Mathematics

Junior High

Grade 10

Teacher Guide

Standards-Based

Department of Education

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Contents

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Acronyms

AAL	Assessment As Learning
AFL	Assessment For Learning
AOL	Assessment Of Learning
BoS	Board of Studies
CDD	Curriculum Development Division
CP	Curriculum Panel
CRS	Classroom Response System
DA	Diagnostic Assessment
HOD	Head Of Department
IHD	Integral Human Development
MTDG	Medium Term Development Goals
NGO	Non-Government Organisations
PBA	Performance Based Assessments
PNG	Papua New Guinea
SAC	Subject Advisory Committee
SBC	Standards-Based Curriculum
SBE	Standards-Based Education
SCG	Subject Curriculum Group
SRS	Student Response System
STEAM	Science, Technology, Engineering, Arts and Mathematics
STEM	Science, Technology, Engineering, and Mathematics

Secretary's Message

The aims and goals of SBC is to identify the important knowledge, skills, values and attitudes that all students are expected to acquire and master in order to effectively function in society and actively function in society and actively contribute to its development, student's welfare and enable them to acquire and apply 21st Century.

The aims of teaching and learning mathematics is to encourage and enable students to recognise that mathematics permeates the world around us. Students should be encouraged to appreciate the usefulness, power and beauty of mathematics and become confident in using mathematics to analyse and solve problems both in school and in real-life situations.

A variety of teaching and learning activities provides students with ideas to motivate students to learn, and make learning relevant, interesting and enjoyable. Teachers should provide students opportunity to develop mathematical curiosity and use inductive and deductive reasoning when solving problems and develop the knowledge, skills and attitudes necessary to pursue further studies in Mathematics.

Learning Mathematics enable students to develop abstract, logical and critical thinking and the ability to reflect critically upon their work and the work of others, develop a critical appreciation of the use of information and communication technology in mathematics appreciate the international dimension of mathematics and its multicultural and historical perspectives.

Teachers are encouraged to integrate Mathematics activities with other subjects, where appropriate, so that students can see the interrelationship between subjects and that the course they are studying provides a holistic education and a pathway for the future.

I commend and approve this Grade 10 Mathematics Teacher Guide to be used in all High Schools throughout Papua New Guinea.

……...

UKE W. KOMBRA, (PhD)
Secretary for Education

Introduction

The aims of teaching and learning mathematics are to encourage and enable students to recognize that mathematics permeates the world around us. Students should be encouraged to appreciate the usefulness, power and beauty of mathematics and become confident in using mathematics to analyse and solve problems both in school and in real-life situations.

The curriculum is designed to ensure that students build a solid foundation in mathematics by connecting and applying mathematical concepts in a variety of ways and situations. To support this process, teachers should provide students opportunity to develop mathematical curiosity and use inductive and deductive reasoning when solving problems and develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics.

Mathematics aims to provide a meaningful pedagogical framework for teaching and learning essential and in demand knowledge, skills, values, and attitudes that are required for the preparation of students for careers, higher education and citizenship in the 21st Century.

Students should be prepared to gather and understand information, analyse issues critically, learn independently or collaboratively, organize and communicate information, draw and justify conclusions, create new knowledge, and act ethically.

Students' employability will be enhanced through the study and application of STEAM principles. STEAM is an integral component of the core curriculum. All students are expected to study STEAM and use STEAM related skills to solve problems relating to both the natural and the physical environments. The aim of STEAM education is to create a STEAM literate society. It is envisioned that the study of STEAM will motivate students to pursue and take up academic programs and careers in STEAM related fields. STEAM has been embedded in the Mathematics curriculum. Equal opportunities should be provided for all students to learn, apply and master STEAM principles and skills.

Time allocation for Mathematics is **200** minutes for Grade 10.

Structure of the Teacher Guide

There are four main components to this teacher guide. They provide essential information on what all teachers should know and do to effectively implement the Mathematics curriculum.

Part 1 provides generic information to help the teachers to effectively use the teacher guide and the syllabus to plan, teach and assess students' performance and proficiency on the national content standards and grade-level benchmarks. The purpose of the teacher guide, syllabus and teacher guide alignment, and the four pillars of PNG SBC, which are, morals and values education, cognitive and high level thinking, and 21st Century thinking skills, STEAM, and core curriculum. These are explained to inform as well as guide the teachers so that they align SBE/SBC aims and goals, overarching and SBC principles, content standards, grade-level benchmarks, learning objectives and best practice when planning lessons, teaching, and assessing students.

Part 2 provides information on the strands, units, topics and learning objectives. How topics and learning objectives are derived is explained to the teachers to guide them to use the learning objectives provided for planning, instruction and assessment. Teachers are encouraged to develop additional topics and learning objectives to meet the learning needs of their students and communities where necessary.

Part 3 provides information on SBC planning to help guide the teachers when planning SBC lessons. Elements and standards of SBC lesson plans are described as well as how to plan for underachievers, use evidence to plan lessons, and use differentiated instruction, amongst other teaching and learning strategies.

Part 4 provides information on standards-based assessment, inclusive of performance assessment and standards, standards-based evaluation, standards-based reporting, and standards-based monitoring. This information should help the teachers to effectively assess, evaluate, report and monitor demonstration of significant aspects of a benchmark.

The above components are linked and closely aligned. They should be connected to ensure that the intended learning outcomes and the expected quality of education standards are achieved. The close alignment of planning, instruction and assessment is to the attainment of learning standards.

Purpose of the Teacher Guide

This teacher guide describes what all teachers should know and do to effectively plan, teach, and assess the Grade 10 Mathematics content to attain the required learning and proficiency standards. The overarching purpose of this teacher guide is to help teachers to effectively plan, teach, assess, evaluate, report and monitor students' learning and mastery of national and grade-level expectations.
That is, the essential knowledge, skills, values and attitudes (KSVAs) described in the content standards and grade-level benchmarks, and their achievement of the national and grade-level proficiency standards.
Ample information with thorough guidelines is provided for the teacher.

Thus, the teacher is expected to;

- understand the significance of aligning all the elements of Standards-Based Curriculum (SBC) as the basis for achieving the expected level of education quality,
- effectively align all the components of SBC when planning, teaching, and assessing students' learning and levels of proficiency,
- effectively translate and align the Biology syllabi and teacher guide to plan, teach and assess different Biology units and topics, and the KSVAs described in the grade-level benchmarks,
- understand the Biology national content standards, grade-level benchmarks, and evidence outcomes,
- effectively make sense of the content (KSVAs) described in the Biology national content standards and the essential components of the content described in the grade-level benchmarks,
- effectively guide students to progressively learn and demonstrate proficiency on a range of Scientific skills, processes, concepts, ideas, principles, practices, values and attitudes,
- confidently interpret, translate and use Biology content standards and benchmarks to determine the learning objectives and performance standards, and plan appropriately to enable all students to achieve these standards,
- embed the core curriculum in their Biology lesson planning, instruction, and assessment to permit all students to learn and master the core KSVAs required of all students,
- provide opportunities for all students to understand how STEAM has and continues to shape the social, political, economic, cultural, and environment contexts and the consequences, and use STEAM principles, skills, processes, ideas and concepts to inquire into and solve problems relating to both the natural and physical (man-made) worlds as well as problems created by STEAM,
- integrate cognitive skills (critical, creative, reasoning, decision-making, and problem- solving skills), high level thinking skills (analysis, synthesis and evaluation skills), values (personal, social, work, health, peace, relationship, sustaining values), and attitudes in lesson planning, instruction and assessment,

- meaningfully connect what students learn in Biology with what is learnt in other subjects to add value and enhance students' learning so that they can integrate what they learn and develop in-depth vertical and horizontal understanding of subject content,

- formulate effective SBC lesson plans using learning objectives identified for each of the topics,

- employ SBC assessment approaches to develop performance assessments to assess students' proficiency on a content standard or a component of the content standard described in the grade-level benchmark,

- effectively score and evaluate students' performance in relation to a core set of learning standards or criteria, and make sense of the data to ascertain students' expected proficiency status of progress towards me standards, and use evidence from the assessment of students' performance to develop effective evidence-based intervention strategies to help students' making inadequate or slow progress towards meeting the grade-level and national expectations to improve their learning and performance.

How to use the Teacher Guide

Teacher Guide provides essential information about what the teacher needs to know and do to effectively plan, teach and assess students learning and proficiency on learning and performance standards. The different components of the teacher guide are closely aligned with SBC principles and practice, and all the other components of PNG SBC. It should be read in conjunction with the syllabus in order to understand what is expected of teachers and students to achieve the envisaged quality of education outcomes.

The first thing teachers should do is to read and understand each of the sections of the teacher guide to help them understand the key SBC concepts and ideas, alignment of PNG SBC components, alignment of the syllabus and teacher guide, setting of content standards and grade-level benchmarks, core curriculum, STEAM, curriculum integration, essential knowledge, skills, values and attitudes, strands, units and topics, learning objectives, SBC lesson planning, and SBC assessment. A thorough understanding of these components will help teachers meet the teacher expectations for implementing the SBC curriculum, and therefore the effective implementation of the Grade 10 Mathematics Curriculum. Based on this understanding, teachers should be able to effectively use the teacher guide to do the following

Identifying topics from benchmarks

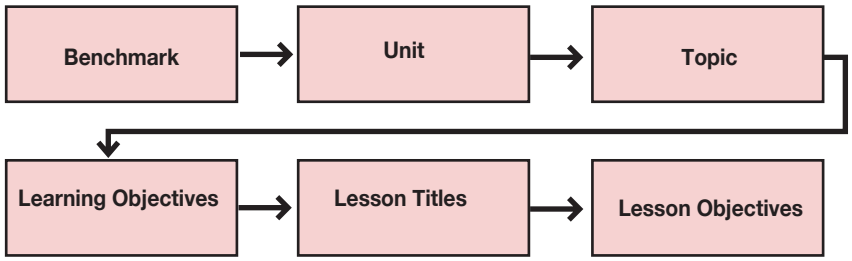
In order to identify the topic from the benchmark, the benchmark needs to be unpack. When unpacking a benchmark, identify what students will be able to know and do in order to mastered the benchmark.

Below is a description of how topics and learning objectives were derived from the grade-level benchmarks.

1. Write out the benchmark that you want to unpack.
2. Write the verbs (skills/actions) – Higher order thinking skills.
3. Underline or highlight the big idea (content) in the benchmark. The big idea (content) is the topic derived from the benchmark.
4. Write essential questions that would be engaging for students.
5. Develop sub-topics from the big idea (topic).
6. Write learning objectives according to the sub-topics.
7. Write lesson topics from the learning objectives.

Determine Lesson Objectives and Lesson Titles

Topics and learning objectives have been identified and described in the Teacher Guide. Learning objectives are derived from topics that are extracted from the grade-level benchmarks. Lesson titles are deduced from the learning objectives. Teachers should familiarise themselves with this process as it is essential for lesson planning, instruction and assessment. However, depending on the context and students' learning abilities, teachers would be required to determine additional lesson objectives and lesson titles. Teachers should use the examples provided in this teacher guide to formulate additional lesson objectives and lesson titles to meet the educational or learning needs of their students.



Identify and Teach Grade Appropriate Content

Grade appropriate content has been identified and scoped and sequenced using appropriate content organisation principles. The content is sequenced using the spiralling sequence principles. This sequencing of content will enable students to progressively learn the essential knowledge, skills, values and attitudes as they progress further into their schooling. What students learn in previous grades is reinforced and deepens in scope with an increase in the level of complexity and difficulty in the content and learning activities. It is important to understand how the content is organised so that grade appropriate content and learning activities can be selected, if not already embedded in the benchmarks and learning objectives, to not only help students learn and master the content, but ensure that what is taught is rigorous, challenging, and comparable.

Integrate the Core Curriculum in Lesson Planning, Instruction and Assessment

Teachers should use this teacher guide to help them integrate the core curriculum – values, cognitive and high level skills, 21st Century skills, STEAM principles and skills, and reading, writing, and communication skills in their lesson planning, instruction and assessment. All students in all subjects are required to learn and master these skills progressively through the education system.

Integrate Cognitive, High Level, and 21st Century Skills in Lesson Planning, Instruction and Assessment

Teachers should integrate the cognitive, high level and 21st Century skills in their annual teaching programs, and give prominence to these skills in their lesson preparation,teaching and learning activities, performance assessment, and performance standards for measuring students' proficiency on these skills. Mathematics addresses the skills and processes of solving problems arising in everyday life, society and the workplace. Thus,students will be able to make informed decisions, problem-solving and management knowledge, skills, values and attitudes in Mathematics. This enables them to function effectively in the work and higher education environments as productive and useful citizens of a culturally diverse and democratic society in an interdependent world.

In addition, it envisaged all students attaining expected proficiency levels in these skills and will be ready to pursue careers and higher education academic programs that demand these skills, and use them in their everyday life after they leave school at the end of Grade 12. Teachers should use the teacher guide to help them to effectively embed these skills, particularly in their lesson planning and in the teaching and learning activities as well as in the assessment of students' application of the skills.

Integrate Mathematics Values and Attitudes in Lesson Planning, Instruction and Assessment

In Mathematics, students are expected to learn, promote and use work,relationship, peace, health, social, personal, family, community, national and global values in the work and study environments as well as in their conduct as community, national and global citizens. Teachers should draw from the information and suggestions provided in the syllabus and teacher guide to integrate values and attitudes in their lesson planning, instruction, and assessment. They should report on students' progression towards internalizing different values and attitudes and provide additional support to students who are yet to reach the internalization stage to make positive progress towards this level.

Integrate Science, Technology, Engineering, Arts and Mathematics (STEAM)

Principles and Skills in Lesson Planning, Instruction and Assessment Teachers should draw from both the syllabus and teacher guide in order to help them integrate STEAM principles and skills, and methodologies in their lesson planning, instruction and assessment. STEAM teaching and learning happens both inside and outside of the classroom. Effective STEAM teaching and learning requires both the teacher and the student to participate as core investigators and learners, and to work in partnership and collaboration with relevant stakeholders to achieve maximum results.
Teachers should use the syllabus, teacher guides and other resources to guide them to plan and implement this and other innovative and creative approaches to STEAM teaching and learning to make STEAM principles and skills learning fun and enjoyable and, at the same time, attain the intended quality of learning outcomes.

Identify and Use Grade and Context Appropriate, Innovative, Differentiated and Creative Teaching and Learning Methodologies

SBC is an eclectic curriculum model. It is an amalgam of strengths of different curriculum types, including behavioural objectives, outcomes, and competency. Its emphasis is on students attaining clearly defined, measurable, observable and attainable learning standards, i.e., the expected level of education quality. Proficiency (competency) standards are expressed as performance standards/criteria and evidence outcomes, in real life or related situations) to indicate that they are meeting, have met or exceeded the learning standards. The selection of grade and contextually appropriate teaching and learning methodologies is critical to enabling all students to achieve the expected standard or quality of education. Teaching and learning methodologies must be aligned to the content, learning objective, and performance standard in order for the teacher to effectively teach and guide students towards meeting the performance standard for the lesson. They should be equitable and socially inclusive, differentiate, student-centred, and lifelong. They should enable STEAM principles and skills to be effectively taught and learned by students. Teachers should use the teacher guide to help them make informed decisions when selecting the types of teaching and learning methodologies to use in their teaching of the subject content, including STEAM principles and skills.

Plan Standards-Based Lessons

SBC lesson planning is quite difficult to do. However, this will be easier with more practice and experience over time. Effective SBC lesson plans must meet the required standards or criteria so that the learning objectives and performance standards are closely aligned to attain the expected learning outcomes. Teachers should use the guidelines and standards for SBC lesson planning and examples of SBC lesson plans provided in the teacher guide to plan their lessons. When planning lessons, it is important for teachers to ensure that all SBC lesson planning standards or criteria are met. If standards are not met, instruction will not lead to the attainment of intended performance and proficiency standards. Therefore, students will not attain the national content standards and grade- level benchmarks.

Use Standards-Based Assessment

Standards-Based Assessment has a number of components. These components are intertwined and serve to measure evaluate, report, and monitor students' achievement of the national and grade-level expectations, i.e., the essential knowledge, skills, values and attitudes they are expected to master and demonstrate proficiency on.

Teachers should use the information and examples on standards-based assessment assess, record, evaluate, report and monitor students' performance in relation to the learning standards.

Make informed Judgments About Students' Learning and Progress Towards Meeting Learning Standards

Teachers should use the teacher guide to effectively evaluate students' performance and use the evidence to help students to continuously improve their learning as well as their classroom practice.

It is important that teachers evaluate the performance of students in relation to the performance standards and progressively the grade-level benchmarks and content standards to make informed judgments and decisions about the quality of their work and their progress towards meeting the content standards or components of the standards. Evaluation should not focus on only one aspect of students' performance. It should aim to provide a complete picture of each student's performance. The context, inputs, processes, including teaching and learning processes, and the outcomes should be evaluated to make an informed judgment about each student's performance, Teachers should identify the causal factors for poor performance, gaps in students learning, gaps in teaching, teaching and learning resource constraints, and general attitude towards learning. Evidence-based decisions can then be made regarding the interventions for closing the gaps to allow students to make the required progress towards meeting grade-level and national expectations.

Prepare Students' Performance Reports

Reporting of students' performance and progress towards the attainment of learning standards is an essential part of SBC assessment. Results of students' performance should be communicated to particularly the students and their parents to keep them informed of students' academic achievements and learning challenges as well as what needs to be done to enable the students' make positive progress towards meeting the proficiency standards and achieve the desired level of education quality. Teachers should use the information on the reporting of students' assessment results and the templates provided to report the results of students' learning.

Monitor Students' Progress Towards Meeting the National Content Standards and Grade-Level Benchmarks

Monitoring of student's progress towards the attainment of learning standards is an essential component of standards-based assessment. It is an evidence-based process that involves the use of data from students' performance assessments to make informed judgements about students' learning and proficiency on the learning standards or their components, identify gaps in students' learning and the causal factors, set clear learning improvement targets, and develop effective evidence-based strategies (including pre-planning and re-teaching of topics), set clear time frames, and identify measures for measuring students' progress towards achieving the learning targets.

Teachers should use the teacher guide to help them use data from students' performance assessments to identify individual students' learning weaknesses and develop interventions, in collaboration with each student and his/her parents or guardians, to address the weaknesses and monitor their progress towards meeting the agreed learning goals.

Develop additional Benchmarks

Teachers can develop additional benchmarks using the examples in the teacher guide to meet the learning needs of their students and local communities. However, these benchmarks will not be nationally assessed as these are not comparable. They are not allowed to set their own content standards or manipulate the existing ones. The setting of national content standards is done at the national level to ensure that required learning standards are maintained and monitored to sustain the required level of education quality.

Avoid Standardisation

The implementation of Grade 10 Mathematics curriculum must not be standardised. SBC does not mean that the content, lesson objectives, teaching and learning strategies, and assessment are standardised. This is a misconception and any attempt to standardise the components of curriculum without due consideration of the teaching and learning contexts, student's backgrounds and experiences, and different abilities and learning styles of students will be counter productive. It will hinder students from achieving the expected proficiency standards and hence, high academic standards and the desired level of education quality. That is, they should not be applied across all contexts and with all students, without considering the educational needs and the characteristics of each context.
Teachers must use innovative, creative, culturally relevant, and differentiated teaching and learning approaches to teach the curriculum and enable their students to achieve the national content standards and grade-level benchmarks. And enable all students to experience success in learning the curriculum and achieve high academic standards.

What is provided in the syllabus and teacher guide are not fixed and can be changed. Teachers should use the information and examples provided in the syllabus and the teacher guide to guide them to develop, select, and use grade, context, and learner appropriate content, learning objectives, teaching and learning strategies, and performance assessment and standards. SBC is evidence-based hence decisions about the content, learning outcomes, teaching and learning strategies, students' performance, and learning interventions should be based on evidence. Teaching and learning should be continuously improved and effectively targeted using evidence from students' assessment and other sources.

Syllabus and Teacher Guide Alignment

A teacher guide is a framework that describes how to translate the content standards and benchmarks (learning standards) outlined in the syllabus into units and topics,learning objectives, lesson plans, teaching and learning strategies, performance assessment, and measures for measuring students' performance (performance standards). It expands the content overview and describes how this content identified in the content standards and their components (essential KSVAs) can be translated into meaningful and evidence-based teaching topics and learning objectives for lesson planning, instruction and assessment. It also describes and provides examples of how to evaluate and report on students' attainment of the learning standards, and use evidence from the assessment of students' performance to develop evidence-based interventions to assist students who are making slow progress towards meeting the expected proficiency levels to improve their performance.

Grade 10 Mathematics comprises of the Syllabus and Teacher Guide. These two documents are closely aligned, complimentary and mutually beneficial. They are the essential focal points for teaching and learning the essential Mathematics knowledge, skills, values and attitudes.

Syllabus and Teacher Guide Alignment	
Syllabus Outlines the ultimate aim and goals, and what to teach and why teach it • Overarching and SBC principles • Content overview • Core curriculum • Essential knowledge, skills, values and attitudes • Strands and units • Evidence outcomes • Content standards and grade-level • benchmark • Overview of assessment, evaluation, and Reporting	**Teacher Guide** Describes how to plan, teach, and assess students' performance • Determine topics for lesson planning, instruction and assessment • Formulate learning objectives • Plan SBC lesson plans • Select teaching and learning strategies • Implement SBC assessment and evaluation • Implement SBC reporting and monitoring

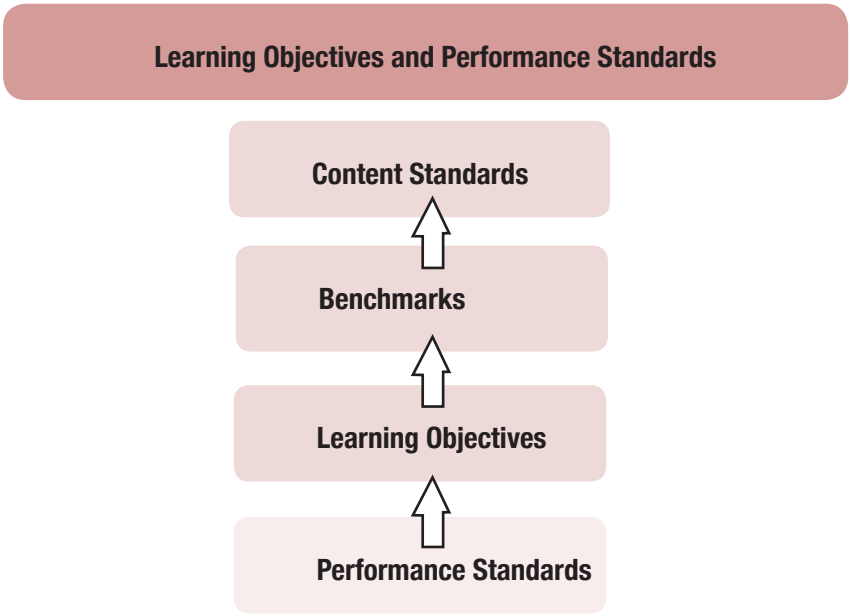
The syllabus outlines the ultimate aim and goals of SBE and SBC, what is to be taught and why it should be learned by students, the underlying principles and articulates the learning and proficiency standards that all students are expected to attain. On the other hand, the teacher guide expands on what is outlined in the syllabus by describing the approaches or the how of planning, teaching, learning, and assessing the content so that the intended learning outcomes are achieved.

This teacher guide should be used in conjunction with the syllabus. Teachers should use these documents when planning, teaching and assessing Grade 10 Mathematics content.

Teachers will extract information from the syllabus (e.g., content standards and grade-level benchmarks) for lesson planning, instruction and is for measuring students' attainment a content standard as well as progress to the next grade of schooling.

Learning and Performance Standards Alignment

Content Standards, Benchmarks, Learning Objectives, and Performance Standards are very closely linked and aligned (see below). There is a close linear relationship between these standards. Students' performance on a significant aspect of a benchmark (KSVA) is measured against a set of performance standards or criteria to determine their level of proficiency using performance assessment. Using the evidence from the performance assessment, individual student's proficiency on the aspect of the benchmark assessed and progression towards meeting the benchmark and hence the content standard are then determined.



Effective alignment of these learning standards and all the other components of PNG SBE and SBC (ultimate aim and goals, overarching, SBC and subject-based principles, core curriculum, STEAM, and cognitive, high level, and 21st Century skills) is not only critical but is also key to the achievement of high academic standards by all students and the intended level of education quality. It is essential that teachers know and can do standards alignment when planning, teaching, and assessing students' performance so that they can effectively guide their students towards meeting the grade-level benchmarks (grade expectations) and subsequently the content standards (national expectations).

Learning and Performance Standards

Standards-Based Education (SBE) and SBC are underpinned by the notion of quality. Standards define the expected level of education quality that all students should achieve at a particular point in their schooling. Students' progression and achievement of education standard (s) are measured using performance standards or criteria to determine their demonstration or performance on significant aspects of the standards and therefore their levels of proficiency or competency. When they are judged to have attained proficiency on a content standard or benchmark or components of these standards, they are then deemed to have met the standard(s) that is, achieved the intend level of education quality.

Content standards, benchmarks, and learning objectives are called learning standards while performance and proficiency standards (evidence outcomes) can be categorised as performance standards. These standards are used to measure students' performance, proficiency, progression and achievement of the desired level of education quality. Teachers are expected to understand and use these standards for lesson planning, instruction and assessment.

Content Standards

Content standards are evidence-based, rigorous and comparable regionally and globally. They have been formulated to target critical social, economic, political, cultural, environment, and employable skills gaps identified from a situational analysis. They were developed using examples and experiences from other countries and best practice, and contextualized to PNG contexts.

Content standards describe what (content - knowledge, skills, values, and attitudes) all students are expected to know and do (how well students must learn and apply what is set out in the content standards) at each grade-level before proceeding to the next grade. These standards are set at the national level and thus cannot be edited or changed by anyone except the National Subject-Based Standards Councils.

Content Standards;

- are evidenced-based,
- are rigorous and compards,
- are set at the national level,
- state or describe the expected levels of quality or achievement,
- are clear, measurable and attainable,
- are linked to and aligned with the ultimate aim and goals of SBE and SBC and overarching and SBC principles,
- delineate what matters, provide clear expectations of what students should progressively learn and achieve in school, and guide lesson planning, instruction assessment,
- comprise knowledge, skills, values, and attitudes that are the basis for quality education,
- provide teachers a clear basis for planning, teaching, and assessing lessons, and
- provide provinces, districts, and schools with a clear focus on how to develop and organise their instruction and assessment programs as well as the content that will include in their curriculum.

