information on students' strengths and weaknesses, learning styles, interests, and attitudes.
PEER ASSESSMENT	Assessment by peers is a powerful way to gather information about students and their understanding. Students can use set criteria to assess the work of their classmates.
PERFORMANCE TASKS	During a performance task, students create, produce, perform, or present works on "real world" issues. The performance task may be used to assess a skill or proficiency, and provides useful information on the process as well as the product.
PORTFOLIOS	A portfolio is a collection of samples of a student's work, and is focused, selective, reflective, and collaborative. It offers a visual demonstration of a student's achievement, capabilities, strengths, weaknesses, knowledge, and specific skills, over time and in a variety of contexts.

QUESTIONS AND ANSWERS (ORAL)	In the question–and-answer strategy, the teacher poses a question and the student answers verbally, rather than in writing. This strategy helps the teacher to determine whether students understand what is being, or has been, presented, and helps students to extend their thinking, generate ideas, or solve problems.
QUIZZES, TESTS, EXAM-INATIONS	A quiz, test, or examination requires students to respond to prompts in order to demonstrate their knowledge (orally or in writing) or their skills (e.g., through performance). Quizzes are usually short; examinations are usually longer. Quizzes, tests, or examinations can be adapted for exceptional students and for re-teaching and retesting.
QUESTIONNAIRES	Questionnaires can be used for a variety of purposes. When used as a formative assessment strategy, they provide teachers with information on student learning that they can use to plan further instruction.
RESPONSE JOURNALS	A response journal is a student's personal record containing written, reflective responses to material he or she is reading, viewing, listening to, or discussing. The response journal can be used as an assessment tool in all subject areas.
SELECTED RESPONSES	Strictly speaking a part of quizzes, tests, and examinations, selected responses require students to identify the one correct answer. The strategy can take the form of multiple-choice or true/false formats. Selected response is a commonly used formal procedure for gathering objective evidence about student learning, specifically in memory, recall, and comprehension.
STUDENT SELF-ASSESSMENTS	Self-assessment is a process by which the student gathers information about, and reflects on, his or her own learning. It is the student's own assessment of personal progress in terms of knowledge, skills, processes, or attitudes. Self-assessment leads students to a greater awareness and understanding of themselves as learners.

APPENDIX 5: Standard-based Lesson Plan Template

Strand: .....

Unit: .....

Content Standard: .....

Benchmark: .....

Topic : .....

Lesson Title: .....

Lesson Objective (s): By the end of the lesson, students will be able to;

- .....
- .....

Materials:

Key Concepts(ASK-MT)	
Attitudes / Values	
Skills	
Knowledge	
Mathematics Thinking	

Lesson Procedure

Teacher Activity	Student Activity
Introduction  (time in minutes)	
Body  (time in minutes)	
Modeling	
Guided Practice	
Independent Practice	
Conclusion  (time in minutes)	

Assessment/Lesson Evaluation

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APPENDIX 6: Standard based Lesson Plan template - Integrating STEAM

Strand: .....

Unit: .....

Content Standard: .....

Benchmark: .....

Topic : .....

Lesson Title: .....

Lesson Objective (s): By the end of the lesson, students will be able to;

- .....

Essential Questions

- .....

Materials:

Key Concepts(ASK-MT)	
Attitudes / Values	
Skills	
Knowledge	
Mathematics Thinking	
STEAM Knowledge and Skills	
Skills	
Knowledge	
STEAM Performance Indicator:	

Lesson Procedure

Teacher Activity	Student Activity
Introduction	(time in minutes)
Body	(time in minutes)
Modeling	
Guided Practice	
Independent Practice	
Conclusion	(time in minutes)

Assessment/Lesson Evaluation

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APPENDIX 7: Time Allocation

Grades 9 and 10	No. lesson/ wk	Min/week	Grades 11 and 12	No. lessons/ wk	Min/week
English	6	6 x 40 = 240	Applied English	6	6 x 40 = 240
Mathematics	5	5 x 40 = 200	L & L	6	6 x 40 = 240
Science	5	5 x 40 = 200	Advanced Math	8	5 x 80 = 400
Social Science	5	5 x 40 = 200	General Math	8	8 x 40 = 320
PD	5	5 x 40 = 200	Physics	6	6 x 40 = 240
Business Studies	5	5 x 40 = 200	Biology	6	6 x 40 = 240
Design & Technology	5	5 x 40 = 200	Chemistry	6	6 x 40 = 240
Arts	5	5 x 40 = 200	Applied Science	6	6 x 40 = 240
CCVE	3	3 x 40 = 120	Geology	6	6 x 40 = 240
RI	1	1 x 60 = 60	Geography	6	6 x 40 = 240
Agriculture	5	5 x 40 = 200	History	6	6 x 40 = 240
TOTALS	50	2020min/wk	Legal Studies	6	6 x 40 = 240
			HPE	6	6 x 40 = 240
			PE	6	6 x 40 = 240
			RE	1	1 x 60 = 60
			Business Studies	6	6 x 40 = 240
			Accounting	6	6 x 40 = 240
			Economics	6	6 x 40 = 240
			Design & Tech	6	6 x 40 = 240
			Computer Studies	6	6 x 40 = 240
			ICT	6	6 x 40 = 240
			CCVE	2	3 x 40 = 120
			ANRM	6	6 x 40 = 240
			TOTALS	128 lessons/ wk	5,460 min/wk