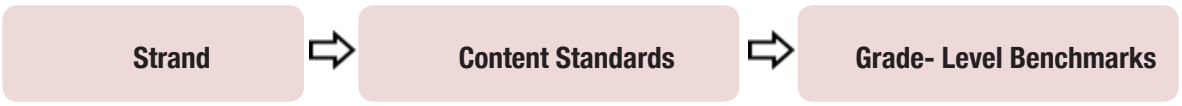
Benchmarks

Benchmarks are derived from the content standards and benchmarked at the grade-level. Benchmarks are specific statements of what students should know (i.e., essential knowledge, skills, values or attitudes) at a specific grade-level or school level. They provide the basis for measuring students' attainment of a content standard as well as progress to the next grade of schooling.

Grade-level benchmarks;

- are evidenced-based,
- are rigorous and comparable to regional and global standards;
- are set at the grade level,
- are linked to the national content standards,
- are clear, measurable, observable and attainable,
- articulate grade level expectations of what students are able to demonstrate to indicate that they are making progress towards attaining the national content standards,
- provide teachers a clear basis for planning, teaching, and assessing lessons,
- state clearly what students should do with what they have learned at the end of each school-level,
- enable students' progress towards the attainment of national content standards to be measured, and
- enable PNG students' performance to be compared with the performance of PNG students with students in other countries.

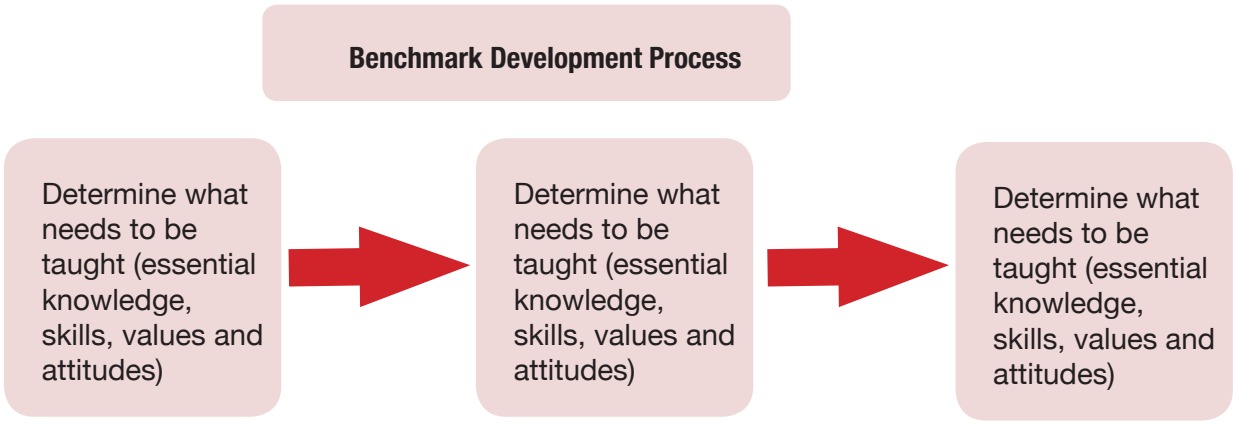
Approach for Setting National Content Standards and Grade-Level Benchmarks



Development of Additional Benchmarks

Teachers should develop additional benchmarks to meet the learning needs of their students. They should engage their students to learn about local, provincial, national and global issues that have not been catered for in the grade-level benchmarks but are important and can enhance students' understanding and application of the content. However, it is important to note that these benchmarks will not be nationally examined as they are not comparable. Only the benchmarks developed at the national level will be tested. This does not mean that teachers should not develop additional benchmarks. An innovative, reflect, creative and reflexive teacher will continuously reflect on his/her classroom practice and use evidence to provide challenging, relevant, and enjoyable learning opportunities for his/her students to build on the national expectations for students.

Teachers should follow the following process when developing additional grade-level benchmarks.



Learning Objectives

Learning or instructional Objectives are precise statements of educational intent. They are formulated using a significant aspect or a topic derived from the benchmark, and is aligned with the educational goals, content standards, benchmarks, and performance standards. Learning objectives are stated in outcomes language that describes the products or behaviours that will be provided by students. They are stated in terms of measurable and observable student behaviour. For example, students will be able to explore the idea of direct and indirect proportion.

Performance Standards

Performance Standards are concrete statements of how well students must learn what is set out in the content standards, often called the **"be able to do"** of "what students should know and be able to do." Performance standards are the indicators of quality that specify how competent a students' demonstration or performance must be. They are explicit definitions of what students **must do to demonstrate proficiency or competency at a specific level on the content standards.**

Performance standards;

- measure students' performance and proficiency **(using performance indicators)** in the use of a specific knowledge, skill, value, or attitude in real life or related situations,
- provide the basis **(performance indicators)** for evaluating, reporting and monitoring students' level of proficiency in use of a specific knowledge, skills, value, or attitude,
- are used to plan for individual instruction to help students not yet meeting expectations **(desired level of mastery and proficiency)** to make adequate progress towards the full attainment of benchmarks and content standards, and
- are used as the basis for measuring students' progress towards meeting grade-level benchmarks and content standards.

Proficiency Standards

Proficiency standards describe what all students in a particular grade or school level can do at the end of a strand, or unit. These standards are sometimes called evidence outcomes because they indicate if students can actually apply or use what they have learned in real life or similar situations. They are also categorized as benchmarks because that is what all students are expected to do before exiting a grade or are deemed ready for the next grade.

Core Curriculum

A core set of common learnings (knowledge, skills, values, and attitudes) are integrated into the content standards and grade-level benchmarks for all subjects. This is to equip all students with the most essential and in-demand knowledge, skills, and dispositions they will need to be successful in modern/postmodern work places, higher-education programs and to be productive, responsible, considerate, and harmonious citizens. Common set of learning are spirally sequenced from Preparatory to Grade 12 to deepen the scope and increase the level of difficulty in the learning activities so that what is learned is reinforced at different grade levels.

The core curriculum includes:

- cognitive (thinking) skills (refer to the syllabus for a list of these skills),
- reasoning, decision-making and problem-solving skills,
- high level thinking skills (analysis, synthesis and evaluation skills),
- 21st Century skills (refer to illustrative list in the appendix 2),
- reading, writing and communication skills,
- STEAM principles and skills,
- essential values and attitudes (core personal and social values, and sustaining values), and
- spiritual values and virtues.

The essential knowledge, skills, values and attitudes comprising the core curriculum are interwoven and provide an essential and holistic framework for preparing all students for careers, higher education and citizenship.

All teachers are expected to include the core learnings in their lesson planning, teaching, and assessment of students in all their lessons. They are expected to foster, promote and model the essential values and attitudes as well as the spiritual values and virtues in their conduct, practice, appearance, and their relationships and in their professional and personal lives. In addition, teachers are expected to mentor, mould and shape each student to evolve and possess the qualities envisioned by society.

Core values and attitudes must not be taught in the classroom only; they must also be demonstrated by students in real life or related situations inside and outside of the classroom, at home, and in everyday life. Likewise, they must be promoted, fostered and modelled by the school community and its stakeholders, especially parents. A holistic of school approach to values and attitudes in teaching, promoting and modelling is critical to students and the whole school community to internalise the core values and attitudes and make them habitual in their work and school place, and in everyday life. Be it work values, relationship values, peace values, health values, personal and social values, or religious values, teachers should give equal prominence to all common learnings in their lesson planning, teaching, assessment, and learning interventions. Common learnings must be at the heart of all teaching and extra-curricular programs and activities.

Science, Technology, Engineering, Arts and Mathematics

STEAM education is an integrated, multidisciplinary approach to learning that uses science,technology, engineering, arts and mathematics as the basis for inquiring about how STEAM has and continues to change and impact the social, political, economic, cultural and environmental contexts and identifying and solving authentic (real life) natural and physical environment problems by integrating STEAM-based principles, cognitive, high level and 21st Century skills and processes, and values and attitudes.

Mathematics is focused on both goals of STEAM rather than just the goal of problem-solving. This is to ensure that all students are provided opportunities to learn, integrate, and demonstrate proficiency on all essential STEAM principles, processes, skills, values and attitudes to prepare them for careers, higher education and citizenship.

Through STEAM education students will be able to:

(i) examine and use evidence to draw conclusions about how STEAM has and continues to change the social, political, economic, cultural and environmental contexts.
(ii) Investigate and draw conclusions on the impact of STEAM solutions to problems on the social, political, economic, cultural and environmental contexts.
(iii) Identify and solve problems using STEAM principles, skills, concepts, ideas and process.
(iv) Identify, analyse and select the best solution to address a problem.
(v) build prototypes or models of solutions to problems.
(vi) replicate a problem solution by building models and explaining how the problem was or could be solved.
(vii) test and reflect on the best solution chosen to solve a problem.
(viii) collaborate with others on a problem and provide a report on the process of problem solving used to solve the problem.
(ix) use skills and processes learnt from lessons to work on and complete STEAM projects.
(x) demonstrate STEAM principles, skills, processes, concepts and ideas through simulation and modelling.
(xi) explain the significance of values and attitudes in problem-solving.

STEAM is a multidisciplinary and integrated approach to understanding how science,technology, engineering, arts and mathematics shape and are shaped by our material, intellectual, cultural, economic, social, political and environmental contexts. And for teaching students the essential in demand cognitive, high level and 21st Century skills, values and attitudes, and empower them to effectively use these skills and predispositions to identify and solve problems relating to the natural and physical environments as well as the impact of STEAM-based solutions on human existence and livelihoods, and on the social, political, economic, cultural, and environmental systems.

STEAM disciplines have and continue to shape the way we perceive knowledge and reality, think and act, our values, attitudes, and behaviours, and the way we relate to each other and the environment. Most of the things we enjoy and consume are developed using STEAM principles, skills, process, concepts and ideas.

Things humans used and enjoyed in the past and at present are developed by scientists, technologists, engineers, artists and mathematicians to address particular human needs and wants. Overtime, more needs were identified and more products were developed to meet the ever changing and evolving human needs. What is produced and used is continuously reflected upon, evaluated, redesigned, and improved to make it more advanced, multi-purpose, fit for purpose, and targeted towards not only improving the prevailing social, political, economic, cultural and environmental conditions but also to effectively respond to the evolving and changing dynamics of human needs and wants. And, at the same time, solutions to human problems and needs are being investigated and designed to address problems that are yet to be addressed and concurred. This is an evolving and ongoing problem-solving process that integrates cognitive, high level, and 21st Century skills, and appropriate values and attitudes.

STEAM is a significant framework and focal point for teaching and guiding students to learn, master and use a broad range of skills and processes required to meet the skills demands of PNG and the 21st Century. The skills that students will learn will reflect the demands that will be placed upon them in a complex, competitive, knowledge-based, information-age, technology- driven economy and society. These skills include cognitive (critical, synthetic, creative, reasoning, decision-making, and problem-solving) skills, high level (analysis, synthesis and evaluation) skills and 21st Century skills (see Appendix 4). Knowledge-based, information, and technology driven economies require knowledge workers not technicians. Knowledge workers are lifelong learners, are problem solvers, innovators, creators, critical and creative thinkers, reflective practitioners, researchers (knowledge producers rather than knowledge consumers), solutions seekers, outcomes oriented, evidence-based decision makers, and enablers of improved and better outcomes for all.

STEAM focuses on the skills and processes of problem solving. These skills and processes are at the heart of the STEAM movement and approach to not only problem solving and providing evidence-based solutions but also the development and use of other essential cognitive, high level and 21st Century skills. These skills are intertwined and used simultaneously to gain a broader understanding of the problems to enable creative, innovative, contextually relevant, and best solutions to be developed and implemented to solve the problems and attain the desired outcomes. It is assumed that by teaching students STEAM-based problem-solving skills and providing learning opportunities inside and outside the classroom will motivate more of them to pursue careers and academic programs in STEAM related fields thus, closing the skills gaps and providing a pool of cadre of workers required by technology, engineering, science, and mathematics-oriented industries.

Although, STEAM focuses on the development and application of skills in authentic (real life) contexts, for example the use of problem- solving skills to identify and solve problems relating to the natural and physical worlds, it does not take into account the significant influence values and attitudes have on the entire process of problem solving. Values and attitudes are intertwined with knowledge and skills. Knowledge, skills, values and attitudes are inseparable. Decisions about skills and processes of skills development and application are influenced by values and attitudes (mindset) that people hold. In the same light, the use of STEAM principles, processes and skills to solve problems in order to achieve the outcomes envisaged by society are influenced by values and the mindset of those who have identified and investigated the problem as well as those who are affected by the problem and will benefit from the outcome.

STEAM Problem-Solving Methods and Approaches

Problem-solving involves the use of problem-solving methods and processes to identify and define a problem, gather information to understand its causes, draw conclusions, and use the evidence to design and implement solutions to address it. Even though there are many different problem-solving methods and approaches, they share some of the steps of problem-solving, such as;

- identifying the problem,
- understanding the problem by collecting data,
- analyse and interpret the data,
- draw conclusions,
- use data to consider possible solutions,
- select the best solution,
- test the effectiveness of the solution by trialling and evaluating it, and
- review and improve the solution.

STEAM problem solving processes go from simple and technical to advance and knowledge-based processes. However, regardless of the type of process used, students should be provided opportunities to learn the essential principles and processes of problem solving and, more significantly, to design and create a product that addressed a real problem and meets a human need.

The following are some of the STEAM problem solving processes.

1. Engineering and Technology Problem Solving Methods and Approaches

Engineering and technology problem-solving methods are used to identify and solve problems relating to the physical world using the design process. The following are some of the methods and approaches used to solve engineering and technology related problems.

Parts Substitution
It is the most basic of the problem-solving methods. It simply requires the parts to be substituted until the problem is solved.

Diagnostics
After identifying a problem, the technician would run tests to pinpoint the fault. The test results would be used either as a guide for further testing or for replacement of a part, which also need to be tested. This process continues until the solution is found and the device is operating properly.

Troubleshooting
Troubleshooting is a form of problem solving, often applied to repair failed products or processes.

Reverse Engineering
Reverse engineering is the process of discovering the technological principles underlying the design of a device by taking the device apart, or carefully tracing its workings or its circuitry. It is useful when students are attempting to build something for which they have no formal drawings or schematics.

Divide and Conquer
Divide and conquer is the technique of breaking down a problem into sub-problems, then breaking the sub-problems down even further until each of them is simple enough to be solved. Divide and conquer may be applied to all groups of students to tackle sub-problems of a larger problem, or when a problem is so large that its solution cannot be visualised without breaking it down into smaller components.

Extreme Cases
Considering "extreme cases" – envisioning the problem in a greatly exaggerated or greatly simplified form, or testing using extreme condition – can often help to pinpoint a problem. An example of the extreme-case method is purposely inputting an extremely high number to test a computer program.

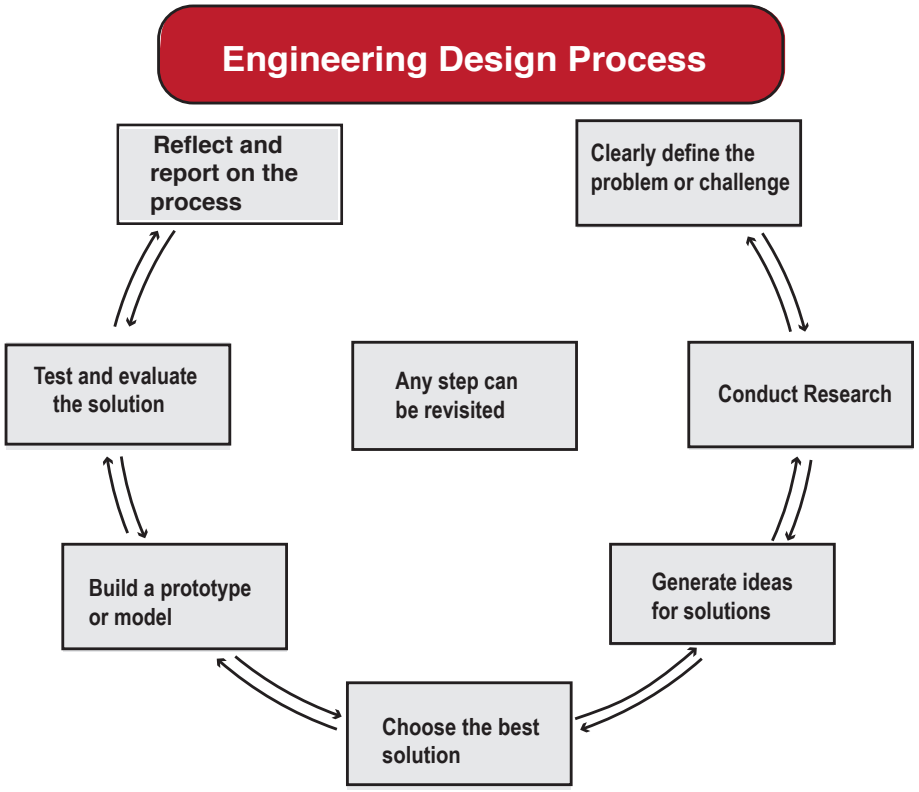
Trial and Error
The trial and error method involve trying different approaches until a solution is found. It is often used as a last resort when other methods have been exhausted.

- Test and evaluate the solution.
- Repeat steps as necessary to modify the design or correct faults.
- Reflect and report on the process.

2. Engineering Design Process

Technological fields use the engineering design process to identify and define the problem or challenge, investigate the problem, collect and analyse data, and use the data to formulate potential solutions to the problem, analyse each of the solutions in terms of its strengths and weaknesses, and choose the best solution to solve the problem. It is an open-ended problem-solving process that involves the full planning and development of products or services to meet identified needs. It involves a sequence of steps as illustrated.

1) Analyse the context and background, and clearly define the problem.
2) Conduct research to determine design criteria, financial or other constraints, and availability of materials.
3) Generate ideas for potential solutions, using processes such as brainstorming and sketching.
4) Choose the best solution.
5) Build a prototype or model.
6) Test and evaluate the solution.
7) Repeat steps as necessary to modify the design or correct faults.
8) Reflect and report on the process.



STEAM-Based Lesson Planning

Effective STEAM lesson planning is key to the achievement of expected STEAM outcomes. STEAM skills can be planed and taught using separate STEAM-based lesson plans or integrated into the standards-based lesson plans. To effectively do this, teachers should know how to write effective standards and STEAM-based lesson plans.

An example of a STEAM-based lesson plan is provided in appendix. Teachers should use this to guide them to integrate STEAM content and teaching, learning and assessment strategies into their standards-based lesson plans.

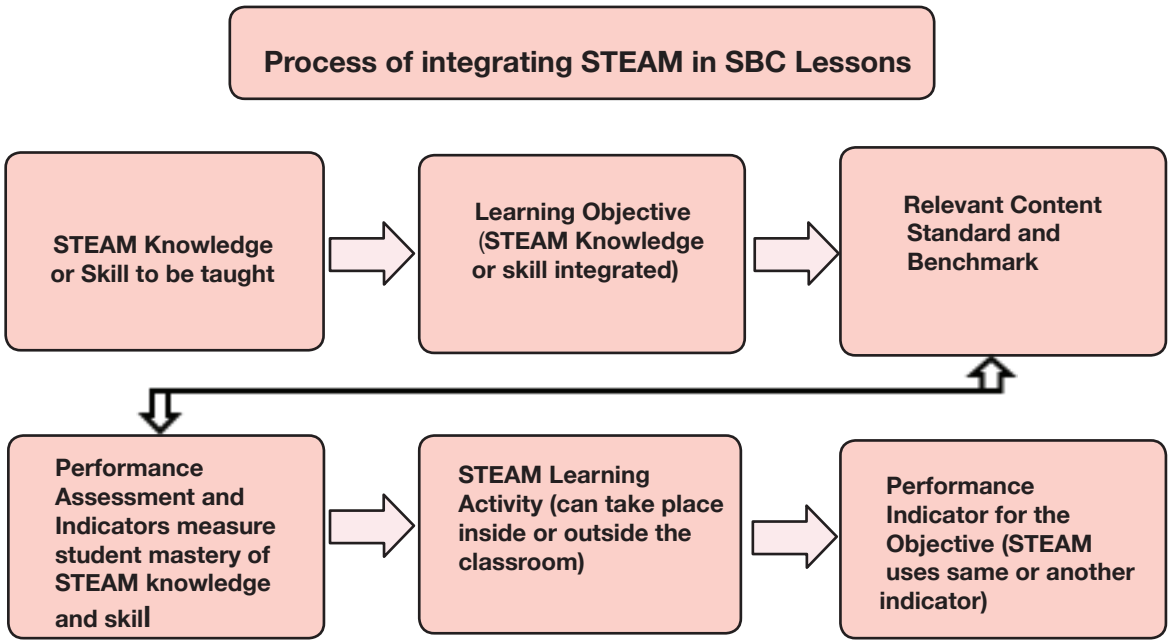
Integration of STEAM problem-solving skills into standards-based lesson plans

Knowing how to integrate STEAM problem-solving skills, principles, values and attitudes as well as STEAM teaching, learning, and assessment strategies into standards-based lesson plans is essential for achieving the desired STEAM learning outcomes. When integrating STEAM problem-solving skills into the standards-based lesson plans, teachers should ensure that these skills are not only effectively aligned to the learning objective and performance standards, they must also be effectively taught and assessed.

STEAM principles and problem-solving skills are integrated into the content standards and grade-level benchmarks. A list of these skills, including 21st Century skills, is provided in the syllabus. Teachers should ensure that these skills are integrated in their standards-based lesson plans, taught and assessed to determine students' level of proficiency on each skill or specific components of the skill.

Teachers are expected to integrate the essential STEAM principles, processes, skills, values and attitudes described in the grade 10 benchmarks when formulating their standards-based lesson plans. Opportunities should be provided inside and outside of the classroom for students to learn, explore, model and apply what they learn in real life or related situations. These learning experiences will enable students to develop a deeper understanding of STEAM principles, processes, skills, values and attitudes and appreciate their application in real life to solve problems.

Teachers should use the following process as guide to integrate STEAM principles and problem-solving skills into the standards-based lesson plans.



Steps for integrating STEAM problem-solving principles and skills into standards-based lesson plans.

Step 1: Identify the STEAM knowledge or skill to be taught (from the table of KSVAs for each content standard and benchmark). This is captured in the learning objective stated in the standards-based lesson plan.

Step 2: Develop and include a performance standard or indicator for measuring student mastery of the STEAM knowledge or skill (e.g. level of acceptable competency or proficiency) if this is different from the one already stated in the lesson plan.

Step 3: Develop student learning activity (An activity that will provide students the opportunity to apply the STEAM knowledge or skill specified by the learning objective and appropriate statement of the standards). Activity can take place inside or outside of the classroom, and during or after school hours.

Step 4: Develop and use performance descriptors (standards or indicators) to analyse students' STEAM related behaviours and products (results or outcomes), which provide evidence that the student has acquired and mastered the knowledge or skill of the learning objective specified by the indicator (s) of the standard(s).

STEAM Teaching Strategies

STEAM education takes place in both formal and informal classroom settings. It takes place during and after school hours. It is a continuous process of inquiry, data analysis, making decisions about interventions, and implementing and monitoring interventions for improvements.

There are a variety of STEAM teaching strategies. However, teaching strategies selected must enable teachers to guide students to use the engineering and artistic design processes to identify and solve natural and physical environment problems by designing prototypes and testing and refining them to effectively mitigate the problems identified. The following are some of the strategies that could be used to utilise the STEAM approach to solve problems and coming up with technological solutions.

1. Inquiry-Based Learning
2. Problem-Based Learning
3. Project-based learning
4. Collaborative Learning

Collaborative learning involves individuals from different STEAM disciplines and expertise in a variety of STEAM problem solving approaches working together and sharing their expertise and experiences to inquire into and solve a problem.
Teachers should plan to provide students opportunities to work in collaboration and partnership with experts and practitioners engaged in STEAM related careers or disciplines to learn first-hand about how STEAM related skills, processes, concepts, and ideas are applied in real life to solve problems created by natural and physical environments. Collaborative learning experiences can be provided after school or during school holidays to enable students to work with STEAM experts and practitioners to inquiry and solve problems by developing creative, innovative and sustainable solutions. Providing real life experiences and lessons, e.g., by involving students to actually solve a scientific, technological, engineering, or mathematical, or Arts problem, would probably spark their interest in a STEAM career path. Developing STEAM partnerships with external stakeholders e.g., high education institutions, private sector, research and development institutions, and volunteer and community development organizations can enhance students' learning and application of STEAM problem solving principles and skills.

Some examples of STEAM-related partnership experiences may include:

- Participatory Learning
- Group-Based Learning
- Task Oriented Learning
- Action Learning
- Experiential Learning
- Modelling
- Simulation

STEAM Learning Strategies

Teachers should include in their lesson plans STEAM learning activities. These activities should be aligned to principle or a skill planned for students to learn and demonstrate proficiency at the end of the lesson to expose students to STEAM and giving them opportunities to explore STEAM-related concepts, they will develop a passion for it and, hopefully, pursue a job in a STEAM field.

Providing real life experiences and lessons, e.g., by involving students to actually solve a scientific, technological, engineering, or mathematical, or arts problem, would probably spark their interest in a STEAM career path. This is the theory behind STEAM education.

STEAM-Based Assessment

STEAM-based assessment is closely linked to standards-based assessment where assessment is used to assess students' level of competency or proficiency of a specific knowledge, skill, value, or attitude taught using a set of performance standards (indicators or descriptors). The link also includes the main components such as the purpose, the assessment principles and assessment strategies and tools.
In STEAM-based assessment, assessments are designed for what students should know and be able to do. In STEAM learning, students are assessed in a variety of ways including portfolios, project/problem-based assessments, backwards design, authentic assessments, or other student-centered approaches.
When planning and designing the assessment, teachers should consider the authenticity of the assessment by designing an assessment that relates to a real world task or discipline specific attributes such as simulation, role play, placement assessment, live projects and debates. These tasks should make the activity meaningful to the student, and therefore be motivating as well as developing employability skills and discipline specific attributes.

Effective STEAM-Based Assessment Strategies

The following are the six assessment tools and strategies to impact teaching and learning as well as help teachers foster 21st Century learning environment in their classrooms.

1. Rubrics
2. Performance-Based Assessments (PBAs)
3. Portfolios
4. Student self-assessment
5. Peer-assessment
6. Student Response Systems (SRS).

Although the list does not include all innovative assessment strategies, it includes what we think are the most common strategies, and ones that may be particularly relevant to the educational context of developing countries in this 21st Century. Many of the assessment strategies currently in use fit under one or more of the categories discussed. Furthermore, it is important to note that these strategies also connect in a variety of ways.

1. Rubrics

Rubrics are both a tool to measure students' knowledge and ability as well as an assessment strategy. A rubric allows teachers to measure certain skills and abilities not measurable by standardized testing systems that assess discrete knowledge at a fixed moment in time. Rubrics are also frequently used as part of other assessment strategies including; portfolios, performances, projects, peer-review and self-assessment which are also elaborated in this section.

2. Performance-Based Assessments

Performance-Based Assessments (PBA), also known as project-based or authentic assessments, are generally used as a summative evaluation strategy to capture not only what students know about a topic, but if they have the skills to apply that knowledge in a "real-world" situation.

By asking them to create an end product. PBA pushes students to synthesize their knowledge and apply their skills to a potentially unfamiliar set of circumstances that is likely to occur beyond the confines of a controlled classroom setting.
The implementation of performance-based assessment strategies can also impact other instructional strategies in the classroom.

3. Portfolio Assessment

Portfolios are a collection of student work gathered over time that is primarily used as a summative evaluation method. The most salient characteristic of the portfolio assessment is that rather than being a snapshot of a student's knowledge at one point in time (like a single standardized test), it highlights student effort, development, and achievement over a period of time; portfolios measure a student's ability to apply knowledge rather than simply regurgitate. They are considered both student-centred and authentic assessments of learning.

4. Self-assessment

While the previous assessment tools and strategies listed in this report generally function as summative approaches, self-assessment is generally viewed as a formative strategy, rather than one used to determine a student's final grade. Its main purpose is for students to identify their own strengths and weakness and to work to make improvements to meet specific criteria.
Self-assessment occurs when students judge their own work to improve performance as they identify discrepancies between current and desired performance. In this way, self-assessment aligns well with standards-based education because it provides clear targets and specific criteria against which students or teachers can measure learning.

Self-assessment is used to promote self-regulation, to help students reflect on their progress and to inform revisions and improvements on a project or paper. In order for self-assessment to be truly effective four conditions must be in place: the self- assessment criteria is negotiated between teachers and students, students are taught how to apply the criteria, students receive feedback on their self-assessments and teachers help students use assessment data to develop an action plan.

5. Peer assessment

Peer assessment, much like self-assessment, is a formative assessment strategy that gives students a key role in evaluating learning. Peer assessment approaches can vary greatly but, essentially process develops both the assessor and assessee's skills and knowledge.

The primary goal for using peer assessment is to provide feedback to learners. This strategy may be particularly relevant in classrooms with many students per teacher since student time will always be more plentiful than teacher time. Although any single student's feedback may not be rich or in-depth as tea her's feedback, the research suggests that peer assessment can improve learning.

6. Student Response System

Student response system (SRS), also known as classroom response (CRS), audience response system (ARS) is a general term that refers to a variety of technology-based formative assessment tools that can be used to gather student-level data instantly in the classroom. Through the combination of hardware, (voice recorders, PC, internet connection, projector and screen) and software.

Teachers can ask students a wide range of questions (both closed and open ended), where students can respond quickly and anonymously, and the teacher can display the data immediately and graphically. The use of technology also includes a use of video which examines how a range of strategies can be used to assess students' understanding.

The value of SRS comes from teachers analysing information quickly and then devising real-time instructional solutions to maximize student learning. This includes a suggested approach to help teachers and trainers assess learning.

Curriculum Integration

What is Curriculum Integration?

Curriculum integration is making connections in learning across the curriculum. The ultimate aim of curriculum integration is to act as a bridge to increase students' achievement and engage in relevant curriculum.

Teachers must develop intriguing curriculum by going beyond the traditional teaching of content based or fragmented teaching to one who is knowledge based and who should be perceived as a 21st Century innovative educator. Curriculum integration is a holistic approach to learning thus curriculum integration in PNG SBC will have to equip students with the essential knowledge, skills, values and attitudes that are deemed 21st Century.

There are three approaches that PNG SBC will engage to foster conducive learning for all its children whereby they all can demonstrate proficiency at any point of exit. Adapting these approaches will have an immense impact on the lives of these children thus they can be able to see themselves as catalyst of change for a competitive PNG. Not only that but they will be comparable to the world standards and as global citizens.

Engaging these three approaches in our curriculum will surely sharpen the knowledge and ability of each child who will foresee themselves as assets through their achievements thus contribute meaningfully to their country. They themselves are the agents of change. Integrated learning will bear forth a generation of knowledge based populace who can solve problems and make proper decisions based on evidence. Thus, PNG can achieve its goals like the Medium Term Development Goals (MTDG) and aims such as the Vision 2050 for a happy, healthy and wealthy society whereby, all its citizens should have access and fair distribution to income, shelter, health, education and general goods and services improving the general standard of living for PNG in the long run.

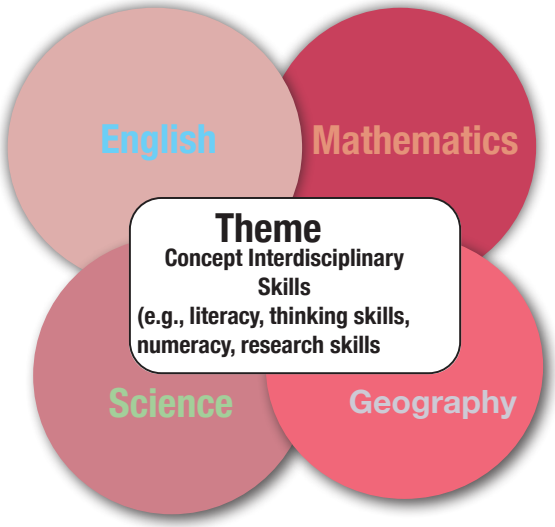
1. (i) Multidisciplinary Approach

In this approach learning involves a theme or concept that will be taught right across all subject area of study by students. That is, content of a particular theme will be taught right across all subjects as shown in the diagram below. For instance, if the theme is global warming, subject areas create lessons or assessment as per their subjects around this theme. Social Science will address this issue, Science and all other subject likewise.

1. (ii) Interdisciplinary Approach

This approach addresses learning similarly to the multidisciplinary approach of integrated learning whereby learning takes place within the subject area. However, it is termed interdisciplinary in that the core curriculum of learning is interwoven into each subject under study by the students. For instance; in Social Science under the strand of geography students write essay on internal migration however, apart from addressing the issues of this topic, they are to apply the skill of writing text types in their essay such as argumentative essay, informative, explanatory, descriptive, expository and narrative essay while writing their essay. They must be able to capture the mechanics of English skills such as grammar, punctuation and so forth. Though these skills are studied under English they are considered as core skills that cut across all subjects under study. For example; if Science students were to write about human development in biology then the application of writing skills has to be captured by the students in their writing. It is not seen as an English skill but a standard essential skill all students must know and do regardless.

Therefore, essential knowledge, skills, values and attitudes comprising the core curriculum are interwoven and provide an essential and holistic framework for preparing all students for careers, higher education and citizenship in this learning.



2. Intradisciplinary Approach

This approach involves teachers integrate sub disciplines within a subject area. For instance, within the subject Social Science, the strands (disciplines) of geography, environment, history, political science and environment will all be captured studying a particular content for Social Science. For example, under global warming, students will study the geographical aspects of global warming, environmental aspect of global warming and likewise for history, political science and economics. Thus, children are well aware of the issues surrounding global warming and can address it confidently at each level of learning.

3. Trans disciplinary Approach

In this approach learning goes beyond the subject area of study. Learning is organized around students' questions and concerns. That is, where there is a need for change to improve lives, students develop their own curriculum to effect these need. The trans-disciplinary approach addresses real-life situations thus giving the opportunity to students to attain real life skills. This learning approach is more to do with
Project–Based Learning also referred to as problem-based learning or place-based learning.

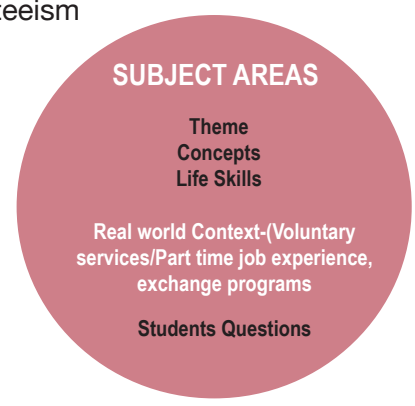
Below are the three steps to planning project based curriculum:

1. Teachers and students select a topic of study based on student interests.
2. curriculum standards, and local resources.
3. The teacher finds out what the students already know and helps them generate questions to explore. The teacher also provides resources for students and
4. opportunities to work in the field
5. Students share their work with others in a culminating activity. Students display the results of their exploration and review and evaluate the project.

For instance; students may come up with slogans for school programs such as 'Our culture – clean city for a healthier PNG'. The main aim could be to curb betel nut chewing in public areas especially around bus stops and local markets. Here, students draw up their own instructions and criteria for assessment which is; they have to clean the nearest bus stop or local market once a week throughout the year. They also design and create posters to educate the general public as their program continues. They can also involve the town council and media to assist them especially to carry out awareness.

Studies have proven that Project based-programs have led to the following:

- Students go far beyond the minimum effort
- Make connections among different subject areas to answer open-ended questions
- Retain what they have learnt
- Apply learning to real-life problems
- Have fewer discipline problems
- Lower absenteeism



These integrated learning approaches will demand for teaches to be proactive in order to improve students learning and achievement. In order for PNG Standards-Based Curriculum to serve its purpose fully, these three approaches must be engaged for better learning for the children of PNG now an in the future

Essential Knowledge, Skills, Values and Attitude and Mathematical Thinking

Students' level of proficiency and progression towards the attainment of content standards will depend on their mastery and application of essential knowledge, skills, values, and attitudes in real life or related situations. Provided here are examples of different types of knowledge, processes, skills, values, and attitudes that all students are expected to learn and master as they progress through the grades. These are expanded and deepen in scope and the level of difficulty and complexity are increased to enable students to study in-depth the subject content as they progress from one grade to the next.

These knowledge, skills, values and attitudes have been integrated into the content standards and benchmarks. They will also be integrated into the performance standards. Teachers are expected to plan and teach essential knowledge, skills, values and attitudes in their lessons, and assess students' performance and proficiency, and progression towards the attainment of content standards.

Types of Knowledge

There are different types of knowledge. These include;	
• Public and private (privileged) knowledge • Specialised knowledge • Good and bad knowledge • Concepts, processes, ideas, skills, values, attitudes • Theory and practice • Fiction and non-fiction • Traditional, modern, and postmodern knowledge	• Subject and discipline-based knowledge • Lived experiences • Evidence and assumptions • Ethics and Morales • Belief systems • Facts and opinions • Wisdom • Research evidence and findings • Solutions to problems

Types of Processes

There are different types of processes. These include;	
• Problem-solving • Logical reasoning • Decision-making • Reflection	• Cyclic processes • Mapping (e.g. concept mapping) • Modelling • Simulating
Mathematics Inquiry processes include: • Gathering information • Analysing information • Evaluating information • Making judgements • Taking actions	

Mathematical Thinking Processes

The five Mathematical process skills that can help the students improve their mathematical thinking.

1. Mathematical Problem Solving

- Understand the meaning of the problem and look for entry points to its solution
- Analyse information (givens, constrains, relationships, goals)
- Make conjectures and plan a solution pathway
- Monitor and evaluate the progress and change course as necessary
- Check answers to problems and ask, “Does this make sense?

2. Mathematical Communication

- Use definitions and previously established causes/effects (results) in constructing arguments
- Make conjectures and use counter examples to build a logical progression of statements to explore and support their ideas
- Communicate and defend mathematical reasoning using objects, drawings, diagrams, actions
- Listen to or read the arguments of others
- Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments.

3. Mathematical Reasoning

- Make sense of quantities and relationships in problem situations
- Represent abstract situations symbolically and understand the meaning of quantities
- Create a coherent representation of the problem at hand
- Consider the units involved
- Flexibly use properties of operations.

4. Mathematical Connections

- Look for patterns or structure, recognizing that quantities can be represented in different ways
- Recognize the significance in concepts and models and use the patterns or structure for solving related problems
- View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems
- Notice repeated calculations and look for general methods and short cuts
- Continually evaluate the reasonableness of intermediate results (comparing estimates) while attending to details and make generalizations based on finding.

5. Mathematical Representation

- Apply prior knowledge to solve real world problems
- Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
- Make assumptions and approximations to make a problem simpler
- Check to see if an answer makes sense within the context of a situation and change a model when necessary.

Types of Skills

There are different types of skills. These include:

1. Cognitive (Thinking) Skills

Thinking skills can be categorized into **critical thinking** and **creative thinking** skills.

i. Critical Thinking Skills

A person who thinks critically always evaluates an idea in a systematic manner before accepting or rejecting it. Critical thinking skills include;	
• Attributing • Comparing and contrasting • Grouping and classifying • Sequencing • Prioritising • Analysing	• Detecting bias • Evaluating • Meta-cognition (Thinking about thinking) • Making informed conclusions.

ii. Creative Thinking Skills

A person who thinks creatively has a high level of imagination, able to generate original and innovative ideas, and able to modify ideas and products. Creative thinking skills include;	
• Generating ideas • Deconstruction and reconstruction • Relating • Making inferences • Predicting • Making generalisations • Visualizing	• Synthesising • Making hypothesis • Making analogies • Invention • Transformation • Modelling • Simulating

2. **Reasoning Skills** - Reason is a skill used in making a logical, just, and rational judgment.

3. **Decision-Making Skills** - Decision-making involves selection of the best solution from various alternatives based on specific criteria and evidence to achieve a specific aim.

4. **Problem Solving Skills** – These skills involve finding solutions to challenges or unfamiliar situations or unanticipated difficulties in a systematic manner.

5. Literacy Skills

A strong emphasis must be placed on various types of literacy, from financial to technological, from media to mathematical, from content to cultural. Literacy may be defined as the ability of an individual to use information to function in society, to achieve goals and to develop her or his knowledge and potential. Teachers emphasize certain aspects of literacy over others, depending on the nature of the content and skills they want students to learn.

The following literacy skills are intended to be exemplary rather than defi nitive	
• Listens, read, write, and speak with comprehension and clarity • Define and apply discipline-based conceptual vocabulary • Describe people, places, and events, and the connections between and among them • Arrange events in chronological sequence • Differentiate fact from opinion • Determine an author's purpose • Determine and analyse similarities and differences • Analyse cause and effect relationships • Explore complex patterns, interactions and relationships • Differentiate between and among various options	• Listens, read, write, and speak with comprehension and clarity • Define and apply discipline-based conceptual vocabulary • Describe people, places, and events, and the connections between and among them • Arrange events in chronological sequence • Differentiate fact from opinion • Determine an author's purpose • Determine and analyse similarities and differences • Analyse cause and effect relationships • Develop an ability to use and apply abstract principals • Explore and/or observe, identify, and analyse how individuals and/or societies relate to one another

6. **High Level Thinking Skills -** These skills include analysis, synthesis, and evaluation skills.

i. Analysis Skills – Analysis skills involve examining in detail and breaking information into parts by identifying motives or causes,underlying assumptions, hidden messages; making inferences and finding evidence to support generalisations, claims, and conclusions.

Keywords				
Analyse	Differences	Find	List	Similar to
Appraise	Discover	Focus	Motivate	Simplify
Arrange	Discriminate	Function	Omit	Take part in
Assumption	Discussion	Group	Order	Test for
Breakdown	Distinction	Highlight	Organize	Theme
Categorize	Distinguish	In-depth	Point out	
Cause & effect	Dissect	Inference	Research	
Choose	Divide	Inspect	See	
Classify	Establish	Isolate	Select	
Comparing	Examine	Investigate	Separate	

ii. Synthesis Skills – Synthesis skills involve changing or creating something new, compiling information together in a different way by combining elements in a new pattern proposing alternative solutions.

iii. Evaluation Skills – Evaluation skills involve justifying and presenting and defending opinions by making judgments about information, validity of ideas or quality of work based on set criteria.

Types of Values

Personal engagement and civic engagement strategies help young people to acquire and apply skills and dispositions that will prepare them to become competent and responsible citizens.

1. Personal Values (importance, worth, usefulness, etc.)

Core values	Sustaining values
• Sanctity of life • Truth • Aesthetics • Honesty • Human • Dignity • Rationality • Creativity • Courage • Liberty • Affectivity • Individuality	• Self-esteem • Self-reflection • Self-discipline • Self-cultivation • Principal morality • Self-determination • Openness • Independence • Simplicity • Integrity • Enterprise • Sensitivity • Modesty • Perseverance

2. Social Values

Core values	Sustaining values
• Equality • Kindness • Benevolence • Love • Freedom • Common good • Mutuality • Justice • Trust • Interdependence • Sustainability • Betterment of human kind • Empowerment	• Plurality • Due process of law • Democracy • Freedom and liberty • Common will • Patriotism • Tolerance • Gender equity and social inclusion • Equal opportunities • Culture and civilisation • Heritage • Human rights and responsibilities • Rationality • Sense of belonging • Solidarity • Peace and harmony • Safe and peaceful communities

Types of Attitudes

Attitudes - Ways of thinking and behaving, points of view	
• Optimistic • Participatory • Critical • Creative • Appreciative • Empathetic • Caring and concern • Positive • Confident • Cooperative	• Responsible • Adaptable to change • Open-minded • Diligent • With a desire to learn • With respect for self, life, equality and excellence, evidence, fair play, rule of law, different ways of life, beliefs and opinions, and the environment.

Teaching and Learning Strategies

Mathematics teaching emphasises and embraces the use of cognitive, reasoning, decision-making, problem solving and higher level thinking skills to teach to enhance students' understanding of inter-disciplinary concepts and issues in relation to environment, geography, history, politics and economic within PNG and globally. It aims to provide a meaningful pedagogical framework for teaching and learning essential and in demand knowledge, skills, values, and attitudes that are required for the preparation of students for careers, higher education and citizenship in the 21st Century.

Students must be prepared to gather and understand information, analyse issues critically, learn independently or collaboratively, organize and communicate information, draw and justify conclusions, create new knowledge, and act ethically.

These teaching and learning strategies will help teachers to;

- familiarize themselves with different methods of teaching in the classroom.
- develop an understanding of the role of a teacher for application of various methods in the classroom.

Successful teachers always keep in view that teaching must "be dynamic, challenging and in accordance with the learner's comprehension. He/she does not depend on any single method for making his/her teaching interesting, inspirational and effective.

A detailed table of Teaching and Learning Strategies are outlined below:

STRATEGY	TEACHER	STUDENTS
CASE STUDY Used to extend students' understanding of real life issues	Provide students with case studies related to the topic of the lesson and allow them to analyse and evaluate.	Study the case study and identify the problem addressed. They analyse the problem and suggest solutions supported by conceptual justifications and make presentations. This enriches the students' existing knowledge of the topic.
DEBATE A method used to increase students' interest, involvement and participation	Provide the topic or question of debate on current issues affecting a bigger population, clearly outlining the expectations of the debate. Explain the steps involved in debating and set a criteria/ standard to be achieved.	Conduct researches to gather supporting evidence about the selected topic and summarising the points.
DISCUSSION The purpose of discussion is to educate students about the process of group thinking and collective decision.	The teacher opens a discussion on certain topic by asking essential questions. During the discussion, the teacher reinforces and emphasises on important points from students responses.	Students ponder over the question and answer by providing ideas, experiences and examples.

STRATEGY	TEACHER	STUDENTS
	Teacher guide the direction to motivate students to explore the topic in greater depth and the topic in more detail. Use how and why follow-up questions to guide the discussion toward the objective of helping students understand the subject and summarise main ideas.	
GAMES AND SIMULATIONS Encourages motivation and creates a spirit of competition and challenge to enhance learning	Being creative and select appropriate games for the topic of the lesson. Give clear instructions and guidelines. The game selected must be fun and build a competitive spirit to score more than their peers to win small prices.	Go into groups and organize. Follow the instructions and play to win.
OBSERVATION Method used to allow students to work independently to discover why and how things happen as the way they are. It builds curiosity.	Give instructions and monitor every activity students do	Students possess instinct of curiosity and are curious to see the things for themselves and particularly those things which exist around them. A thing observed and a fact discovered by the child for himself becomes a part of mental life of the child. It is certainly more valuable to him than the same fact or facts learnt from the teacher or a book. Students • Observe and ask essential questions • Record • Interpret
PEER TEACHING & LEARNING (power point presentations, pair learning) Students teach each other using different ways to learn from each other. It encourages; team work, develops confidence, feel free to ask questions, improves communication skills and most importantly develop the spirit of inquiry.	Distribute topics to groups to research and teach others in the classroom. Go through the basics of how to present their peer teaching	Go into their established working groups. Develop a plan for the topic. Each group member is allocated a task to work on. Research and collect information about the topic allocated to the group. Outline the important points from the research and present their findings in class

STRATEGY	TEACHER	STUDENTS
PERFORMANCE-RELATED TASKS (dramatization, song/lyrics, wall magazines) Encourages creativity and take on the overarching ideas of the topic and are able to recall them at a later date	Students are given the opportunity to perform the using the main ideas of a topic. Provide the guidelines, expectations and the set criteria	Go into their established working groups. Being creative and create dramas, songs/lyrics or wall magazines in line with the topic.
PROJECT (individual/group) Helps students complete tasks individually or collectively	Teacher outline the steps and procedures of how to do and the criteria	Students are involved in investigations and finding solutions to problems to real life experiences. They carry out researches to analyse the causes and effects of problems to provide achievable solutions. Students carefully utilise the problem-solving approach to complete projects.
USE MEDIA & TECHNOLOGY to teach and generate engagement depending on the age of the students	Show a full movie, an animated one, a few episodes form documentaries, you tube movies and others depending on the lesson. Provide questions for students to answer before viewing	Viewing can provoke questions, debates, critical thinking, emotion and reaction. After viewing, students engage in critical thinking and debate

Strands, Units, Topics and Suggested Lesson Titles

This section contains the overview of Mathematics content to be taught in grade 10. The table below outlines strands, units,topics with suggested lesson titles. Teachers will use this to develop their own termly and yearly programs.

STRAND	UNIT	TOPIC	SUGGESTED LESSON TITLES
Number, Operation and Computation	Ratios and Proportions	Ratios	Simplifying Ratios as Fractions
			Ratios in Measurement
			Ratios involving Rational Numbers
			Dividing a Quantity in a Given Ratio
		Proportion	Proportional Parts
			Direct Proportion
			Inverse Proportion
			Measure of Rate
		Percentage	Express quantity as a percentage
			Application of percentage
	Exponentials	Operations with exponents	Addition and Subtraction of Exponents
			Multiplication and Division of Exponents
			Positive and negative exponents
		Roots	Square roots
			Cube roots
			n^{th} root
			Problem solving on nth root
		Logarithm	Convert index to log
			Convert log to index
			Base 10
			Laws of logarithms
			Simplify using the laws of logarithms
	Financial Mathematics	Earning, Saving and Spending Money	Earning Money
			Budgeting
			Saving money
			Spending money
		Consumer Arithmetic	Borrowing money
			Simple interest
			Simple interest and applications
			Compound interest and application
		Basic Microsoft Excel	Introduction to basic Microsoft Excel
			Use of Microsoft Excel to calculate simple interest Problems
			Use of Microsoft Excel to calculate compound interest Problems

Geometry, Measurement and Transformation	Trigonometry	Pythagorean Theorem	Right angle triangle
			Pythagorean Theorem
			Calculating the length of the hypotenuse using Pythagoras rule
			Calculating shorter sides using Pythagoras rule
			Problem solving involving right angle triangle
		Trigonometric ratios	Introduction to Trigonometric ratios
			Using the sine ratio to find the length of sides
			Using the sine ratio to find angle values
			Using the cosine ratio to find the length of sides
			Using the cosine ratio to find angle values
		Trigonometric problems	Mixed Problems with diagrams on Trigonometric Ratios
			Word Problems on Trigonometric Ratios
		Direction and angles of elevation and depression	Direction (compass direction)
			Bearing
			Angle of elevation and depression
			Problem solving
		Sine, cosine and area rule for non-right angled triangle	Sine rule
			cosine rule
			area rule
			Problem Solving
Patterns and Algebra	Linear and Quadratic Equations	Simultaneous equations	Solution by Substitution Method
			Solution by Elimination Method
			Solution by Graph
		Quadratic equations	Factorization
			Quadratic formulae
			Completing Squares
			Graphs
			Solve linear and quadratic equation by substitution and graph
		Parabola	Sketch, describe, interpret and parabolas
		Representations of relationship between simple quadratic and straight line	Graph using Microsoft Excel
			Interpret connection between simple quadratic and straight line

	Equations and inequality	Linear Equations and inequalities with two variables	Sketch using table of values
			Derive the linear equation from the Graphs
			Inequalities
			Sketch the inequality
			Linear Equations and inequalities with two variables
			Problem solving with two variables
			Sketch and verify shaded regions
		Linear and quadratic inequalities	Solve linear and quadratic inequality
			Sketching the quadratic inequality
			Sketching linear and quadratic inequalities.
			Sketch and verify shaded regions
	Polynomials	Polynomials	Introduction of Polynomials
			Properties of Polynomial expressions
			Adding and Subtracting Polynomials with first and second degrees
			Multiplication and Division of Polynomials with first and second degrees
			Represent Polynomials with concrete or pictorial models.
			Solve problems using algebraic manipulations.
		Operations with Polynomial	Division of Polynomials
			Remainder Theorem
			Factor Theorem
			Word problems on factor theorem.
		Sketching polynomials	Intercepts of polynomial using factor theorem
			Sketch polynomials
			Describe features of polynomial curves
Statistics and Probability	Probability	Probability Events Independent and Dependent Events	Simple Probability and Meaning of Probability
			Independent events,
			Dependent events
		With Replacement and Without Replacement Events	With replacement
			Without replacement
	Data Analysis	Measure of dispersions	Measures of Central Tendency
			Measures of Dispersion
			The Range
			Lower Quartiles
			Upper Quartiles
			Variance
			The Standard Deviation

Grade 10
Mathematics
Teaching Content

Strand 1: Number, Operations and Computation

Content Standard: Students will be able to represent numbers in various situation and forms, develop fluency in calculations through operations, use base ten as key for extending numbers and operations, and apply numbers in practical situations to develop numbers sense.

Units	Benchmark	Topics	Lesson Titles
Ratios and Proportions	**10.1.1.1** Apply ratios, proportions and percent to represent the relationship between two quantities and solve.	Ratios	Simplifying Ratios as Fractions
			Ratios in Measurement
			Ratios involving Rational Numbers
			Dividing a Quantity in a Given Ratio
		Proportion	Proportional Parts
			Direct Proportion
			Inverse Proportion
			Measure of Rate
		Percentage	Express quantity as a percentage
			Application of percentage
Exponentials	**10.1.1.2** Add, subtract, multiply, and divide numbers with positive and negative exponents.	Operations with exponents	Addition and Subtraction of Exponents
			Multiplication and Division of Exponents
			Positive and negative exponents
	10.1.1.3 Solve problems involving square root, cube root and so on.	Roots	Square roots
			Cube roots
			n^{th} root
			Problem solving on nth root
	10.1.1.4 Use the definition of logarithm to establish and apply the laws	Logarithm	Convert index to log
			Convert log to index
			Base 10
			Laws of logarithms
			Simplify using the laws of logarithms
Financial Mathematics	**10.1.1.5** Apply the skills of managing money in authentic situations	Earning, Saving and Spending Money	Earning Money
			Budgeting
			Saving money
			Spending money
	10.1.1.6 Demonstrate the concepts of simple and compound interest	Consumer Arithmetic	Borrowing money
			Simple interest
			Simple interest and applications
			Compound interest and application
	10.1.1.7 Solve problems involving simple interest and compound interest even by using appropriate digital technologies.	Basic Microsoft Excel	Introduction to basic Microsoft Excel
			Use of Microsoft Excel to calculate simple interest Problems
			Use of Microsoft Excel to calculate compound interest Problems

Unit: Ratio and Proportion — Topic: Ratio

Benchmark	**10.1.1.1** Apply ratios, proportions and percentages to represent the relationship between two quantities and solve problems.

Learning Objective: By the end of this topic, students will be able to;

- understand and explain the concept of ratio,
- apply the concept of ratio as a tool to solve problems, and
- solve Ratio problems involving rational numbers.

Essential Questions:

1. What is the concept of ratio?
2. How is ratio applied in real life situation?

Key Concepts(ASK-MT)	
Attitudes / Values	Show confidence in calculating and simplifying ratios in relation to authentic situation.
Skills	Calculate and solve ratio and fractional ratios into a given quantity and simplify using the HCF.
Knowledge	Relationships between ratio, proportion and percentage.
Mathematical Thinking	Think about how to apply ratios in solving problems.

Content Background

This unit on ratio is a revision of what has been learnt in the previous grades. The students will use their prior knowledge, skills and process learnt to simplify and solve problems in relation to ratio and proportion.

Ratio

A ratio is a comparison of two or more quantities of the same kind.
We can say that the comparison or simplified form of two quantities of the same kind is referred to as ratio. This relation gives us how many times one quantity is equal to the other quantity. In simple words, the ratio is the number which can be used to express one quantity as a fraction of the other ones.

- The ratio $a:b$ may be written as the fraction $\frac{a}{b}$.
- To compare two quantities in a ratio, they must be expressed in the same unit. A ratio has no unit.

The two numbers in a ratio can only be compared when they have the same unit. We make use of ratios to compare two things. The sign used to denote a ratio is ':'.
A ratio can be written as a fraction, say 2/5, or can be represented by using "to", as "2 to 5." We happen to see various comparisons or say ratios in our daily life.

EXAMPLE:

There are 16 boys and 20 girls in a class. Find the ratio of;

a The number of boys to the number of girls

b The number of girls to the total number of students in the class.

Solution:

a Ratio of the number of boys to the number of girls = 16:20 = 4:5

which is $16:20 = \frac{16}{20} = \frac{4 \text{ x } 4}{4 \text{ x } 5} = \frac{4}{5}$

b Total number of students = 16 + 20 = 36

∴ Ratio of the number of girls to the total number of students = 20:36 = 5:9

Since a ratio can be expressed as a fraction, we will reduce the ratio to its simplest form $a:b$ where a and b have no common factors except for 1.

Dividing a Quantity into Given Ratio

Example 1: Divide K18 in the ratio 2:3

Solution : A ratio of 2:3 means a total of 5 parts.

$\therefore$ the smaller share = $\frac{2}{5}$ x $18 = 7.20$ and using larger share $\frac{3}{5}$ x $\text{K}18 = \text{K}10.80$

Example 2 : A cake is cut into pieces in the ratio 3:5. If the larger piece has a mass of 425 g, find the mass of the smaller piece of cake.

Solution : Suppose the smaller piece has a mass of $x\,g$, then

$$x:425 = 3:5 \qquad i.e.\ \frac{x}{425} = \frac{3}{5}$$

$$x = \frac{3}{5} \text{ x } 425 = 255$$

Therefore, the smaller piece of cake has a mass of 255 g.

Problems Involving Map Scales

A map scale can be expressed in the form of 1: n, where n is a whole number.
E.g. 1:50 000 000 means 1 cm represents 50 000 000 cm.

Example: A map has a scale of 1: 50 000.

a What is the distance, in km does a length of 6 cm on the map represent?
b What length on the map represents a railway track of 12.5 km?
c Calculate the area of the lake (in km^2) which represents by an area of 4 cm^2 on the map.

Solution:

a 1:50 000 = 1 cm : 50 000 cm
= 1 cm : 500 m
= 1 cm : 0.5 km
Since 1 cm represents 0.5 km, $\therefore$ 6 cm represents 6 x 0.5 km = 3 km
$\therefore$ 6 cm on the map represents 3 km on the ground

b Let x cm on the map represent 12.5 km on the ground. $\therefore$ x cm represents 12.5 km and 1 cm represents 0.5 km

$\therefore \frac{x}{1} = \frac{12.5}{0.5} \Longrightarrow x = 25$
$\therefore$ the railway track is represented by 25 cm on the map.

c 1 cm represents 0.5 km
$\therefore$ $(1\,cm)^2$ Represents $(0.5\,cm)^2$
i.e. $(1\,cm)^2$ represents 0.25 km^2
Let 4 cm^2 represents y km^2
$\therefore \frac{y}{0.25} = \frac{1}{4}$
$\therefore$ The actual area of the lake is 1 km^2.

Unit: Ratio and Proportion — Topic: Proportion

Benchmark	**10.1.1.1** Apply ratios, proportions and percentages to represent the relationship between two quantities and solve problems.

Learning Objective: By the end of this topic, students will be able to;

- explore the idea of direct and indirect proportion, and
- distinguish how to use direct and indirect proportion to solve authentic situations.

Essential Questions:

1. What is the relationship between inverse and direct proportion?
2. How can this relationship be represented?

Key Concepts(ASK-MT)	
Attitudes / Values	Show confidence in expressing proportional relationships between two quantities.
Skills	Solve direct and indirect proportional problems.
Knowledge	Proportional relationships of two quantities changing together.
Mathematics Thinking	Think about how to analyse proportional relationship.

Content Background

Ratio and Proportion

Ratio and proportions are said to be faces of the same coin. When two ratios are equal in value, then they are said to be in **proportion**. In simple words, it compares two ratios. Proportions are **denoted by the symbol '::' or '='**.

Now, let us learn the Maths ratio and proportion formulas here. Assume that, we have two quantities or two numbers or two entities and we have to find the ratio of these two, then the formula for ratio is defined as;

$$a:b \Rightarrow \frac{a}{b}$$

where a and b could be any two quantities.
Here, "a" is called the first term or antecedent, and "b" is called the second term or consequent.

Now, let us assume that, in proportion, the two ratios are $a:b$ & $c:d$. The two terms 'b' and 'c' are called **'means** or **mean term**,' whereas the terms 'a' and 'd' are known as **'extremes** or **extreme terms**.

Example:

Are the ratios $4:5$ and $8:10$ said to be in Proportion?

Solution:

$4:5 = \frac{4}{5} = 0.8$ and $8:10 = \frac{8}{10} = 0.8$

Since both the ratios are equal, they are said to be in proportion.

Are the two ratios $8:10$ and $7:10$ in proportion?

Solution:

$8:10 = \frac{8}{10} = 0.8$ and $7:10 = \frac{7}{10} = 0.7$

Since both the ratios are not equal, they are not in proportion.

Direct Proportion

The term direct proportion means that two (or more) quantities increase or decrease in the same ratio.

Example The ratio of pizza to people is 1:3. You are having a party and there will be 12 there. How many pizzas do you need for the party?
Ratio of pizza to people is 1:3

1 pizza feeds 3 people, we have 12 people. How many groups of 3 are in 12? 12 ÷ 3 = 4
∴ We would need 4 pizzas to feed 12 people

Inverse Proportion

For inverse proportions, when one quantities increase, the other quantity is decreased by the same ratio.

Example
It takes 12 days for 3 painters to paint a house. How long will it take for 9 painters to do the same job?
First, we need to determine which kind of proportionality it is. It takes 12 days for 3 painters. 9 painters…will they take more or less time to do the job?
More painters mean that it'll take less time for them to finish the job. So, we're dealing with an inverse proportionality. Now, we just need to apply the rule of three inverses:

3 painters = 12 days
9 painters = x

$$x = \frac{3 \text{ x} 12}{9} = 4$$

It takes 4 days for 9 painters to paint the house.

Measure of Rate

In describing the units of a rate, the word "per" is used to separate the units of the two measurements used to calculate the rate (for example a heart rate is expressed "beats per minute").

Example

a A plane travels at an average speed of 513 km/h. How far does the plane travel in one and half hour?

b Joe walked to his school from his home in 37 minutes. If his school is 3 km away from his home, calculate:

i) Joe's average speed in meters per second (m/s)
ii) How far Joe would walk in an hour if he maintained the same average speed.

Solution: a

$513\,km = 1h$

$x = 1\frac{1}{2}$

$x = 513 \text{ x} 1.5$

$x = 769.15\,km$

b

$3\,km = 37\text{mins}$

$3 \text{ x} 1000\,m = 37 \text{ x } 60s$

$3000m = 2220s$

$\text{Average speed} = \frac{3000\,m}{2220s}$

$Averagespeed = 1.35m/s$

$i \quad 1.35m = 1s$

$x = 1 \text{ x } 60 \text{ x } 60 = 3600s$

$x = 3600 \text{ x } 1.35x$

$x = 4860m$

Unit: Ratio and Proportion — Topic: Percentages

Benchmark **10.1.1.1** Apply ratios, proportions and percentages to represent the relationship between two quantities and solve problems.

Learning Objective: By the end of this topic, students will be able to;

- express quantity as a percentage, and
- apply percentage to solve authentic situations.

Essential Questions:

1. What is percentage?
2. What are the different concepts in percentages?
3. How is it applied in authentic situations?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate and display confidence in calculating percentage.
Skills	Appropriate Calculations of percentages in any given situation.
Knowledge	Concept of percentage and its usefulness in different context or situations in daily life.
Mathematics Thinking	Think about how to solve percentage problem in real life situations.

Content Background

Percentage

Percentage deals with a group of decimal fractions whose denominators are out of 100. That is fractions of two decimal places.
Since hundredths were used so frequently the decimal point was dropped and the symbol % was placed after the number and read as percentage (per 100) thus, 0.15 and 15% represent the same value, $\frac{15}{100}$.

The first is read 15 hundredths and the second is read as 15 percent. Both means 15 parts out of 100. By converting all decimal fractions so that they had a common denominator 100.
Since percentage means hundredths any decimal may be changed to convey an idea of relative value or relationship.

Express quantity as a percentage

To express one Quantity as a Percentage of another from a fraction, use the two numbers of the fraction and multiply by 100. The number expressed as a percentage is placed as the numerator of the fraction. For example if we need to express 5 as a percentage of 20, we put 5 in the numerator's place and 20 in the denominator's place of the fraction ($\frac{5}{20}$)

Example: Express 12 as a percentage of 25.

Form a fraction by placing 12 as the numerator and 25 the denominator. $\frac{12}{25}$

To change the fraction into percentage, multiply by 100. $\frac{12}{25} \times 100 = \frac{12}{25} \times \frac{100}{1}$ (write 100 as a fraction by putting it over 1. 1200/25 Multiply numerators together and denominators together. Divide the numerator by the denominator to find the answers as percentage or giving the answer correct to 2 decimal places.

Application of percentage

Example
In an election, a candidate got 70 % of the total valid votes. 20 % of the total votes were declared invalid. If the total number of votes is 600 000, find the number of valid votes polled in favour of the candidate.

Solution: Total number of invalid votes = 20 % of 600 000
= 20/60× 600 000
= 120 000

Total number of valid votes polled = 600 000 – 120 000 = 480 000
Percentage of valid votes polled in favour of the candidate = 70 %
Therefore, the Number of valid votes polled in favour of the candidate = 70 % of 480 000
$= \frac{70}{100} \text{ x } 480\,000 = 336\,000$ votes

Therefore, the number of valid votes polled in favour of the candidate is 336 000.

Example
Mother's weight is 25 % more than that of daughter. What percentage is daughter's weight less than mother's weight?

Solution:
Let daughter's weight be 100 kg.
Then mother's weight = (100 + 25) kg = 125 kg
If mother's weight is 125 kg, then daughter's weight is 100 kg.
If mother's weight is 1 kg, then daughter's weight is 100/125 kg
If mother's weight is 100 kg, then daughter's weight = (100/125 × 100) kg Therefore, daughter's weight is 20 % less than that of mother.

Unit: Exponentials	Topic: Operations with Exponents

Benchmark	**10.1.1.2** Add, subtract, multiply, and divide numbers with positive and negative exponents.

Learning Objective: By the end of this topic, students will be able to;

- add, subtract, multiply and divide positive and negative exponents using the laws of exponents, and
- solve problems and simplify using the laws of exponents.

Essential Questions:

1. What are exponentials?
2. What are the laws of exponential?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate the laws of exponents and its usefulness in solving problems.
Skills	Process and simplify through adding, subtracting, multiplying, dividing positive and negative exponents using the laws of exponents.
Knowledge	Laws of exponents.
Mathematics Thinking	Think about how to solve exponential problems.

Content Background

Exponents

In this topic, we review the laws of indices and learn how to apply the laws for calculations involving multiplication and division of numbers in index form. Exponents and powers mean the same as indices.

- **First Law of indices** – when expressions are written in index form and having the same base are multiplied, the indices are added together. $a^m \times a^n = a^{m+n}$

Example:

a $f^5 \times f^8 = f^{5+8} = f^{13}$

b $2x^3y^2 \times 3xy^4 = 2 \times x^3 \times y^3 \times 3 \times x \times y^4$

$= (2 \times 3) \times (x^3 \times x^1) \times (y^2 \times y^4) = 6 \times x^{3+1} \times y^{2+4}$

$= 6 \times x^4 \times y^6$

$= 6x^4y^6$

- **Second Laws of Indices** – When expressions written in index form and having the same non-zero base are divided, the index of the quotient is obtained by subtracting the index of the denominator from the numerator.

$$a^m \div a^n = \frac{a^m}{a^n} = a^{(m-n)}, \quad a \neq 0$$

Example: **a** $f^8 \div f^6 = \frac{f^8}{f^6} = f^{8-6} = f^2$

b $\frac{16h^9p^6}{8h^4p^3} = (16 \div 8)h^{9-4}p^{6-3}$

$= 2h^5p^3$

- **Third Law of Indices** – When an expression written in index form is raised to another power index, the indices are multiplied. $(a^m)^n = a^{mxn} = a^{mn}$

Example:

a $(k^6)^5 = k^{6x5} = k^{30}$

b $\frac{(y^3)^4}{(y^2)^5} = \frac{y^{3x4}}{y^{2x5}} = \frac{y^{12}}{y^{10}}$

$= y^{12-10}$

$= y^2$

- **Fourth Law of Indices** – For any unknown a, b and n

$(a \text{ x } b)^n = a^n \text{x } b^n$

Example:

a $(2a^2b)^3 = 2^3 \text{x } a^{2x3} \text{x } b^3 = 8a^6b^3$

b $(h^2k^3)^4 \div k^5 = h^8k^{12} \div k^5 = h^8k^{12-5} = h^8k^7$

- **Fifth Law of Indices** - For any unknown a, b and n

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example:

$\left(\frac{j^2}{s^4}\right) = \frac{j^{2 \text{ x } 3}}{s^{4 \text{ x } 3}} = \frac{j^6}{s^{12}}$

- **Zero Index Law** – When any non-zero expressions is raised to the power of zero, the answer is equal to 1.

$a^0 = 1 (a \neq 0)$

Example:

$7^0 \text{x } 7^4 \div 7^2 = 1 \text{x } 7^{4-2}$

$= 1 \text{x } 7^2$

$= 7^2$

$= 49$

- **Negative Index Law** – for all unknowns a and n $(a \neq 0)$ $a^{-n} = \frac{1}{a^n}$

Example:

a $t^{-4} \text{x } t^3 = t^{-4-3} = t^{-1} = \frac{1}{t}$

b $(3r^2)^{-3} = 3^{-3}r^{-6} = \frac{1}{3^3} \text{x} \frac{1}{3^3} \text{x} \frac{1}{r^6} = \frac{1}{27r^6}$

Note: that all the laws that have been used for positive indices can also be used for negative indices.

Unit: Exponentials — Topic: Roots

Benchmark **10.1.1.3** Solve problems involving square roots and cube roots, and so on.

Learning Objective: By the end of this topic, students will be able to;

- apply and explain the concept of roots,
- recognize that numbers have rational and irrational roots, and
- apply square roots, cube roots and nth root in problem solving.

Essential Questions:

1. What is the concept of roots?
2. What are rational and irrational roots?
3. What is the difference between square roots, cube roots and n^{th} root?

Key Concepts(ASK-MT)

Attitudes / Values	Display confidence in using square root, cube root and nth root.
Skills	Analyse the differences between square root, cube root and nth root and communicate their methods of simplifying.
Knowledge	The concept of nth roots , square roots and cube roots.
Mathematics Thinking	Think about how to apply square and cube roots in authentic situations.

Content Background

n^{th} Roots

Definition: Let n be any positive integer, greater than 1, and let A be any real number.
Suppose that n is an even number. If A is negative, then the n^{th} root of A; denoted by $\sqrt[n]{A}$, is undefined. If A is non-negative, then $\sqrt[n]{A}$ is the non-negative number that, when raised to the n^{th} power, the result is A.
Suppose that n^{th} is an odd number. Then the n^{th} root of A, denoted by $\sqrt[n]{A}$, is the number that, when raised to the n^{th} power, the result is A.

Example: Simplify

a $(\sqrt[5]{3})^5$ **b** $(\sqrt[4]{7})^8$ **c** $(\sqrt[3]{2})^{12}$ **d** $(\sqrt[5]{10})^{15}$

Solution:

a By definition, $\sqrt[5]{3}$ is the number that, when raised to the 5th power, the result is 3. So the answer is 3

b By definition, $\sqrt[4]{7}$ is the number that, when raised to the 4th power, the result is 7. Also recall the rule of repeated exponentiation: $(a^m)^n = a^{mn}$

$$(\sqrt[4]{7})^8 = (\sqrt[4]{7})^{4.2} = [(\sqrt[4]{7})^4]^2 = 7^2 = 49$$

Square Roots

Let A be a non-negative number. Then the square root of A (notation: $\sqrt{A}$) is the non-negative number that, if squared, the result is A. If A is negative, then $\sqrt{A}$ is undefined.

For example, consider $\sqrt{25}$. The square-root of 25 is a number, that, when we square, the result is 25. There are two such numbers, 5 and -5. The square root is defined to be the non-negative such number, and so $\sqrt{25}$ is 5. On the other hand, $\sqrt{-25}$ is undefined, because there is no real number whose square is negative.

Example
Evaluate each of the following expressions

a $\sqrt{49}$ **b** $-\sqrt{49}$ **c** $\sqrt{-49}$ **d** $-\sqrt{-49}$

Solution:

a $\sqrt{49}=7$
b $-\sqrt{49}=-1.\sqrt{49}=-1.7=-7$
c $-\sqrt{-49}=\text{undefined}$
d $-\sqrt{-49}=-1.\sqrt{-49}=\text{undefined}$

Square roots, when stretched over entire expressions, also function as grouping symbols.

Example:

a $\sqrt{25}-\sqrt{16}$ **b** $\sqrt{25}-\sqrt{16}$ **c** $\sqrt{144+25}$

Solution:

a $\sqrt{25}-\sqrt{16}=5-4=1$
b $\sqrt{25-16}=\sqrt{9}=3$
c $\sqrt{144+25}=\sqrt{169}=13$
d $\sqrt{144}+\sqrt{25}=12+5=7$

Cube Roots

Let A be any real number. Then the third root of A (notation: $\sqrt[3]{A}$) is the number that, if raised to the third power, the result is A. (We also refer to $\sqrt[3]{A}$ as cube root of A).

Notice that this definition is much simpler than the previous one. If we square 3 and -3, we get 9 in both cases. For this reason, there are two candidates for $\sqrt{9}$ and no candidate for $\sqrt{-9}$. Third roots behave more pleasantly. If we cube (same as raise to the third power) 3 and -3, we get 27 and -27. Thus, cube roots exist of both positive and negative numbers, and there is no ambiguity on the choice of the cube root. Simply $\sqrt[3]{8}$ is 2 and $\sqrt[3]{-8}$ is -2.

Example: Evaluate each of the given numerical expressions.

a $\sqrt[3]{125}$ **b** $-\sqrt[3]{8}$ **c** $\sqrt[3]{-27}$ **d** $-\sqrt[3]{-64}$

Solution:

a $\sqrt[3]{125}=5$ **b** $-\sqrt[3]{8}=-1.\sqrt[3]{8}=-1.2=-2$ **c** $\sqrt[3]{-27}=-3$
d $-\sqrt[3]{-64}=-1.\sqrt[3]{64}=-1.(-4)=4$

e By definition, $\sqrt[3]{2}$ is the number that, when raised to the 3rd power, the result is 2. Also recall the rule of repeated exponentiation:
$(a^m)^n=a^{mn}$
$(\sqrt[3]{2})^{12}=(\sqrt[3]{2})^{3.4}=[(\sqrt[3]{2})^3]^4=2^4=16$

By definition, $\sqrt[5]{10}$ is the number that, when raised to the 5th power, the result is 10. Also recall the rule of repeated exponentiation:
$(a^m)^n=a^{mn}$
$(\sqrt[5]{10})^{15}=(\sqrt[5]{10})^{5.3}=[(\sqrt[5]{10})^5]^3=10^3=1000$

Unit: Exponentials — Topic: Logarithms

Benchmark **10.1.1.4** Use the definition of logarithm to establish and apply the laws.

Learning Objective: By the end of this topic, students will be able to;

- define logarithms and its laws,
- convert index to log and vice versa,
- apply base 10 and laws of logarithms, and
- simplify expressions using the laws of logarithms.

Essential Questions:

1. What is base 10?
2. What are the laws of logarithms?

Key Concepts(ASK-MT)

Attitudes / Values	Confidently use the laws of logarithms in simplifying logarithmic expressions.
Skills	Convert indices to logarithm (vice-versa) and simplify using laws of logarithms.
Knowledge	Laws of logarithms.
Mathematics Thinking	Think about how to apply logarithm laws in solving problems.

Content Background

This topic will help students to understand the link between logarithms and exponents. It will also show them how to switch between the logarithmic and exponential forms of a particular piece of mathematics. Logarithms apply the laws of indices to transform multiplication into addition and division into subtraction.
Logarithms and exponents are closely linked. Just as subtraction is the inverse operation of addition and division is the inverse operation of multiplication, logarithms are the inverse operation of exponentiation, which is raising a number to a power.

Logarithm

Definition of a logarithm: If $x > 0$ and b is a constant $b \neq 1$, then $y = \log_b x$, if and only if $b^y = x$. In the equation $y = \log, x, y$ is referred to as the logarithm, b is the base, and x is the argument.
The notation $\log_b x$ is read "the logarithm (or log) base b of x" The definition of a logarithm indicates that a logarithm is an exponent.

$y = \log_b x$ is the logarithm form of $b^y = x$
$b^y = x$ is the exponential form of $y = \log_b$

Example

1. Write each equation in its exponential form.

a $2 = \log_7 x$ **b** $3 = \log_{10}(x + 8)$ **c** $\log_5 125 = x$

Solution:

Use the definition if and only if

a $2 = \log_7 x$ if and only if $7^2 = x$

b $3 = \log_{10}(x + 8)$ if and only if $10^3 = (x + 8)$

c $\log_5 125 = x$ if and only if $5x = 125$

1. Write the following in its logarithmic form: $x = 25\,(\frac{1}{2})$

Solution:

Use $b_y = x$ if and only if if $y = \log_b x$ if and only if $(\frac{1}{2}) = \log_{25} x$

Equality of exponent theorem: if b is positive real number (b≠1) such that $b^x = b^y$, then $x = y$

Example: Evaluate: $\log_2 32 = x$

Solution: $\log_2 32 = x$ if and only if $2^x = 32$
Since $32 = 2^5$ we have $2^x = 2^5,\ \therefore x = 5$

Properties of Logarithm

If b, a, and c are positive real numbers, $b \neq 1$, and n is a real number, then:

1. Product: $\log_b(a \cdot c) = \log_b a + \log_b c$
2. Quotient: $\log_b(\frac{a}{c}) = \log_b a - \log_b c$
3. Power: $\log_b a^n = n.\log_b a$
4. $\log_b 1 = 0$

Example: Use the properties of logarithms to simplify the following

a $\log_{10} 5 + \log_{10} 4 = \log_{10}(5.4) = \log_{10} 20$

b $\log_{10} 12 - \log_{10} 2 = \log_{10}(\frac{12}{2}) = \log_{10} 6$

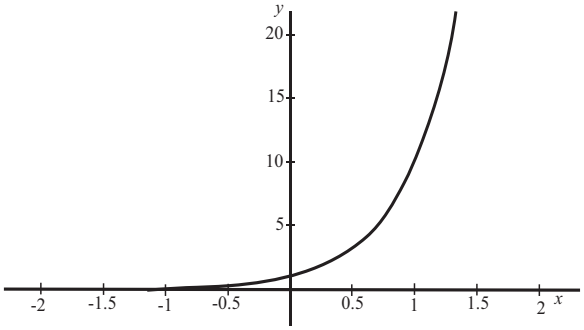
c $\log_{10} 5^3 = 3\log_{10} 5$

d $\log 4x + \log x = \log(4x \cdot x) = \log 4x^2$

e $3\log x - \log x^2 = \log\left(\frac{x^3}{x^2}\right) = \log x$

Logarithms to Base 10 (Common Logarithms)

We will begin by considering the function $y = 10^x$ in the graphed below. Given any number x, we can raise 10 to the power of x to obtain another number which we write as 10^x.



What of the reverse procedure? Suppose we begin with a number and we wish to find the power to which 10 must be raised to obtain that number.
For example, suppose we begin with the number 7 and we wish to find the power to which10 must be raised to obtain 7. This number is called the logarithm to the base 10 of 7 and is written $\log_{10} 7$. Similarly, $\log_{10} 15$ is equal to the power to which 10 must be raised to obtain 15.

For a general number x, $\log_{10} x$ is equal to that power to which 10 must be raised to obtain the number x.
i.e. for any real number $x, \log_{10} 10^x = x$.

Example: Calculate $\log_{10} 10^3$
Solution:
According to the definition, $\log_{10} 10^3$ is equal to that power to which 10 must be raised to obtain 10^3. To what power must we raise 10 to obtain 10^3 ? Or, $10^? = 10^3$. Surely the answer is 3. Notice that 10^3=1000, so we have worked out $\log_{10} 1000$ and without using a calculator! We have been able to work this out because we have understood the meaning of the logarithm of a number.
Similarly; $\log_{10} 1000 = \log_{10} 10^2 = 2$ and $\log_{10} 0{\cdot}1 = \log_{10} 10^{-1} = -1$

Unit: Financial Mathematics — Topic: Earning, Saving and Spending Money

Benchmark **10.1.1.5** Apply the skills of managing money in authentic situations.

Learning Objective: By the end of this topic, students will be able to;

- discuss and identify ways of earning money,
- identify and explain the importance of budgeting, and
- discuss and explain ways of saving and spending money.

Essential Questions:

1. Why is understanding the concept of earning, saving and spending money important?

Key Concepts(ASK-MT)

Attitudes / Values	Display confidence in applying the skills of managing money and appreciate its usefulness.
Skills	Managing money through earning, budgeting, spending and saving money.
Knowledge	Money management.
Mathematics Thinking	Think about how to manage financial needs.

Content Background

Earning money

In order to spend money, money has to be earned and there are many ways in which people earn and spend money. These are classified into 5 main categories: Marketing, Piece work, Wages, Salaries, and Commission.

Example : Marketing
Rea sells Digicel flex cards and earns 15% for every card she sells. In a month she sells the following cards. 400 K3.00 cards 550 K5.00 cards, 60 K10 cards, 30 K20 cards, and 8 K50 cards. All cards were sold at per printed price.

Copy and complete the table, the first one is done for you.

Cards	No sold	Amount earn	Rea's 15%	Digicel 85%
K3.00	400	K1200.00	K180.00	K1020.00
K5.00				
K10.00				
K20.00				
K50.00				
Total	**1048**			

Wages and Salaries

Salaries are fixed amount for each payday over a period of time. There is no over time or commission or any increase in the pay for quality or quantity of work. Salaries are paid every two weeks or monthly income is always the same

Wages, on the other hand, are calculated on the number of hours worked in a week, fortnight or month. Employers pay wages either weekly, fortnightly or monthly, and are linked to how many hours the employee worked.

Example:
Three people were paid the following salaries in a year after other deductions were made.

Douglas - K45 800
Raka - K39 600
Kila - K55 500

Find their; fortnightly salary sp b) monthly salary
e.g.: Douglas K45 800

Fortnightly $= \frac{\text{K45 800}}{26} = \text{K1761.54}$ monthly $= \frac{\text{K45 800}}{12} = \text{K3,816.67}$

Use the example provides to workout Rake and Kila's fortnightly and monthly salaries.

Example:
Mea works for a company from Monday to Saturday, 8am to 5pm. He is paid an hourly of K8.50. He did not go to work on a Wednesday due to family problem. Work out how much he will received at the end of two weeks.

Hourly rate = K8.50
Number of working hours = 9
K 8.50 × 9 = K76.50

For 12 days wages
12 × K76.50 =K918
He missed one day of work so it is deducted from his pay.
K918 – K76.50 = K841.50

Budgeting

A budget is a written plan showing all the income and expenses of an individual, family, group or a business. Budgeting helps individual and businesses to spend money wisely and for the right purpose. Personal budgets usually have fewer items on the income list and many items on the expense list. Businesses have more income and expenditures. Budgets can be made for weekly, Fortnightly, Monthly or even yearly.

Example:
Kippa is a subsistence farmer. He estimated weekly budget

Income	Amount	Estimated expenses Needs	Amount
Kaukau sales	K80		
Potato sales	K60	Store goods	K50.00
Banana sales	K50	Clothes	K40.00
Sugar cane sales	K50	Children's lunch money	K10.00
Vegetable sales	K30	Wants	
		PMV fare	K10
		Medical fee	K20
		Weekend party	K30
		Savings	K60
		Others	K50
Total	**K270**		K270

Saving Money

The best time to buy big and something expensive and still save some money is when discounts are offered. Wise people save money during ordinary times and spend money during discount periods. Christmas, Easter, Mother or Father's days are periods when many business houses offer discounts. Clearance sales of old goods, promotion of new products and business closure are other times when prices of goods and services are reduced. Another good time to buy goods at a reduced or discount price is when the supply of certain items are high and the demand is low. This is when goods can be purchase in bulk and money is saved

Example:

28 % Christmas special discount on the following stock in Brain Bell stores nationwide.

Stock	Old price	Calculation	New Price	Save
Television	K1500	0.72 x 1500	K1,080	K420
Gas stoves	K850	0.72 x 850	K612	K238
Generator	K2400	0.72 x 2400	K1,728	K672

Best and Wise Buying (Spending Money)

Best or wise buying means spending wisely on goods and service for a maximum benefit. That is comparing of prices, comparing of quantity and qualities of goods and services, so that money spent is beneficial.

Example:

Comparing items for price at different shops	Comparing same item unit price and quantity of goods	Comparing quality, size , economical value, durability and purpose
Example i Cost of a sandal Melpa LTD, K25 | Best Buys K18 | Best Buys K21 Same design and brand A wise buyer compares the same goods and services in different shops and then buys the cheapest one. Which of the above is the best and state your reason? Best buy at Best Buys trading because it cost less.	**Example ii** Some goods are packed in different units. Compare unit price before buying. Buying by unit price is cheaper and wise. K10 for 3 biscuits K3 for 1 biscuit Which of the above is the best buy and state your reason? *Solution:* 3 biscuits = K10 which means 1 biscuit = K3.33 Compare K3.33 for 1 biscuit with K5 for 1 biscuit. K5 – K3.33 = K1.67 Best buy is 3 biscuits because more quantity and cost K1.67 less.	**Example iii** Use vehicles and Car 5 passengers and PMV bus 25 passengers Sale K25,990 Sale K30,990 Best buying is also comparing the quality, size, durability and whether or not it is economical to own. Which of the above is best buy and state your reason. Best buy is the bus because it is big, economical, or can earn income, durable, can travel to places cars cannot go, carry more people than the car and the engine and parts are more durable than cars.

Unit: Financial Mathematics **Topic: Consumer Arithmetic**

Benchmark **10.1.1.6** Demonstrate the concepts of simple and compound interest.

Learning Objective: By the end of this topic, students will be able to;

- understand the concept of borrowing and investing money,
- explain what simple interest and compound interest is mean and demonstrate how it is calculated using the formula, and
- apply the simple and compound interest formula to calculate given problems and situations in real life.

Essential Questions:

1. What is simple and compound interest?
2. How is simple and compound interest calculated?

Key Concepts(ASK-MT)

Attitudes / Values	Confidently calculate simple and compound interest.
Skills	Calculate simple and compound interest.
Knowledge	Application of formula of simple and compound interest.
Mathematics Thinking	Think about how to solve problems on simple and compound interest.

Content Background

Borrowing money

Borrowing money means to obtain loan from individuals, business houses, financial schemes or organizations. When money is borrowed an interest is added to the loan during repayments. The amount of interest paid depends on the following;

- The amount of loan known as the principal
- The interest rate – is normally given in percentage
- The type of interest – simple interest or compound interest and
- The length of time on the type of loan, amount loan, loan organization, loan security.

In any financial institution there are certain processes and procedures to follow in order to obtain loans.

Types of loans

Different financial organization offer different types of loan with different interests. Some loans offered in the financial organizations are; Personal loans, School fee loans, Housing loans, Commercial loans, Agriculture loans and Small scale business loans.

Loan Calculations

Example:

Ragan paid K400.00 simple interest for 4 years at 10%.

Calculate:

a. amount loaned.

b. total repayment at the end of 4 years.

c. yearly , monthly and fortnightly loan repayments. Use $I = \frac{prt}{100}$

Solutions:

a. $L = \frac{K400}{4 \times \frac{10}{100}}$, $L = 400 \div \frac{K1400}{40}$, $L = 400 \times \frac{K100}{40}$, $L = \frac{K4000}{40} = K1{,}000$

b. Total repayments

Yearly

TP = K1000.00 + K400.00

TP = K1400

$\frac{K1000 + K400}{4}$, y = $\frac{K1400}{4}$ = K1,000 , y = K350

Monthly

M = $\frac{K1000 + K400}{48}$,M = $\frac{K1400}{4}$, M = K29.17

Fortnightly

F= $\frac{K1000 + K400}{104}$,F= $\frac{K1400}{104}$ F= K350

Simple interest

Simple interest is calculated on the full amount borrowed or invested over a period of time. The amount invested or borrowed is called the principal. The simple interest rate as a percentage is often called the flat rate of the interest. The term of the loan or investment is usually expressed in years, but may be given as any time period. The formula for calculating simple interest is:

$I = Prn$ where I is the amount of interest in Kina
P is the principal, the amount borrowed or invested
r is the percent interest rate per time period, expressed as decimal
n is the number of time periods

Example

An amount of K4090 is invested in a savings account which pays simple interest for 4 years at a flat rate of 8% per annum.

1. Calculate the balance accumulated by the end of 4 years.

2. The total interest earned after 4 years.

Solution:

$I = Prn$, where $P = 4090$
$r = 8 \div 100 = 0.08$
$n = 4$

Therefore, I = 4090 x 0.08
= K327.20

Interest after 4 years = 4 x K327.20 = K1, 308.8

The value of the investment after 4 years K5398.80

Simple interest Graphs

A mathematical model for simple interest may be developed using a graph that compares the interest and the rate for fixed amount of money over a number of years.

Example:

Draw a graph showing the amount of interest earned over a period of 10 years if K1000 is invested at the rate of 8% *pa*.

1. Find the interest after 8 years.
2. Find the time or period in which the earned interest is K550.

Use the formula that calculate simple interest and substitute the values,

$I = Prn$

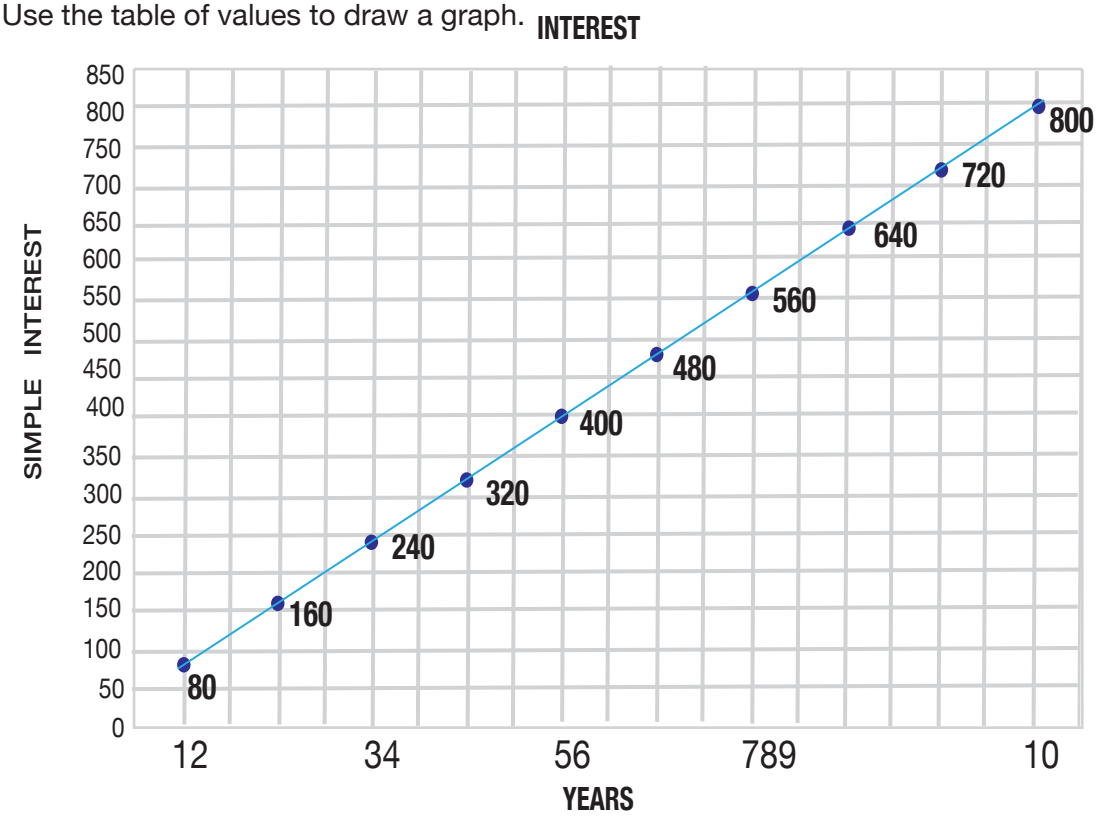
$= 1000 \times 0.08 \times n$

$= 80n$

Use a table to find the values

n	1	2	3	4	5	10
I	80	160	240	320	400	800

Use the table of values to draw a graph.



a. From the graph the interest after 8½ years is K680.

b. The time taken to earn interest of K550 is a little less than 7 years.

Compound interest and application

When money is invested at compound interest, the interest earned at the end of each time period is added to the principal amount. This increase in the principal is used to calculate the interest for the next time period. With compound interest you can earn interest on the interest.

Example:

An amount of K4500 is invested in a savings account which pays simple interest at a rate of 7.5% per annum. Calculate the balance accumulated by the end of 2 years and total interest earned.

Solution:

First year

P = principal amount = K4500

Interest rate = 7.5% of K4500

$$= \frac{75}{100} \times K4500$$

Interest at 7.5% $p.a$ = K337.50

When this interest is reinvest for the 2nd year, the principal is K4500 + K337.50 = K4837.50.

Second year

P = principal amount = K4837.50

Interest rate = 7.5% of K4837.50

$$= \frac{75}{100} \times K4837.50$$

Interest at 7.5% $p.a$ = K362.80

1. Calculate the balance accumulated by the end of 2 years.

2. What is the total interest earned after 2 years?

Solution:

The balance of the investment after 2 years

1. *K4500 + K337.50 + K 362.80 = K5200.30* or *K362.80 + K4837.50 = K5200.30*

2. *K5200.30 - K4500 = K700.30, or K337.50 + K362.80 = K700.30*

Example

K6, 000 is invested in a savings account for 3 years which pays a compound interest rate of 9% $p.a$.

a. Calculate the balance accumulated by the end of 3 years.

b. The amount of interest earned.

Time	Working	Total balance	Interest at the end of year
Start of first year		K6,000	$\frac{9}{100}$ × 6,000 = K540
Start of 2nd year	K6,000 + K540	K6540	$\frac{9}{100}$ × 6,540 = K588.60
Start of 3rd year	K,6540 + K588.60	K7,128.60	$\frac{9}{100}$ × 7 128.60 = K641.57
The accumulated amount is K7,128.60 + K641.57 = K7,770.17			

Compound interest formula is

$A = P\ (= r)2$ where , A The amount is the final balance or future value

P the principal is the initial value invested or present value

r the interest rate per compound period as a decimal

r the number of time periods.

The amount of interest earned is = K7, 770.17 – K6000 = K1, 770.17

Using the compund Interest formula

Example:

a. Use the compound interest formula to calculate an investment of K6000 over 4 years at 5,5% pa interest compounding yearly.

b. Find the total interest earned

Solution:

$A = P(1+r)n$ where

P = K6,000

R= 0.055, as the compounding period is annual, the interest rate is annual rate

N= 4, as the compound period is annual the number of time period is the same as the number of years

$A = 6000 \times (1 + 0.055)^4$

$A = 6000 \times (1.055)^4$

A= K7432.95

a. K7432.95

b. Interest = K7,432.95 – K6,000
= K 1, 432.95

Unit: Financial Mathematics — Topic: Simple and compound interest using digital technologies

Benchmark **10.1.1.7** Solve problems involving simple interest and compound interest even by using appropriate digital technologies.

Learning Objective: By the end of this topic, students will be able to;

- calculate simple interest Problems using Microsoft Excel,
- calculate compound interest Problems using Microsoft Excel.

Essential Questions:

1. What digital technology can be used to calculate simple and compound interest?
2. What is Microsoft Excel?
3. How do you calculate simple and compound interest using Microsoft Excel?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate and enjoy the use of digital software to calculate simple and compound interest.
Skills	Use Microsoft Excel to calculate simple interest and compound interest problems
Knowledge	Use of Microsoft Excel to calculate simple and compound interest.
Mathematics Thinking	Think about how to calculate simple and compound interest using a Microsoft Excel.

Content Background

Calculate simple and compound interest using digital technology

Apply knowledge and skills of Microsoft Excel and calculate simple interest problems.

Example:
An amount of K1500 is invested in a savings account which pays simple interest for 8 years at a flat rate of 12% per annum.

1. Calculate the interest after 8 years.
2. Graph the interest over the 8 years period.
3. Calculate the balance accumulated each year and over the end of 8 years.

Enter the following data and calculations in each of the following cells on the spreadsheet:

- Principle = enter 1500 in cell A3 to A10
- Rate = enter 0.12 in cell B3 to B10 ($\frac{12}{100} = 0.12$)
- Time = enter each number (1 to 8) in each cell (C3 to C10).
- Simple Interest = enter the formulae **(= A3*B3*C3)** in cell D3 and press Enter key on the keyboard. Next enter **(= A4*B4*C4)** in cell D4 and press Enter. Continue entering the other formulas until the last formulae in cell D10.

	A	B	C	D	E	F
1						
2	PRINCIPLE	RATE	TIME	SIMPLE INTEREST	BALANCE	
3	1500	0.12	1	= A3*B3*C3	= A3+D3	
4	1500	0.12	2	= A4*B4*C4	= A4+A4	
5	1500	0.12	3	= A5*B5*C5	= A5+D5	
6	1500	0.12	4	= A6*B6*C6	= A6+D6	
7	1500	0.12	5	= A7*B7*C7	= A7+D7	
8	1500	0.12	6	= A8*B8*C8	= A8+D8	
9	1500	0.12	7	= A9*B9*C9	= A9+D9	
10	1500	0.12	8	= A10*B10*C10	= A10+D10	
11						

Solution:

1. Calculate the interest after 8 years.

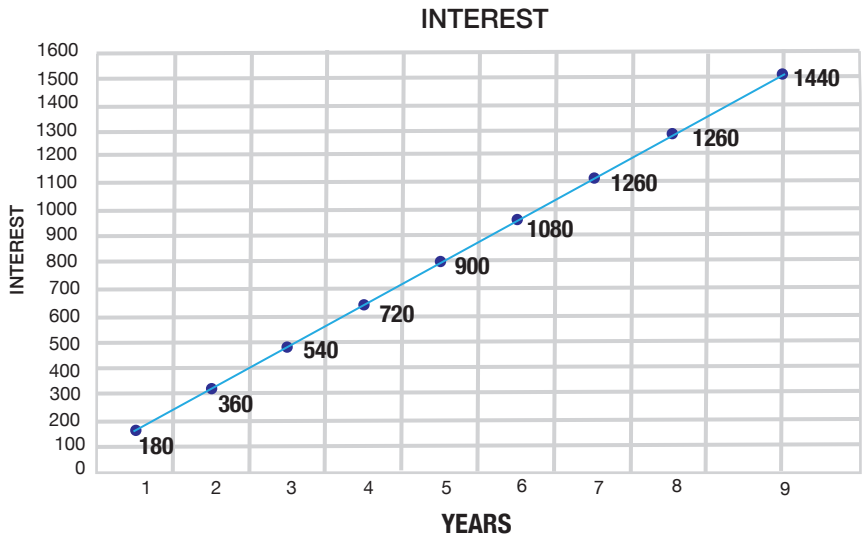
	A	B	C	D	E	F
1						
2	PRINCIPLE	RATE	TIME	SIMPLE INTEREST	BALANCE	
3	1500	0.12	1	180	1680	
4	1500	0.12	2	360	1860	
5	1500	0.12	3	540	2040	
6	1500	0.12	4	720	2220	
7	1500	0.12	5	900	2400	
8	1500	0.12	6	1080	2580	
9	1500	0.12	7	1260	2760	
10	1500	0.12	8	1440	2940	
11						

Result of the calculation of the accumulated simple Interest per year is shown above. K180 is the simple interest after the first year. K1440 is the total interest after eight (8) years when 1500 is invested at a rate of 12% per annum.

2. Graph the interest over the 8 years period

Time (n)	1	2	3	4	5	6	7	8
Simple Interest (I)	180	360	540	720	900	1080	1260	1440

Note: Graph can also be drawn using Microsoft Excel



3. Calculate the balance accumulated each year and over the end of 8 years. The balance accumulated at the end of the first year is K1680 and after the end of 8 years is K2940.

Strand 2: Geometry, Measurement and Transformation

Content Standard: Students will be able to comprehend the meaning and significant of geometry, measurements and spatial relationship including units and system of measurement and develop and use techniques, tools, and formulas for measuring the properties of objects and relationships among the properties and use

Units	Benchmark	Topics	Lesson Titles
Trigonometry	**10.2.2.1** Investigate and Apply the Pythagorean Theorem to solve problems.	Pythagorean Theorem	Right angle triangle
			Pythagorean Theorem
			Calculating the length of the hypotenuse using pythagoras rule
			Calculating shorter sides using pythagoras rule
			Problem solving involving right angle triangle
	10.2.2.2 Investigate sine, cosine and tangent ratios for a given angle in right-angled triangles.	Trigonometric ratios	Introduction to Trigonometric ratios
			Using the sine ratio to find the length of sides
			Using the sine ratio to find angle values
			Using the cosine ratio to find the length of sides
			Using the cosine ratio to find angle values
			Using the tangent ratio to find the length of sides
			Using the tangent ratio to find the angle values
	10.2.2.3 Apply trigonometry to solve right-angled triangle problems.	Trigonometric problems	Mixed Problems with diagrams on Trigonometric Ratios
			Word Problems on Trigonometric Ratios
	10.2.2.4 Solve right-angled triangle problems including those involving direction and angles of elevation and depression.	Direction and angles of elevation and depression	Direction (compass direction)
			Bearing
			Angle of elevation and depression
			Problem solving
	10.2.2.5 Establish the sine, cosine and area rules for non-right angled triangle and solve related problems.	Sine, cosine and area rule for non-right angled triangle	Sine rule
			Cosine rule
			Area rule
			Problem Solving

Unit: Trigonometry — Topic: Pythagorean Theorem

Benchmark **10.2.2.1** Investigate and apply the Pythagorean Theorem to solve problems.

Learning Objective: By the end of this topic, students will be able to;

- understand the properties of a right angle triangle,
- state pythagoras' theorem,
- calculate the sides of a right angle triangle, and
- use pythagoras' theorem to solve problems involving right-angled triangles.

Essential Questions:

1. What are the properties of a right angle triangle?
2. How do we describe a right angle triangle using the pythagoras' theorem?
3. How do we calculate unknown sides of a right angle triangle using the pythagoras' theorem?

Key Concepts(ASK-MT)

Attitudes / Values	Appreciate the application of pythagorean theorem to real life problems.
Skills	Apply concept of pythagorean theorem to calculate unknown sides and apply in problem solving.
Knowledge	Properties of a right-angle triangle and pythagorean Theorem.
Mathematics Thinking	Think about how to solving problems using right angled-triangle.

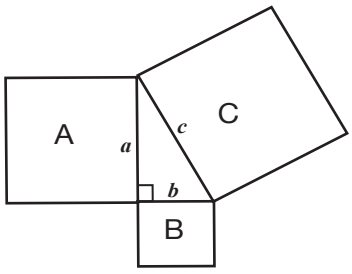
Content Background

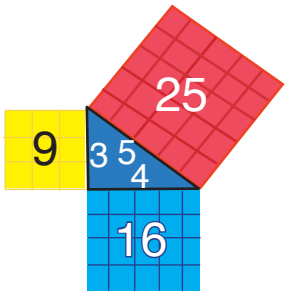
Trigonometry

Trigonometry is the study of triangles and application of trigonometric functions to determine unknown angles and sides of a triangle. It is important for measuring and calculating inaccessible height, length and angles in real life situations such as surveying and navigation.

Pythagorean Theorem

Pythagoras was a Greek philosopher and mathematician who lived during the 7th Century ***bc***. The Pythagorean Theorem, however, was known much earlier, in the 17th Century ***bc***.
What is this famous theorem and what do we use it for?
The Pythagorean Theorem states that when a triangle has a right angle (90°) and squares are made on each of the three sides then the biggest square has the exact same area as the other two square put together as shown. The square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides. A triangle with side lengths of a, b, c which satisfy $c^2 = a^2 + b^2$ is a right –angled triangle.





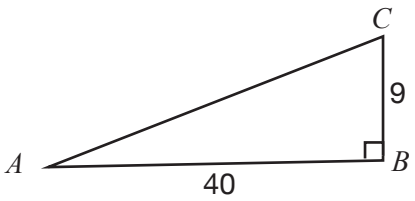
Example
A "3, 4, 5" triangle has a right angle in it. Let's check to see the areas are the same: $3^2+4^2=5^2$
When this is calculated = 9 + 16 = 25
Proofing can be done this way.

Using Pythagoras rule to find the hypotenuse

Example: In the right angled triangle

AB = 40 cm. BC = 9 cm.

What is the length of the hypotenuse AC?



Solution: $AC^2 = AB^2 + BC^2$

$AC^2 = 40^2 + 9^2$

$AC^2 = 1600 + 81$

$AC^2 = 1681$

$AC = \sqrt{1681}$

$AC = 41$

Using Pythagoras rule to find the shorter side

Example: In the right triangle PQR

PQ = 17 cm and QR = 8 cm

What is the length of RP?

Solution:

$PQ^2 = QR^2 + RP^2$

$17^2 = 8^2 + RP^2$

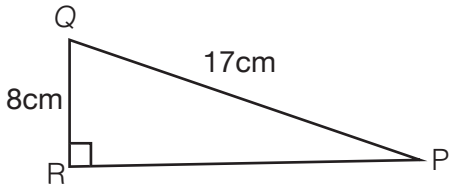
$289 = 64 + RP^2$

$RP^2 = 289 - 64$

$RP^2 = 225$

$RP = 225$

$RP = 15$



So the side RP is 15 cm in length.

Unit: Trigonometry — Topic: Trigonometric Ratios

Benchmark **10.2.2.2** Investigate sine, cosine and tangent ratios for a given angle in right-angled triangles.

Learning Objective: By the end of this topic, students will be able to;

- understand the properties of a right-angled triangle in terms of the 90^0 and hypotenuse which is opposite the right angle,
- understand the properties of sine, cosine, and tangent ratios, and
- apply sine and cosine ratios to find length of sides and apply tangent ratio to find angle values.

Essential Questions:

1. What is a right-angled triangle?
2. What are the properties of sine, cosine and tangent ratios?
3. When do we apply sine and cosine ratios to find length of sides and apply tangent ratio to find angle values?

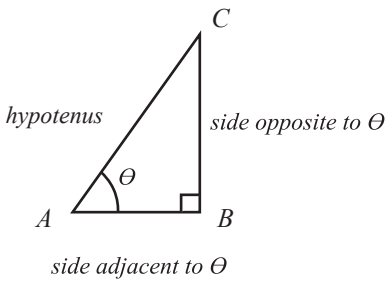
Key Concepts(ASK-MT)

Attitudes / Values	Appreciate the use of trigonometric ratios to find unknown sides and angles.
Skills	Apply sine, cosine and tangent ratios to calculate unknown sides and angles.
Knowledge	Properties of a right angled triangle and trigonometric ratios.
Mathematics Thinking	Think about how to use sine, cosine and tangent ratios in authentic situations.

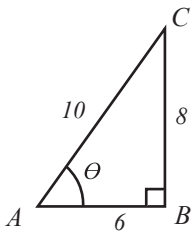
Content Background

SINE, COSINE AND TANGENT RATIOS

Study the right-angled triangle ABC shown below.



The side opposite the right-angle is called the hypotenuse. The side opposite to θ is BC. The remaining side, AB, is said to be adjacent to θ. Suppose we know the lengths of each of the sides as in the figure shown below.



We can then divide the length of one side by the length of one of the other sides. The ratio $\frac{AC}{BC}$ is known as the **sine** of angle θ. This is abbreviated to sin θ. In the triangle shown we see that

$Sin\ \theta = \frac{8}{10} = 0.8$

The ratio $\frac{AC}{BC}$ is known as the **cosine** of angle θ. This is abbreviated to cos θ. In the triangle shown we see that $Cos\ \theta = \frac{6}{10} = 0.6$

The ratio $\frac{BC}{AC}$ is known as the **tangent** of angle θ. This is abbreviated to tan θ. In the triangle shown we see that $Tan\ \theta = \frac{8}{6} = 1.3333$

In any right-angled triangle we define the trigonometrically ratios as follows:

$$Sin\ \theta = \frac{Opposite}{Hypotenuse} = \frac{BC}{AC}$$

$$Cos\ \theta = \frac{Adjacent}{Hypotenuse} = \frac{AB}{AC}$$

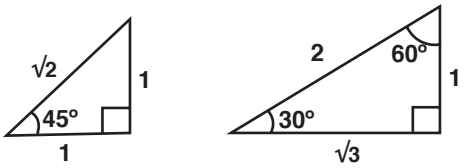
$$Tan\ \theta = \frac{Opposite}{Adjacent} = \frac{BC}{AB}$$

A Helpful Mnemonic for Remembering the Ratios:

SOHCAHTOA

Sine is **O**pposite over **H**ypotenuse
Cosine is **A**djacent over **H**ypotenuse
Tangent is **O**pposite over **A**djacent

Some standard, or common, triangles



$$sin45^\circ = \frac{1}{\sqrt{2}},\ sin30^\circ = \frac{1}{2},\ sin60^\circ = \frac{\sqrt{3}}{2}$$

$$cos45^\circ = \frac{1}{\sqrt{2}},\ cos30^\circ = \frac{\sqrt{3}}{2},\ cos60^\circ = \frac{1}{2}$$

$$tan45^\circ = 1,\ tan30^\circ = \frac{1}{\sqrt{3}},\ tan60^\circ = \sqrt{3}$$

The trigonometric ratios can be used to find unknown length of sides and angles.

Example 1: Calculate the value of sin 50° , cos 50° and tan 50° in the triangle given.

a. hypotenuse = 63mm, opposite = 48mm, adjacent = 40mm

b.

$$sin50^\circ = \frac{O}{H} = \frac{48}{63} = 0.76$$

$$cos50^\circ = \frac{A}{H} = \frac{40}{63} = 0.63$$

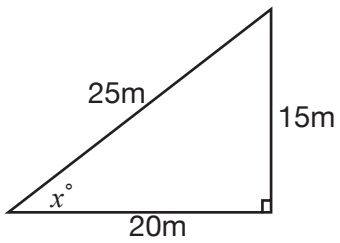
$$tan50^\circ = \frac{O}{A} = \frac{48}{40} = 1.2$$

Example 2: Calculate the value of sin x°, cos x°, and tan x° in the triangle alongside, which is not draw to scale.

$$sin\ x^\circ = \frac{O}{H} = \frac{15}{25} = \frac{3}{5} \text{ or } 0.6$$

$$cos\ x^\circ = \frac{A}{H} = \frac{40}{25} = \frac{4}{5} \text{ or } 0.8$$

$$tan\ x^\circ = \frac{O}{A} = \frac{15}{24} = \frac{3}{4} \text{ or } 0.75$$



Unit: Trigonometry **Topic: Trigonometric Problems**

Benchmark **10.2.2.3** Apply trigonometry to solve right-angled triangle problems.

Learning Objective: By the end of this topic, students will be able to;

- solve right angled triangle problems, and
- apply Pythagoras' Theorem and trigonometric ratios to solve problems involving right triangles.

Essential Questions:

1. How is trigonometric used to solve problems?
2. where is Pythogoras theorem and trigonometry ratios used in solving problem?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate the application of Pythagoras' Theorem and Trigonometric Ratios to solve problems.
Skills	Apply pythagorean theorem and trigonometric ratios to solve problem.
Knowledge	Application of pythagorean theorem and trigonometric ratios in solving problems.
Mathematics Thinking	Think about how to apply trigonometry in real life situations.

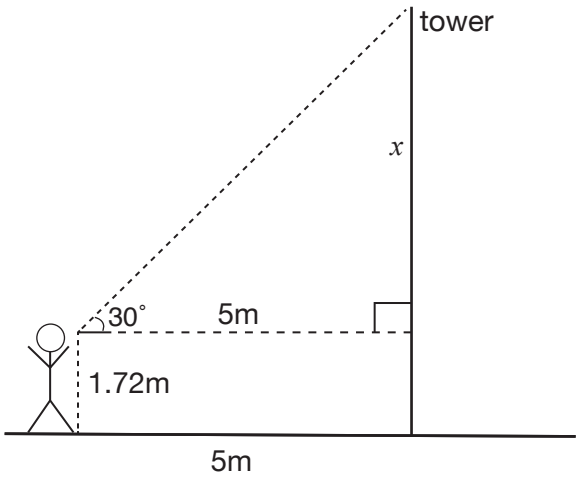
Content Background

Trigonometric Problems

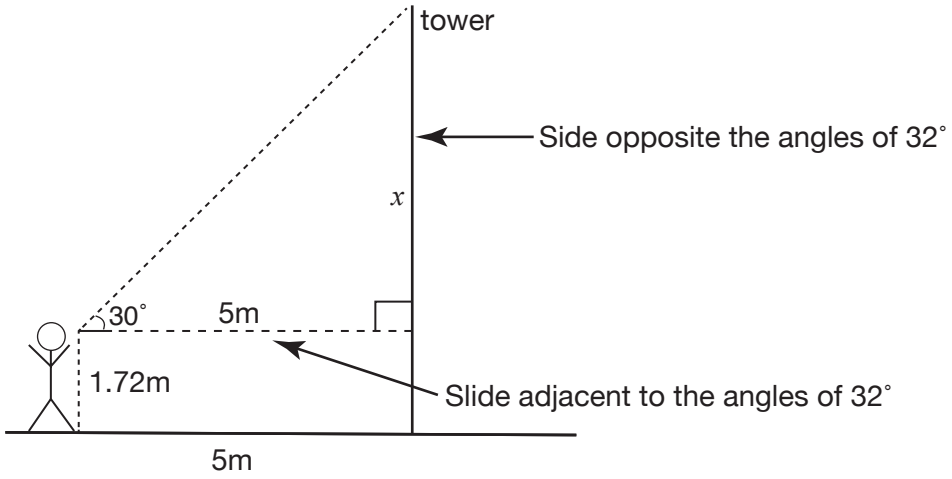
Example

The very practical area of surveying is the problem of finding out the heights or the length of objects, when you cannot actually measure it, perhaps the height of a tower.

So suppose a surveyor wants to know the height of the tower in the diagram below. The angle has been found to be 32° as shown. The surveyor is 5m away from the tower. Suppose the height of the surveyor doing the surveying is 1.72m. How high is the tower?



The side opposite of angle 32° is shown in the Figure below. The adjacent side is 5m. So what trigonometry ratio relates to the opposite and adjacent?' The answer is the tangent ratio.



Let the length of the opposite side be x. Then

$$\frac{OPP}{AJD} = tan32^{\circ}$$

$$\frac{x}{5} = tan32^{\circ}$$

$$x = tan32^{\circ}$$

$$x = 3.124$$

Finally the height of the tower can be found by adding on the height of the surveyor:

Height = 3.124 + 1.72
= 4.844 m
= 4.8 (2 s.f.)

So, the height of the tower is 4.8m.

Unit: Trigonometry | **Topic: Direction and Angles of Elevation and Depression**

Benchmark **10.2.2.4** Solve right-angled triangle problems including those involving direction and angles of elevation and depression.

Learning Objective: By the end of this topic, students will be able to;

- give direction using compass direction as N, S, E, W or NE, SE, SW, NW,
- provide direction using bearing which is measured by a clockwise rotation from the True North direction, and
- identify sides of right angle triangles and calculate angles of elevation and depression.

Essential Questions:

1. How do we read directions using compass?
2. What is bearing?
3. What is angle of elevation and depression?
4. How do we solve problems involving the concepts of compass direction, bearing, and angle of elevation and depression?

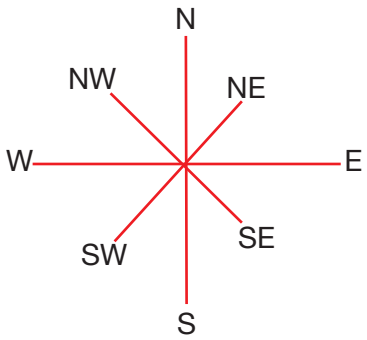
Key Concepts(ASK-MT)

Attitudes / Values	Confidently read directions and enjoy calculating bearings and angles angle of elevation and depression.
Skills	Use a compass to locate directions, measure and provide directions using bearing.
Knowledge	Compass directions, bearing and angle of elevation, and depression.
Mathematics Thinking	Think about how to use the compass bearing in real life situations.

Content Background

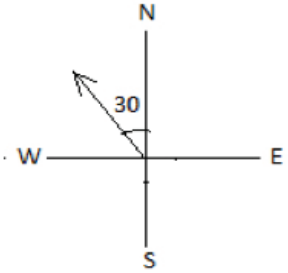
Direction

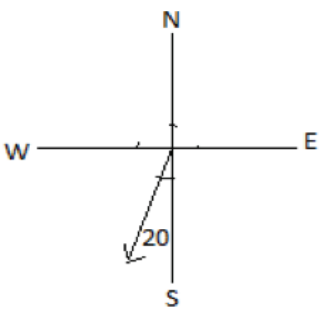
One way of giving direction is to use the main eight points of the compass as N, S, E, W, NE, SE, NW and SW.



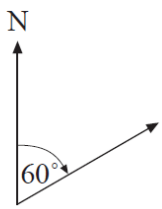
Bearing

More accurate method of giving direction is to use bearing. Direction in bearing can be given using two different ways.
First, bearing is given as an angle measured from north or south in an easterly or a westerly direction such N30°W or S20°W as shown in the diagrams below.

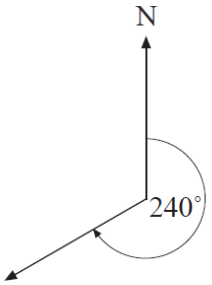




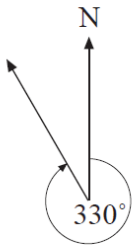
Secondly, bearing is the angle measured from the true north going clockwise and the angle is written using three digits.



Bearing 060°



Bearing 240°



Bearing 330°

Example:

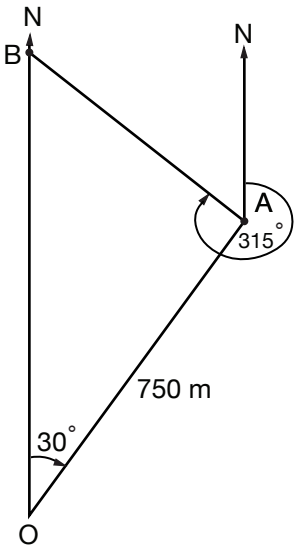
A man walks 750 m on a bearing of 030°. He then walks on a bearing of 315° until he is due north of his starting point, and stops.

a. How far does he walk on the bearing of 315°?

b. How far is he from his starting point when he stops?

Solution:

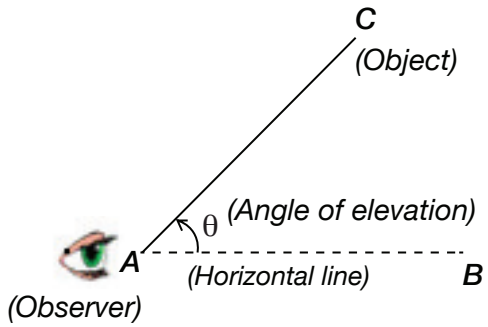
A scale can be produced, using a scale of 1cm to 100 m.

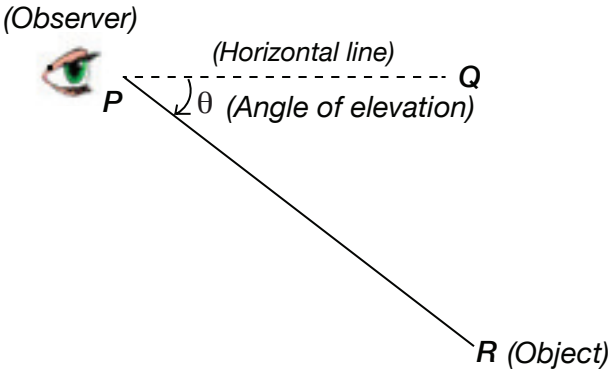


Scale : 1 cm = 100 m

- The distance AB can be measured as 5.4 cm, which represents an actual distance of 540 m.
- The distance OB can be measured as 10.2 cm, which represents an actual distance of 1020 m.

Angle of Elevation and Depression



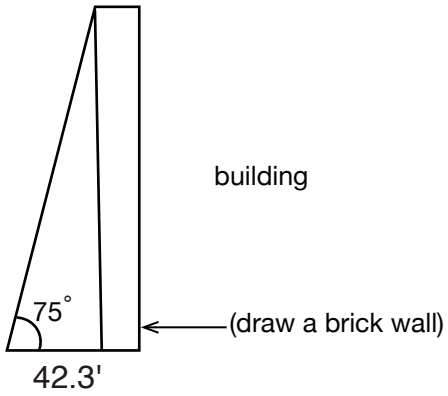


The angle of elevation is the angle between a horizontal line from the observer and the line of sight to an object that is above the horizontal line. In the diagram above, AB is the horizontal line. θ is the angle of elevation from the observer at A to the object at C.

The angle of depression is the angle between a horizontal line from the observer and the line of sight to an object that is below the horizontal line. In the diagram above, PQ is the horizontal line. θ is the angle of depression from the observer at P to the object at R.

Example

At a point 42.3 feet from the base of a building, the angle of elevation of the top is 750°. How tall is the building?



Solution:

$$\tan 75^0 = \frac{Height}{42.3}$$

$$(42.3)(tan75^0) = Height$$

$$(42.3)(3.7321) = Height$$

$$157.87 = Height$$

Therefore the height of the Building is 157.87 feet.

Unit: Trigonometry	Topic: Sine, Cosine and Area Rule for Non-Right angled triangle
Benchmark	**10.2.2.5** Establish the sine, cosine and area rules for non-right angled triangle and solve related problems.

Learning Objective: By the end of this topic, students will be able to;

- use sine rule, cosine rule and area rule solve non-right angled triangle, and
- calculate unknown angles and sides of non-right angled triangle.

Essential Questions:

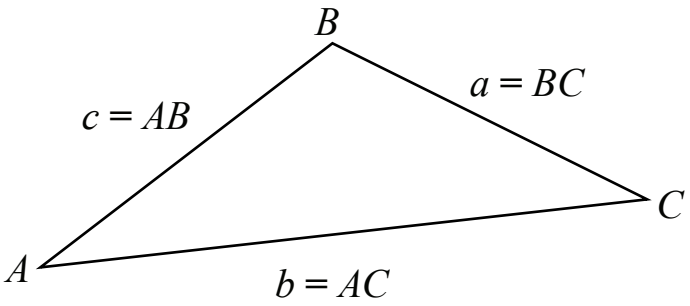
1. What is sine, cosine, and area rule for non-right angled triangle?
2. How is sine, cosine and area rules applied to solve non-right angled triangle problems?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate the application of sine, cosine, and area rule to solve related problems that involved non-right angled triangles.
Skills	Use the appropriate sine, cosine and area rule to solve related problems that involved non-right angled triangles.
Knowledge	Sine rule, cosine rule, and area rule and know when to apply them.
Mathematics Thinking	Think about how to apply sine rule, cosine rule or area rule to solve related problems that involved non-right angled triangles.

Content Background

Sine rule

Study the triangle ABC shown below. Let B stands for the angle at B. Let C stand for the angle at C and so on. Also, let $b = AC$, $a = BC$ and $c = AB$.



In any $\Delta\ ABC$ The SINE rule states:

$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

In any triangle, the sides are proportional to the sine of the opposite angles.

The Sine rule is used to solve problems when given:

i. Two angles and one side **ii.** Two sides and the angle opposite one of them

Example

In triangle ABC, $B = 21°$, $C = 46°$, and $AB = 9$cm. Solve this triangle.

Solution:

We are given two angles and one side and so the sine rule can be used. Furthermore, since the angles in any triangle must add up to180° then angle A must be113°. We know that $c = AB = 9$. Using the sine rule

$$\frac{a}{\sin 113^0} = \frac{b}{\sin 21^0} = \frac{9}{\sin 46^0}$$

So, $\frac{b}{\sin 21^0} = \frac{9}{\sin 46^0}$ from which $b = \sin 21^0 \times \frac{9}{\sin 46^0} = 4.484$ cm (3dp)

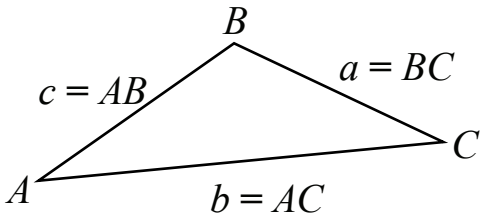
Similarly, $a = \sin 113^0 \times \frac{9}{\sin 46^0} = 11.517$ cm (3dp)

Cosine rule

The Cosine rule is an alternative formula for 'solving a triangle' ABC. It is particularly useful for the case where the Sine rule cannot be used, i.e. when two sides of the triangle are known together with the angle between these two sides.

In any ΔABC The COSINE rule states:

$a^2 = b^2 + c^2 - 2bc\ CosA$	$cosA = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = c^2 + a^2 - 2ca\ CosB$	$cosB = \dfrac{c^2 + a^2 - b^2}{2ca}$
$c^2 = a^2 + b^2 - 2ab\ CosC$	$cosC = \dfrac{a^2 + b^2 - c^2}{2ab}$



Example

In triangle ABC, AB = 42cm, BC = 37cm and AC = 26cm. Solve this triangle.

Solution:

We are given three sides of the triangle and so the cosine rule can be used. Writing a = 37, b = 26 and c = 42.

we have

$$a^2 = b^2 + c^2 - 2bc\ CosA$$

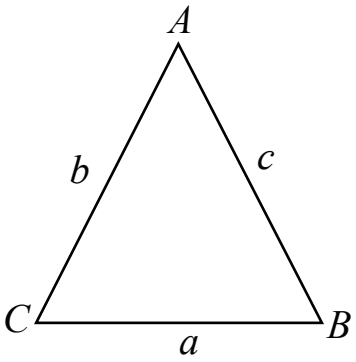
$$37^2 = 26^2 + 42^2 - 2(26)(42)CosA$$

$$\cos A = \frac{26^2+42^2-37^2}{2(26)(42)} = \frac{1071}{2184} = 0.4904$$

And so, $A = \cos^{-1}0.4904 = 60.63^{\circ}$

You should apply the same technique to verify that B=37.76° and C= 81.61°. You should also check that the angles you obtain add up to180°.

The Area of a Triangle



The area of a triangle is defined as:

$$\Delta ABC = \frac{1}{2}ab\ Sin\ C = \frac{1}{2}ac\ Sin\ B = \frac{1}{2}bc\ Sin\ A$$

The area of a triangle is half the product of two sides and the sine of the angle they include.

Example:

Find the area of ΔABC given $\angle B = 51^0, a = 17.6\ cm, c = 23.8\ cm$

Solution:

Area of $\Delta ABC = \frac{1}{2}ac\ \sin B = \frac{1}{2} \times 17.6 \times 23.8 \times \sin 51^0$

=162.755

∴ The area of ΔABC is 162.8 cm^2 correct to 1 d.p

Strand 3: Patterns and Algebra

Content Standard: Students will be able to interpret various types of patterns and functional relationships, use symbolic forms to represent, model, and analyze mathematical situations and collect, organize, and represent data to answer questions.

Units	Benchmark	Topics	Lesson Titles
Linear and Quadratic Equations	**10.3.3.1** Solve simple linear and quadratic equations using a range of strategies.	Simultaneous equations	Solution by Substitution Method
			Solution by Elimination Method
			Solution by Graph
		Quadratic equations	Factorization
			Quadratic formulae
			Completing Squares
			Graphs
			Solve linear and quadratic equation by substitution and graph
	10.3.3.2 Describe, interpret and sketch parabolas.	Parabola	Sketch, describe, interpret and parabolas
	10.3.3.3 Explore the connection between algebraic and graphical representations of relations such as simple quadratics and straight lines using digital technology as appropriate.	Representations of relationship between simple quadratic and straight line	Graph using Microsoft Excel
			Relationship between simple quadratic and straight line
Equations and inequality	**10.3.3.4** Solve and sketch linear and quadratic inequalities with two variables using algebraic methods, manipulative or models.	Linear Equations and inequalities with two variables	Sketch using table of values
			Deriving linear equation from the Graphs
			Inequalities
			Sketch the inequality
			Linear Equations and inequalities with two variables
			Problem solving with two variables
			Sketch and verify shaded regions
		Linear and quadratic inequalities	Solve linear and quadratic inequality
			Sketching the quadratic inequality
			Sketching linear and quadratic Inequalities.
			Sketch and verify shaded regions
Polynomials	**10.3.3.5** Add, subtract, multiply, divide and factor first and second-degree polynomials in one variable using concrete modelling.	Polynomials	Introduction of Polynomials
			Properties of Polynomial expressions
			Addition & Subtraction of Polynomials with first and second degrees
			Multiplication & Division of Polynomials with first and second degrees
			Represent Polynomials with concrete or pictorial models.
			Solve problems using algebraic manipulations.

	10.3.3.6 Use factor and remainder theorems to solve polynomials.		Division of Polynomials
			Remainder Theorem
			Factor Theorem
	10.3.3.7 Apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation.	Sketching polynomials	Word problems on factor theorem.
			Sketch polynomials
			Describe features of polynomial curves

Unit: Linear and Quadratic Equations | **Topic: Simultaneous Equations**

Benchmark **10.3.3.1** Solve simple linear and quadratic equations using a range of strategies.

Learning Objective: By the end of this topic, students will be able to;

- substitution method,
- elimination method, and
- graphical method.

Essential Questions:

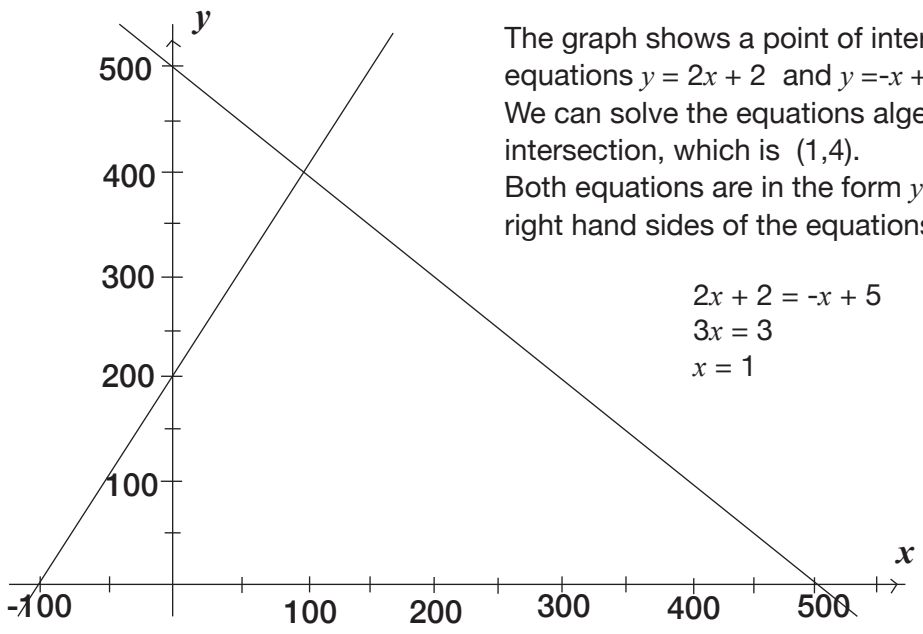
1. What is a simultaneous equation?
2. How is simultaneous equations using substitution, elimination and graphical method solved?
3. What skills are used to solve simultaneous equation using the different methods of solving simultaneous equations?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate the different strategies used to solve problems using substitution, elimination and graphical methods.
Skills	Apply substitution, elimination and graphical method to solve simultaneous linear equations.
Knowledge	Simultaneous linear equations using substitution, elimination and graphical methods.
Mathematics Thinking	Think about how to solve simultaneous linear equations when solving authentic situations.

Content Background

Substitution, Elimination and Graphical Methods for solving simultaneous linear equation with two variables.

The substitution Method



The graph shows a point of intersection for a pair of linear equations $y = 2x + 2$ and $y = -x + 5$.
We can solve the equations algebraically to find this point of intersection, which is (1,4).
Both equations are in the form $y = f(x)$ we can equate the right hand sides of the equations and solve for x.

$$2x + 2 = -x + 5$$
$$3x = 3$$
$$x = 1$$

We can now substitute $x = 1$ into either equation to find y: $y = 2(1) + 2 = 4$. So, we confirm that the point of intersection is (1, 4).

Example:
Solve the pair of simultaneous equations: $3x + y = 13$ and $x + 2y = 1$.

Solution:

$3x + y = 13$ ..... (1)

and

$x + 2y = 1$ ..... (2)

Rewrite equation (1) as:

$y = 13 - 3x$ ..... (3)

Substitute (3) in (2):

$x + 2(13 – 3x) = -5x + 26 = 1$.

So, $-5x = -25$, i.e. $x = 5$.

Now substitute $x = 5$ in equation (1):

$3(5) + y = 15 + y = 13$.

So, $y = -2$.

Check by substituting $x = 5$ and y = -2 into (2): 5 + 2(-2) = 1 as required.

Solving simultaneous equations by the elimination method

A pair of simultaneous equations, $2x – y = -2$ and $x + y = 5$. can solved by taking the sum of the left hand sides and equating it to the sum of the right hand sides as follows:

$2x – y + (x + y) = 3x = 3$.

So $x = 1$

Example:

Solve the pair of equations $5x + 2y = 10$ and $4x + 3y = 15$.

Solution:

$5x + 2y = 10$ .....(1)

$4x + 3y = 15$..... (2)

Choosing to eliminate y, multiply equation (1) by 3 and equation (2) by 2 to get:

$15x + 6y = 30$ ..... (3)

and

$8x + 6y = 30$..... (4)

As the coefficients and the sign of y in each equation are the same, we subtract (4) from (3) to get:

$15x – 8x = 7x = 30 - 30 = 0$.

So, $x = 0$.

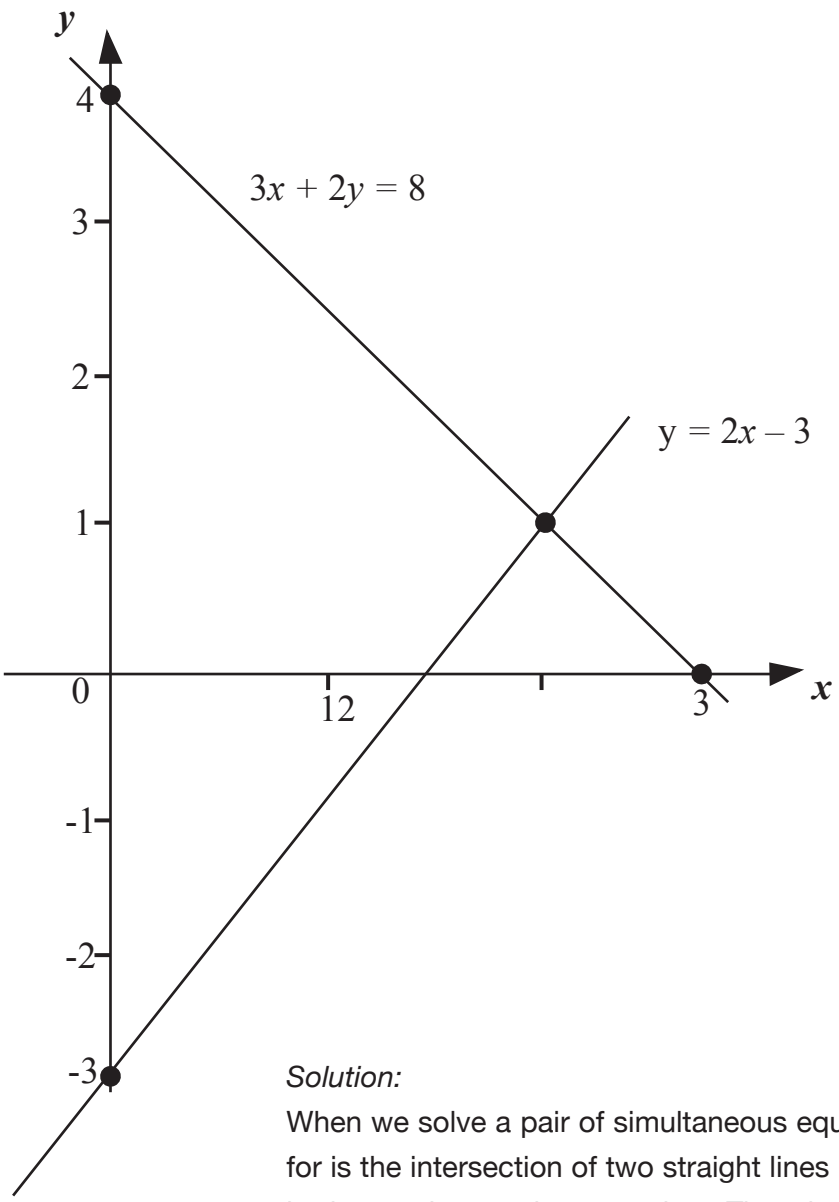
Substituting $x = 0$ into equation (2) we get $4(0) + 3y = 15$ i.e. $y = 5$.

Check by substituting into equation (1): 5(0) + 2(5) = 10 as required.

Solving Simultaneous Equation Graphically

Example:

Solve $3x + 2y = 8$ and $y = 2x - 3$ graphically.



Solution:

When we solve a pair of simultaneous equations what we are actually looking for is the intersection of two straight lines because it is this point that satisfies both equations at the same time. Thus the solution of,

3x + 2y = 8 and y = 2x – 3 is at x = 2, y = 1 or (2, 1)

Unit: Linear and Quadratic Equations | **Topic: Quadratic equations**

Benchmark **10.3.3.1** Solve simple linear and quadratic equations using a range of strategies.

Learning Objective: By the end of this topic, students will be able to;

- expand and factories quadratic expressions,
- solve quadratic expressions, quadratic functions and equations using factorization, quadratic formulae, completing squares and graphs, and
- solve quadratic problems using any method.

Essential Questions:

1. What are quadratic equations and how can they be solve using factorization, quadratic formulae, completing squares and graphs?

Key Concepts(ASK-MT)

Attitudes / Values	Confidently apply strategies in solving simple linear and quadratic equations.
Skills	Calculate and solve linear and quadratic equations, using factorization, quadratic formulae, completing squares and graphs.
Knowledge	Strategies and processes for solving linear and quadratic equations.
Mathematics Thinking	Reflect on the process and procedures applied to solve linear and quadratic equations when solving authentic situations.

Content Background

Factorization

Consider the equation $(x-3)(x-4) = x^2-7x+12$. Going from left to right is called expansion. Going from right to left is called factorization. Once a quadratic is factorized, it is easy to write down the solutions to the corresponding equation.

Example:

Factorize the following

$4z^2 + 6z$ b. $x(x + 3) - 2(x + 3)$

Solution:

$4z^2 + 6z = 2z(2z + 3)$ …(The highest common factor of $4z^2$ and $6z$ is $2z$)

$x(x + 3) - 2(x + 3) = (x + 3)(x - 2)$……(x+3 is the common factor)

Difference of two squares

Recall the identity $(a+b)(a-b) = a^2 - b^2$ which called the difference of two squares. We can use this result other way around to factories an expression that is the difference of two squares.

Example:

Factorize:

a. $3a^2 - 27$ b. $9x^2-16$

Solution:

$3a^2 - 27 = 3(a-9) = 3(a - 3)(a + 3)$

$9x^2-16 = (3x)^2 - 4^2 = (3x + 4)(3x - 4)$

Perfect Squares

A perfect square is an expression such as $(x+3)^2$,$(x-5)^2$ or $(2x+7)^2$. The expansion of a perfect square has a special form

$(x+3)^2 = (x+3)(x+3) = x^2+ 6x + 9 = x^2+ 2 \text{ x } (3x)+3^2$

The sum of the squares of each term plus twice the product of the terms.
In the quadratic $x^2 + 10x + 25$, the coefficient of $x$is 10 and the constant term is 25.
The quadratic $x^2 +10x + 25$ is a perfect square since the constant term is equal to the square of the half of the coefficient of x and so, $x^2+10x + 25= (x+5)^2$
We recognize a perfect square such as this in the following way: The constant term is equal to the square of the half of the coefficient of x.

For example:

$x^2+12x + 36= (x+6)^2$

$x^2-14x + 49= (x-7)^2$

In general:

$a^2 + 2ab + b^2= (a+b)^2$

Similarly

$a^2-2ab + b^2= (a-b)^2$

Factorizing simple quadratics

A simple quadratic expression is an expression of the form $x^2+ bx + c$, where b and c are given numbers
In general, when we expand $(x+p)(x+p)$, we obtain

$x^2+ px + px + pp$

$= x^2+ (p+p)x + pp$

The coefficient of x is the sum of p and q, and the constant term is the product of p and q.

To factorize a simple quadratic, look for two numbers whose sum is the coefficient of x, and whose product is the constant term.

Example:

Factorize $x^2-7x-18$

Solution:

We are looking for two numbers whose product is -18 and whose sum is -7. The numbers -9 and 2 satisfy these conditions. So

$x^2-7x -18 = (x-9)(x+2)$

Sometimes a common factor can be taken out of the quadratic expressions so that the expression inside the brackets becomes a simple quadratic.

Example:

Factorize $2x^2- 4x -96$

Solution:

$2x^2-4x-96 = 2(x^2- 2x- 48)$

$= 2(x-8)(x+6)$

Completing the square

What number must be added to $x^2 + 6x$ to make a perfect square? It is 9, which the square of the half of the coefficient of x. We get $x^2 + 6x + 9 = (x + 3)^2$. This idea is the basis for an important technique called completing square.

The key step is to add and subtract the square of half the coefficient of x. For example to complete the square of $x^2 + 10x - 6$, we add and subtract 25 which of the half of 10.

$$x^2 + 10x - 6 = x^2 + 10x + 25 - 25 - 6$$
$$= (x^2 + 10x + 25) - 31$$
$$= (x + 5)^2 - 31$$

This process is called **completing the square.**

Quadratic Formulae

Using the method of competing squares we can solve the general quadratic equation $ax^2 + bx + c = 0$ and thus develop a formula for solving a quadratic equation in one step.

Solution:

$x^2 + bx + c = 0$

$x^2 + bx = -c$

$x^2 + \frac{b}{a}x = -\frac{c}{a}$ ..... Dividing the coefficient of x^2

$\therefore\ x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$ .....Adding $\left(\frac{b}{2a}\right)^2$ to each side to complete the square

$x + \left(\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a}$ .....Taking square roots on both sides

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$\textit{if } \boldsymbol{ax^2 + bx + c = 0}, \textit{then } \boldsymbol{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

Example:

Solve $x^2 + 2x - 3 = 0$, using the formula.

Solution:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{(2)^2-(4)(1)(-3)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{4--12}}{2}$$
$$= \frac{-1 \pm \sqrt{16}}{2}$$
$$= \frac{-1 \pm 4}{2}$$

$$\therefore x = \frac{3}{2} \quad \text{or} \quad x = -\frac{5}{2}$$

Unit: Linear and Quadratic Equations **Topic: Parabola**

Benchmark **10.3.3.2** Describe, interpret and sketch parabolas.

Learning Objective: By the end of this topic, students will be able to;

- sketch, describe and interpret parabolas.

Essential Questions:

1. What is a parabola and the basic features of parabola?
2. What information is required to sketch a Parabola?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate the ability to solve parabolic equations using graphical strategies.
Skills	Apply graphical method to solve parabolic equations.
Knowledge	Parabolas.
Mathematics Thinking	Think about how to apply graphical strategies to sketch a parabola.

Content Background

Basic Parabolas

A quadratic function is any function of the form $f(x)=Ax^2+Bx+C$ where A,B and C are constants. The thing that sets a quadratic apart is that the degree (the highest power of x) is 2. The graphs of quadratics are parabolas. The highest (or lowest) point of the parabola is called the **vertex.**
The equation $f(x)=Ax^2+Bx+C$ is the standard form for the equation of a quadratic, but the vertex form of a quadratic gives more information.

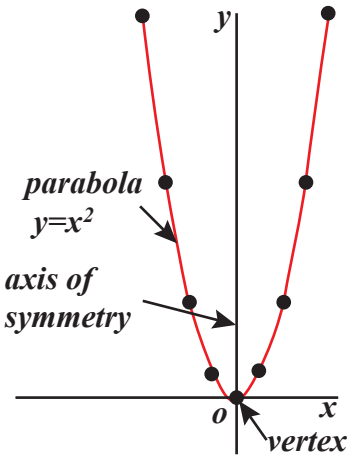
$f(x) = a(x - h)^2+k$

The point (h,k) is the vertex of the parabola. If a > 0 the parabola opens up and if a<0 the parabola opens down.

Example:
Firstly, let us tabulate some pairs of values of x and y which satisfy $y = x^2$:

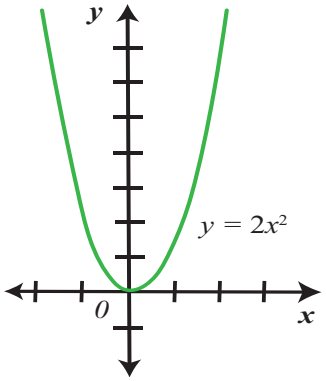
$x =$	-4	-3	-2	-1	0	1	2	3	4
$y = x^2$	16	9	4	1	0	1	4	9	16

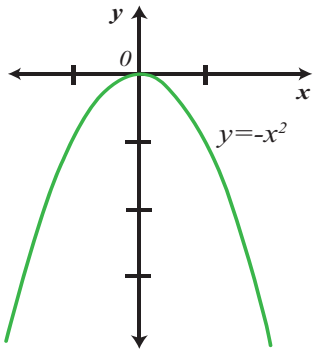
Now plot the points representing these number pairs:

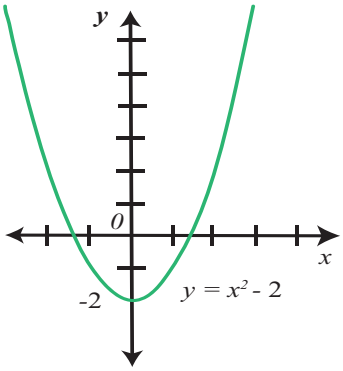


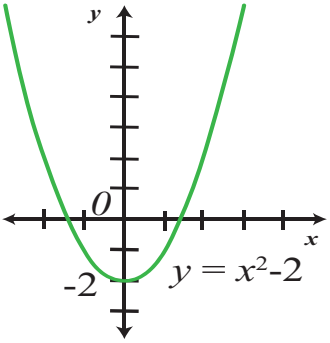
Observe that in the quadratic function $f(x) = x^2$ for the parabola, we have written y instead of $f(x)$ (i.e.,$y = f(x)$.

Some similar parabolas which are clearly related to the standard one drawn in the previous page shown below. Notice that $y = -x^2$ points upwards instead of downward and that $y = -x^2$ has the same general shape and position as $y = x^2$ except that its y-value is always twice as big as the corresponding y value for $y = x^2$, i.e., it is "thinner" (more elongated).









More Parabolas

A graph often gives important information about the function it represents.

Example:

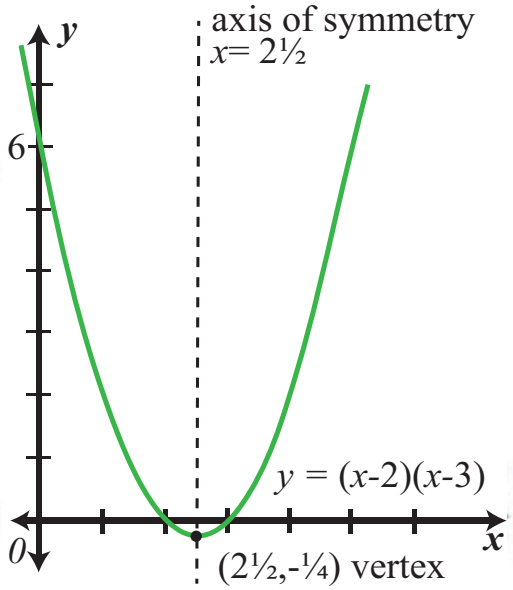
$y = x^2 - 5x + 6 = (x-2)(x-3)$

This expression happens to have factors, but this won't always be the case. As $y = 0$, when $x = 2$ and $x = 3$, this means that the parabola cuts the x-axis ($y = 0$) at x = 2 and $x = 3$. When $x = 0$, the value of y is 6, i.e. the curve cuts the y-axis at $y = 6$.

To get an accurate sketch of the curve, you will have to plot several points. The axis of symmetry will lie between $x = 2$ and $x = 3$, and it is obvious that $x = 2\frac{1}{2}$ is the axis of symmetry. Putting $x = 2\frac{1}{2}$ in the equation of the parabola we obtain

$$y = \left(2\frac{1}{2}\right)^2 - 5\,x\left(2\frac{1}{2}\right) + 6 = -\frac{1}{4}$$

∴ The vertex is the point $\left(2\frac{1}{2}, -\frac{1}{4}\right)$



In general, the equation of the axis of symmetry for a parabola of the form

$y = ax^2 + bx + c$ is $x = -\frac{b}{2a}$

i.e., for $= x^2 - 7x + 6$, the equation of the axis of symmetry is

$x = \frac{-(-7)}{2\,x\,1} = 3\frac{1}{2}$.

Observe that the equation of the parabola drawn may be written $y = \left(x - 2\frac{1}{2}\right)^2 - \frac{1}{4}$ ($y = x^2 - 5x + 6$), which gives us the vertex and axis of symmetry very quickly.

Discriminant of the Quadratic Equation

Discriminant of a quadratic equation: $D = b^2 - 4ac$ is called the discriminant of the quadratic equation $ax^2+bx+c = 0$. The type of root which arises from a quadratic equation depends on the value of the discriminant. Consider the general quadratic equation: $ax^2 + bx + c = 0$

When $b^2 - 4ac > 0$, the equation has two real roots.
When $b^2 - 4ac < 0$, the equation has no real roots.
When $b^2 - 4ac = 0$, the equation has one repeated root.

Unit: Linear and Quadratic Equations	Topic: Representations of relationship between Simple quadratic and straight line

Benchmark	**10.3.3.3** Explore the connection between algebraic and graphical representations of relations such as simple quadratics and straight lines using digital technology as appropriate.

Learning Objective: By the end of this topic, students will be able to;

- graph simultaneous quadratic and linear equation, and
- calculate intersection of linear and quadratic equations using algebra.

Essential Questions:

1. What is simultaneous quadratic and linear equation?
2. What skills and knowledge are required to graph simultaneous quadratic and linear equation and calculate their point of intersection?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate the ability to sketch simultaneous quadratic and linear equation and calculate their point of intersection.
Skills	Solve simultaneous quadratic and linear equations algebraically and graphically.
Knowledge	Graphing Simultaneous quadratic and linear equation.
Mathematics Thinking	Think about how to solve simultaneous quadratic and linear equations algebraically and graphically.

Content Background

Determine whether a Linear Function Intersects a Quadratic Function

Example

Determine algebraically whether the given linear and quadratic functions intersect. If they do intersect, determine the number of points of intersection.

a. $y = 3x + 5$ and $y = 3x^2 - 2x - 4$

b. $y = -x - 2$ and $y = -2x^2 + x - 3$

Solution:

a. Equate the expression and simplify.

$3x^2 - 2x - 4 = 3x + 5$
$3x^2 - 2x - 4 - 3x - 5 = 0$
$3x^2 - 5x - 9 = 0$
$a=3$, $b=-5$ and $c=-9$

Use the discriminant:

$b^2 - 4ac = (-5)^2 - 4(3)(-9) = 25 + 108 = 133$

Since the discriminant is greater than zero, there are two solutions. This means that the linear-quadratic system has two points of intersections.

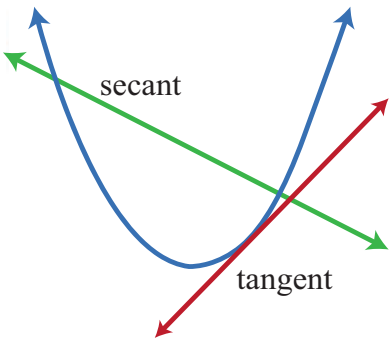
Evaluate the expressions and simplify

$-2x^2 + x - 3 = -x - 2$
$-2x^2 + x - 3 + x + 2 = 0$
$-2x^2 + 2x - 1 = 0$
$a=-2$, $b=2$ and $c=-1$.

Use the discriminant

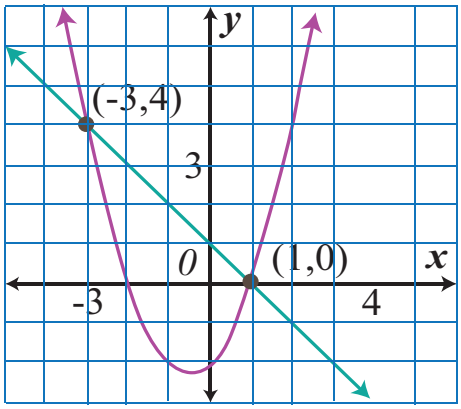
$b^2 - 4ac = (2)^2 - 4(-2)(-1) = 4 - 8 = -4$
Since the discriminant is less than zero, there are no solutions. This means that the linear-quadratic system has no point of intersection.

You have considered how a line can intersect a curve such as a quadratic function. One type of intersection results in a secant and the other results in a tangent line to the quadratic function.



Secant- a line that intersects a curve at two distinct points.
Tangent line - a line that touches a curve at one point and has the slope of the curve at that point.

Solving by Graphing
Solve the following systems by graphing. $y = x^2 + x - 2$
$y = -x + 1$
Graph both equations on the same coordinate plane. Identify the points (s) of intersection, if any.



The points (-3, 4) and (1, 0) are solutions of the systems.

Unit: Equations and Inequalities | **Topic: Linear Equations and Inequalities with two Variables**

Benchmark **10.3.3.4** Solve and sketch linear and quadratic inequalities with two variables using algebraic methods, manipulative or models.

Learning Objective: By the end of this topic, students will be able to;

- sketch using table of values and derive the linear equation from the graphs,
- solve and sketch linear equations and inequalities with two variables, and
- sketch inequalities and verify shaded regions.

Essential Questions:

1. What are linear equations and inequalities?
2. What information is required to sketch inequalities and verify their shaded regions?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate the ability to solve and sketch linear equations and inequalities with two variables and verify their shaded regions.
Skills	Sketch linear equations and inequalities with two variables and verify their shaded regions.
Knowledge	Linear Equations and Inequalities with two Variables.
Mathematics Thinking	Think about how to solve inequalities with two variables.

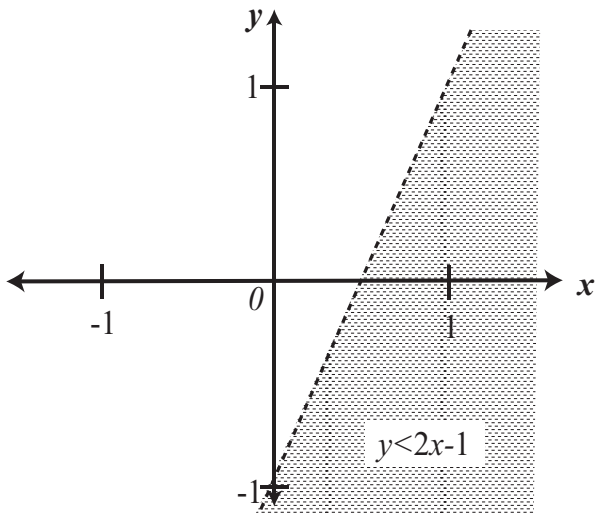
Content Background

Sketch Using tables of values

To sketch the linear equation y=2x-1, we first draw the table of values to get pair of (x,y) coordinates.

x	-3	-2	-1	0	1	2	3
$y = 2x - 1$	-7	-5	-3	-1	1	3	5

The coordinates of the x-intercept and y –intercept are (1/2, 0) and (0, –1) respectively and is shown below.



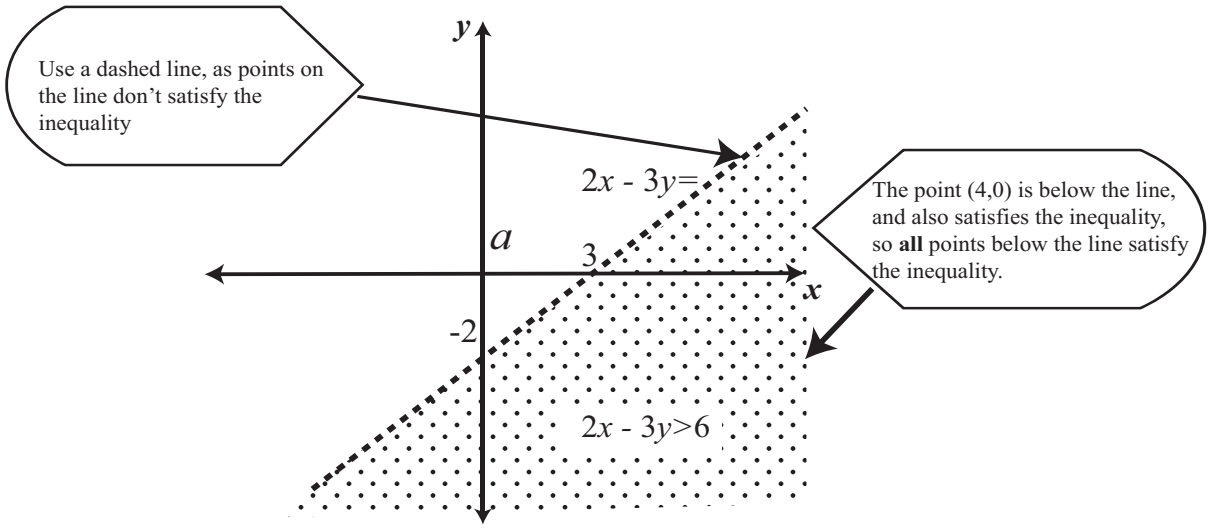
Inequalities

The inequality $y < 2x - 1$ is an example of a linear inequality. The graph of this inequality is the set of all points in the shaded region below. This can bewritten as {(x, y): <2x-1 }.

A straight line divides the coordinate plane into two regions, one below the line and one above it. If you are not sure which region satisfies a linear inequality, just select a point in one region and check if it satisfies the inequality.

Example: Sketch the region satisfying $2x-3y>6$

Solution:
First draw the line which splits the plane into two regions, then decide which region satisfies the inequality.
Put $y = 0$, then $2x = 6 \rightarrow x = 3$.
Put $x = 0$, then $-3y = 6 \rightarrow y = -2$.



The intercepts of $2x - 3y > 6$ are (3, 0) and (0, –2).

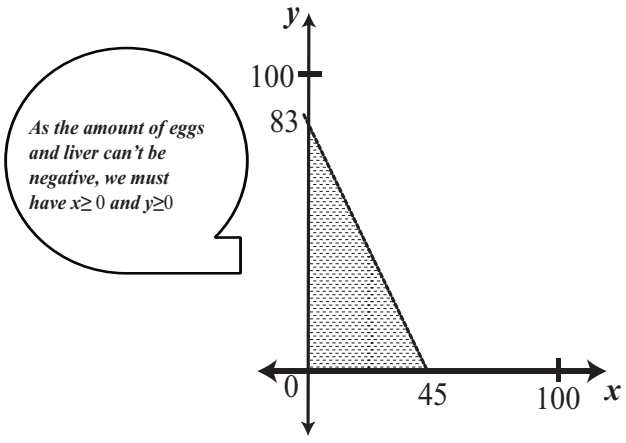
Linear Equations and Inequalities with two variables

Problem Solving

A person on a certain diet should have less than 300 mg of cholesterol per day. It is known that 1gm of whole egg contains 6.6 mg of cholesterol and 1 mg of liver contains 3.6 mg of cholesterol. Find the relationship between the quantities of egg and liver that can be allowed in the diet, assuming that these are the main sources of cholesterol.
Draw a graph showing this relationship.

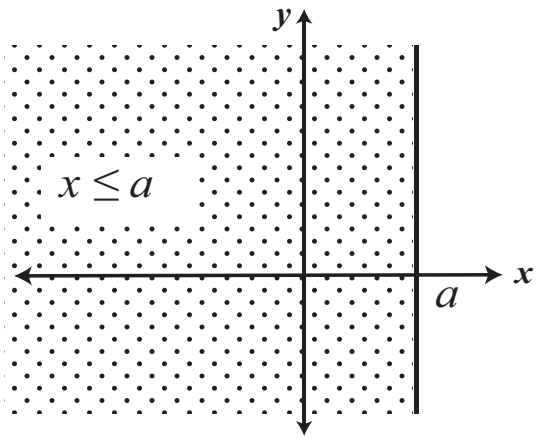
Solution:
a. The relationship.
If x gm of egg and y gm of liver is eaten, then the amount of cholesterol will be $6.6x+3.6y$. This has to be less than 300, so the relationship between the quantities of egg and liver allowed in the diet is be $6.6x+3.6y<300$.

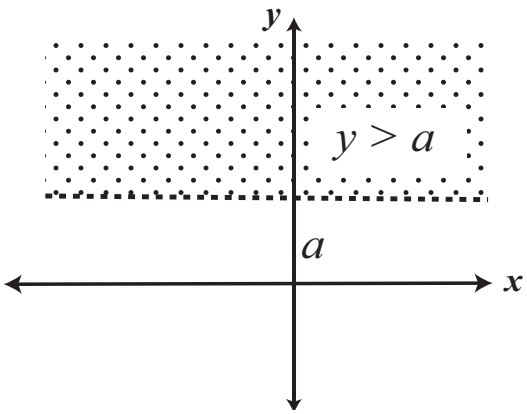
b.The graph of $6.6x + 3.6y < 300$.
Put $x=0$, then $3.6y=300 y=300/3.6 \approx 83$.
Put $y.=0$, then $6.6x=300 x=300/6.6 \approx 45$.
The intercepts are (45, 0) and (0, 83).

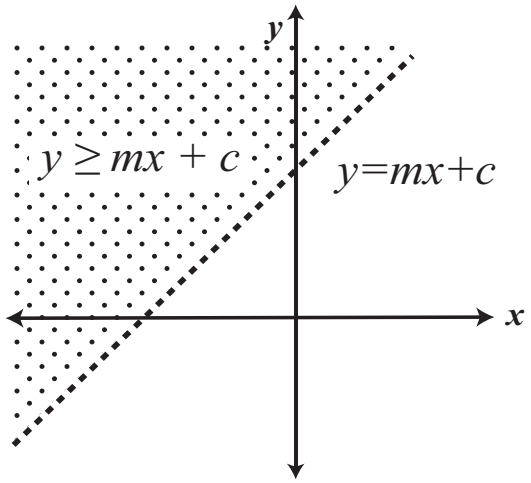


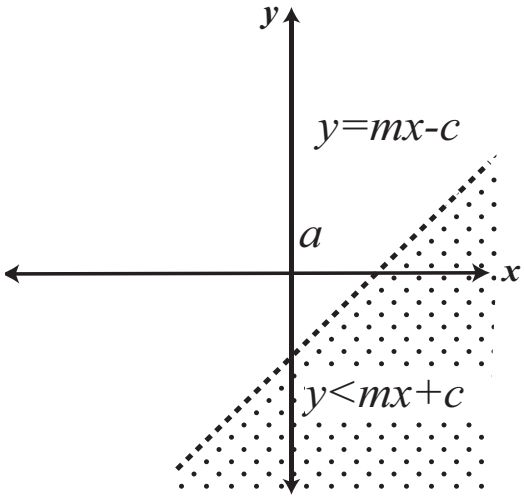
Sketch and Verifying Shaded Region

Shown below are some examples of graphs of linear inequalities.









Unit: Equations and Inequalities	Topic: Linear & Quadratic inequalities

Benchmark **10.3.3.4** Solve and sketch linear and quadratic inequalities with two variables using algebraic methods, manipulative or models.

Learning Objective: By the end of this topic, students will be able to;

- calculate and solve linear and quadratic inequalities, and
- sketch linear and quadratic Inequalities and verify their shaded regions.

Essential Questions:

1. What methods are appropriate to solve linear and quadratic inequalities?
2. What strategies are applied to sketch linear and quadratic inequalities?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate the ability to solve simultaneous linear and quadratic inequalities using different strategies.
Skills	• Apply the different methods of solving simultaneous linear and quadratic inequalities. • Sketching linear and quadratic inequalities and verifying their shaded regions.
Knowledge	Methods, manipulative and model of simultaneous linear and quadratic inequalities.
Mathematics Thinking	Think about how to solve simultaneous linear and quadratic inequalities using different strategies.

Content Background

Quadratic inequalities

There are four types of quadratic inequalities in two variables that we can look at for this topic.

$y< ax^2+bx+c$ $y>ax^2+bx+c$ $y\leq ax^2+bx+c$ $y\geq ax^2+bx+c$

The graph of any such inequality consists of all solutions (x, y) of the inequality. The steps used to graph a quadratic inequality are very much like those used to graph a linear inequality.

To graph one of the four types of quadratic inequalities shown above, follow these steps:

1. Draw the parabola with equation $y=ax^2+bx+c$. Make the parabola dashed for inequalities with $<$ or $>$ and solid for inequalities with $\leq$ or $\geq$.

2. Choose a point (x, y) inside the parabola and check whether the point is a solution of the inequality.

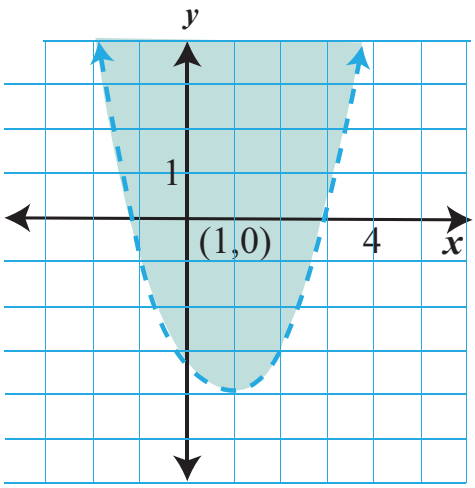
3. If the point from Step 2 is a solution, shade the region inside the parabola. If it is not a solution, shade the region outside the parabola.

Example

Graph $y>x^2-2x-3$

Follow the 3 steps listed above

1. Graph $y>x^2-2x-3$, since the inequality symbol is >, make the parabola dashed
2. Test a point inside the parabola, such as (1, 0). $y>x^2-2x-3 \rightarrow 0>1^2-2(1)-3 \quad 0>-4$ ☑
   So, (1, 0) is a solution of the inequality.
3. Shade the region inside the parabola.



Solving a Quadratic Inequality by Graphing

Example: Solve $x^2-6x+5<0$

Solution:

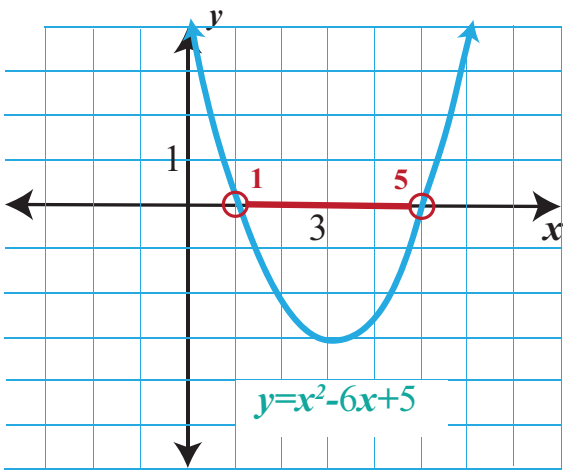
The solution consists of the x-values for which the graph of $y=x^2-6x+5$ lies below the x-axis. Find the graphs x-intercepts by letting $y=0$ and using factoring to solve for x.

$0=x^2-6x+5=(x-1)(x-5)$

therefore, $x=1$ or $x=5$

Sketch a parabola that opens up and has 1 and 5 as x-intercepts. The graph lies below the x-axis between $x = 1$ and $x = 5$.

Therefore, the solution of the given inequality is $1 < x < 5$.



Unit: Polynomials	Topic: Polynomials

Benchmark	**10.3.3.5** Add, subtract, multiply, divide and factor first and second-degree polynomials in one variable using algebraic manipulation.

Learning Objective: By the end of this topic, students will be able to;

- describe properties of Polynomial expressions,
- add, subtract, multiply and divide Polynomials, and
- solve problems using algebraic manipulations.

Essential Questions:

1. What are polynomials?
2. What skills and knowledge are used to apply the four operations on polynomials?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate the ability to apply addition and subtraction , multiplication and division to polynomials.
Skills	Apply addition and subtraction, multiplication and division of polynomials.
Knowledge	Polynomials.
Mathematics Thinking	Think about how to apply addition and subtraction, multiplication and division on polynomials when solving authentic situations.

Content Background

The Language of polynomials

A polynomial is an expression such as x^5-5x^2+7x, $3x^7+2$ *and* $1/5x^2+2x-5$
The function $P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2 x^2 + a_1x + a_0$ is called a polynomial function of degree n.

A polynomial may have any number of terms (the word 'polynomial' means many terms), but each term must be multiple of a whole-number power of x.
The term of highest index amongst then non-zero terms is called the leading term. Its coefficient is called the leading coefficient, and its index is called the degree of the polynomial. Thus

- $x^5 - 5x^2+7x$ has the leading term x^5, and leading coefficient 1 and degree 5
- $3x^7+2$ has leading term$3x^7$, leading coefficient 3 and degree 7
- $15x^2 + 2x-5$ has leading term$15x^2$. Leading coefficient 15 and degree 2.

A monic polynomial has leading coefficient one such as x^5-5x^2+7x.The other two examples are non-monic because neither leading coefficient is 1. A polynomial of degree 2 with leading coefficient 1 such as x^2-6x+2, are called quadratic (or monic quadratic). A polynomial of degree 2 with leading coefficient other than 1 such as ax^2-bx+c, are called quadratic trinomials (or non-monic quadratic), where $a\neq1$

Example:

Which of the following expressions are polynomials? (Give reasons)

a. $5x^3 - 7x^2 + 2x + 1$ **b.** 25
c. $x^2 - 5x + \frac{3}{x} - 2$ **d.** $2x^4 - 6x^3 - 6x + 7\sqrt{x} + 1$

Solution:

a. Because the powers of all terms in x are integers greater than or equal to zero. It is a polynomial.

b. 25= $25x^0$, i.e. It is a polynomial of degree zero.

c. No, as it contains the like term $\frac{3}{x} = 3x^{-1}$ which has a negative index.

d. No, as it contains a term $7\sqrt{x} = 7x^{\frac{1}{2}}$

Solution:

P (-2) is the value of the polynomial when x = -2

i.e. P (-2) $=(-2)^3 - 2(-2)^2 + 5(-2) + 1 = 25$

P (0) = $0^3 - 2(0)^2 + 5(0) + 1 = 1$

Addition and subtraction of polynomials

To add or subtract two polynomials, simply collect like terms

Example

1. Find the sum of polynomial P(x) = $4x^3 - 3x^2 + 7x - 1$ and Q(x) = $2x^3 + 6x^2 - 2x - 5$

2. Find the difference P(x) – Q(x)

Solution:

1. P(x) + Q(x)

= $(4x^3 - 3x^2 + 7x - 1) + (2x^3 + 6x^2 - 2x - 5)$

= $(4x^3 + 2x^3) + (-3x^2 + 6x^2) + (7x - 2x) + (-1 - 5)$

= $6x^3 + 3x^2 + 5x - 6$

2. P(x) - Q(x)

= $(4x^3 - 3x^2 + 7x - 1) - (2x^3 + 6x^2 - 2x - 5)$

= $4x^3 - 3x^2 + 7x - 1 - 2x^3 - 6x^2 + 2x + 5$

= $2x^3 - 9x^2 + 9x + 4$

Multiplication of polynomials

To multiply two polynomials, multiply each term in the first polynomial by the second polynomial and add these expressions together. We then expand the brackets, collect like terms and write the polynomial in standard form.

Example: The polynomials $P(x)$ and $Q(x)$are given by $P(x) = x^3 - x^2 + x - 1$ and $Q(x) = 3x^3 - 2x^2$. Find $P(x)Q(x)$.

Solution: $P(x)Q(x) = (x^3 - x^2 + x - 1)(3x^3 - 2x^2)$

$= x^3(3x^3 - 2x^2) - x^2(3x^3 - 2x^2) + x(3x^3 - 2x^2) - (3x^3 - 2x^2)$

$= 3x^6 - 2x^5 - 3x^5 + 2x^4 + 3x^4 - 2x^3 - 3x^3 + 2x^2$

$= 3x^6 - 5x^5 + 5x^4 - 5x^3 + 2x^2$

Dividing Polynomials

Whenever, we add, subtract or multiply two polynomials, the result is another polynomial. Division of Polynomials, however, does not usually result in a polynomial.

Example: The polynomials $P(x)$ and $Q(x)$are given by $P(x) = 3x^4 - 5x^2 + 7$ and $Q(x) = x^2$. Find $P(x)/Q(x)$.

Solution: $P(x)/Q(x) = \frac{3x^4 - 5x^2 + 7}{x^2} = \frac{3x^4}{x^2} - \frac{5x^2}{x^2} + \frac{7}{x^2} = 3x^2 - 5 + \frac{7}{x^2}$

Unit: Polynomials | **Topic: Operations with Polynomial**

Benchmark **10.3.3.6** Use factor and remainder theorems to solve polynomials.

Learning Objective: By the end of this topic, students will be able to;

- divide Polynomials,
- explain Remainder Theorem,
- divide using Factor Theorem, and
- solve Word problems on factor theorem.

Essential Questions:

1. What are polynomials and their operations?
2. What background information is needed to solve polynomial problems?

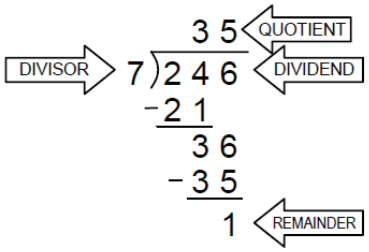
Key Concepts(ASK-MT)	
Attitudes / Values	Confidently solve division of Polynomials using Remainder Theorem and Factor Theorem and solve word problems on factor theorem.
Skills	Apply division of polynomials, Remainder Theorem, Factor Theorem to solve problems on polynomial.
Knowledge	Operations with polynomial.
Mathematics Thinking	Think about how to solve operations with polynomial problem when solving authentic situations.

Content Background

Division of whole numbers

Division of whole numbers

We divide polynomials using a method similar to long division so, let's review that first.



In long division process, you first find the largest multiple that can go into the first part of the dividend, subtract, bring down, and repeat this until you're done. The solution to the problem at the left might be written as $246 \div 7 = 35\frac{1}{7}$ or as $264 \div 7 = 35$ R 1.

Let's look at how polynomials are divided in a similar way.

Division of Polynomials

The key idea when dividing one polynomial by another is to keep working with the leading terms. The Following example shows how it is done.

Example: Divide $(x^3 - 2x^2 + 6x - 6) \div (x - 3)$

Solution:

$$
\begin{array}{r}
x^2 + x + 9 \\
x - 3 \overline{)\, x^3 - 2x^2 + 6x - 6} \\
(-)\ x^3 - 3x^2 \\
\hline
x^2 + 6x \\
(-)\ x^2 - 3x \\
\hline
9x - 6 \\
(-)\ 9x - 27 \\
\hline
21
\end{array}
$$

Our quotient is $x^2 + x + 9$, and the remainder is 21. We can write this answer as:

$x^3 - 2x^2 + 6x - 6 = (x - 3)(x^2 + x + 9) + 21$ or

$x^3 - 2x^2 + 6x - 6 \div (x - 3) = (x^2 + x + 9) + \frac{21}{x-3}$

We can check this by foiling out the quotient times the divisor, plus the remainder, which should equal the dividend.

Factors of Polynomials

When one polynomial is a factor of another, then the remainder after division is zero. For example, 7 is a factor of 42, and when we divide 42 by 7, we obtain 42 = 7 x 6 + 0. We can go on to factorize 42 completely into primes as 42 = 7 x 3 x 2.

Here is an example of dividing a polynomial by one of its factors.

Example: Divide $x^3 + 4x^2 - 5x - 14$ by $x - 2$

Solution:

$$
\begin{array}{r|l}
 & x^2 + 6x + 7 \\
\hline
x-2 & x^3 + 4x^2 - 5x - 14 \\
 & -(x^3 - 2x^2) \\
\hline
 & 6x^2 - 5x \\
 & -(6x^2 - 12x) \\
\hline
 & 7x - 14 \\
 & -(7x - 14) \\
\hline
 & 0
\end{array}
$$

Since the remainder is zero, $x - 2$ is a factor of $x^3 + 4x^2 - 5x - 14$

and

$x^3 + 4x^2 - 5x - 14 = (x - 2)(x^2 + 6x + 7)$

The remainder Theorem

Let $P(x)$ be a polynomial and let α be a constant. When $P(x)$ is divided by $x - \alpha$, the remainder is $P(\alpha)$.

Example:
Find the remainder when $2x^3 + 4x^2 - 5x - 7$ is divided by $x - 3$

Solution:
$P(3) = 2 \text{ x } 27 + 4 \text{ x } 9 - 5 \text{ x } 3 - 7$
= 54 +36 -15 -7
= 68
So, the remainder is 68

The factor Theorem

Suppose we want to know whether $x + 3$ a factor of the polynomial is $P(x) = x^3 + 2x^2 - 5x - 6$. All we need to do is to find the remainder after division by $x + 3$.

- If the remainder is 0, then we know that $x + 3$ is a factor
- If the remainder is not 0, then we know that $x + 3$ is not a factor.

Using the remainder theorem, the remainder is $P(-3)$.
$P(-3) = -27 + 18 + 15 - 6$
=0
And so $x + 3$ is a factor of $P(x)$.
On the other hand, $x + 2$ is not a factor of $P(x)$, because after dividing by $+2$, the remainder is (-2),
$P(-2)$ = - 8 +8 +10 - 6
= $4 \neq 0$ ……And so $x + 2$ is not a factor of $P(x)$.

Unit: Polynomials	Topic: Sketching Polynomials

Benchmark **10.3.3.7** Apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation.

Learning Objective: By the end of this topic, students will be able to;

- sketch Polynomials of degree 0, 1, 2 and 3, and
- describe features including intercepts of polynomial curves.

Essential Questions:

1. What are polynomials and the degrees of polynomials?
2. What factors distinguishes polynomials of different degrees?
3. What are the different features of polynomial curves?
4. What skills and features are required to solve and sketch the different degrees of polynomials?

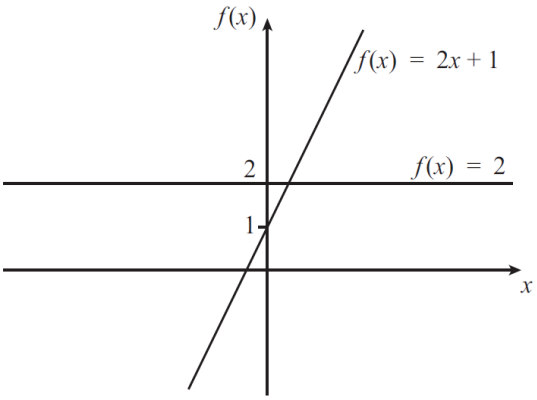
Key Concepts(ASK-MT)

Attitudes / Values	Appreciate the ability to sketch and describe features of polynomials.
Skills	Sketch graphs of Polynomials with different degrees.
Knowledge	Sketching of polynomials.
Mathematics Thinking	Think about how to sketch polynomials.

Content Background

Graphs of polynomial functions

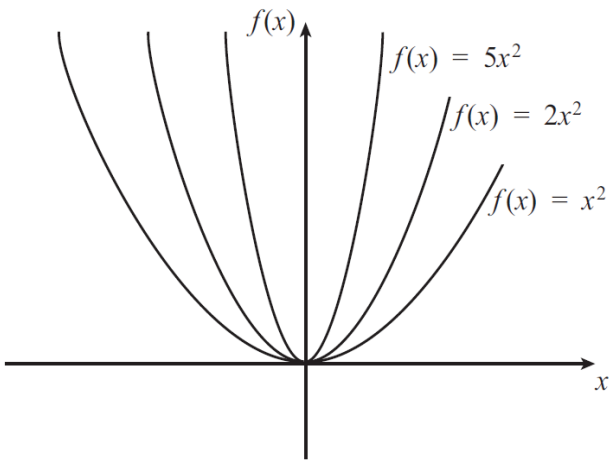
We have met some of the basic polynomials already. For example, $f(x) = 2$ is a constant function and $f(x) = 2x + 1$ is a linear function.



It is important to notice that the graphs of constant functions and linear functions are always straight lines.

A quadratic function is a polynomial of degree 2. Here are some examples of quadratic functions:

$f(x) = x^2$, $f(x) = 2x^2$, $f(x) = 5x^2$.



What is the impact of changing the coefficient of x^2 as we have done in these examples? One way to find out is to sketch the graphs of the functions.
You can see from the graph that, as the coefficient of x^2 is increased, the graph is stretched vertically (that is, in the y direction).

What will happen if the coefficient is negative? This will mean that all of the positive $f(x)$ values will now become negative. So what will the graphs of the functions look like?

The functions are now $f(x) = -x^2$, $f(x) = -2x^2$, $f(x) = -5x^2$.

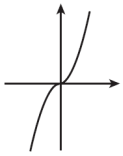
Turning points of polynomial functions

A turning point of a function is a point where the graph of the function changes from sloping downwards to sloping upwards, or vice versa. So the gradient changes from negative to positive or from positive to negative. Generally speaking, curves of degree n can have up to $(n-1)$ turning points.

A quadratic has only one turning point.

A cubic could have up to two turning points, and so would look something like this.

Some cubic have fewer turning points: for example $f(x) = x^3$. But no cubic has more than two turning points.



A quartic could have up to three turning points, and so would look something like this.

***Key Point**: A polynomial of degree n can have up to $(n-1)$ turning points.*

Roots of Polynomial Function

You may recall that when $(x-a)(x-b) = 0$, we know that a and b are roots of the function $f(x) = (x-a)(x-b)$. Now we can use the converse of this, and say that if a and b are roots, then the polynomial function with these roots must be $f(x) = (x-a)(x-b)$, or a multiple of this.

For example, if a quadratic has roots $x = 3$ and $x = -2$, then the function must be $f(x) = (x-3)(x+2)$, or a constant multiple of this. This can be extended to polynomials of any degree.

For example, if the roots of a polynomial are $x = 1, x = 2, x = 3, x = 4$, then the function must be $f(x) = (x - 1)(x - 2)(x - 3)(x - 4)$, or a constant multiple of this.

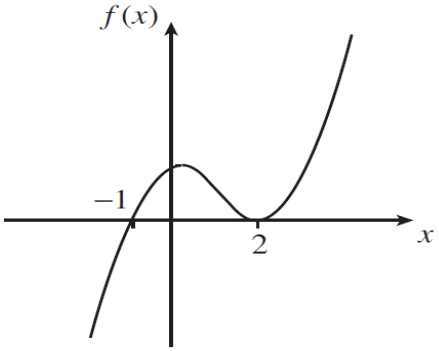
Let us also think about the function $f(x) = (x - 2)^2$. We can see straight away that $x - 2 = 0$, so that $x = 2$. For this function, we have only one root. This is what we call a repeated root, and a root can be repeated any number of times. For example, $f(x) = (x - 2)^3(x + 4)^4$ has a repeated root $x = 2$, and another repeated root $x = -4$. We say that the root $x = 2$ has multiplicity 3, and that the root $x = -4$ has multiplicity 4.
The useful thing about knowing the multiplicity of a root is that it helps us with sketching the graph of the function. If the multiplicity of a root is odd then the graph cuts through the x −axis at the point $(x, 0)$. But if the multiplicity is even then the graph just touches the x-axis at the point $(x, 0)$.

For example; take the function, $f(x) = (x - 3)^2(x + 1)^5(x - 2)^3(x + 2)^4$.

- The root $x = 3$ has multiplicity 2, so the graph touches the x -axis at (3, 0).
- The root $x = -1$ has multiplicity 5, so the graph crosses the x -axis at (–1, 0).
- The root $x = 2$ has multiplicity 3, so the graph crosses the x -axis at (2, 0).
- The root $x = -2$ has multiplicity 4, so the graph touches the x -axis at (–2, 0).

Sketching Polynomials

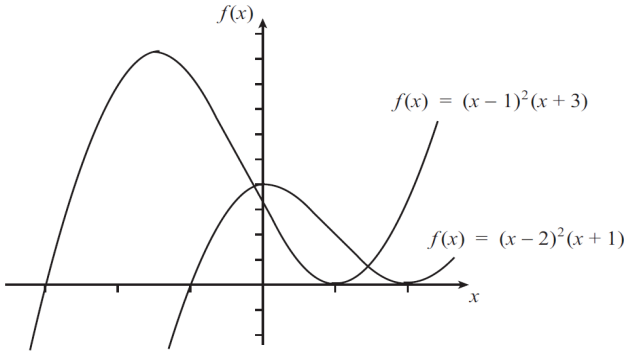
To take another example, suppose we have the function $f(x) = (x - 2)^2(x + 1)$. We can see that the largest power of x is 3, and so the function is a cubic. We know the possible general shapes of a cubic, and as the coefficient of x^3 is positive the curve must generally increase to the right and decrease to the left. We can also see that the roots of the function are $x = 2$ and $x = -1$. The root $x = 2$ has even multiplicity and so the curve just touches the x-axis here, whilst$x = -1$ has odd multiplicity and so here the curve crosses the x-axis. This means we can sketch the graph as follows.



Example: Sketch the following functions,

a. $f(x) = (x - 2)^2(x + 1)$ **b.** $f(x) = (x - 1)^2(x + 3)$

Solution:



Strand 4: Statistics and Probability

Content Standard: Students will be able to investigate how to interpret data using methods of exploratory data analysis, develop and evaluate inferences, predictions and arguments that are based on data and understand and apply basic notions of chance and probability.

Units	Benchmark	Topics	Lesson Titles
Probability	**10.4.4.1** Calculate probabilities for simple events under different relationships, including independent events, dependent events, with replacement and without replacement procedures.	Probability Events	Simple Probability and Meaning of Probability
		Independent and Dependent Events	Independent events,
			Dependent events
		With Replacement and Without Replacement Events	With replacement
			Without replacement
Data Analysis	**10.4.4.2** Calculate and analyse the measure of dispersions.	Measure of Central Tendency and Dispersion	Measures of Central Tendency
			Measures of Dispersion
			The Range
			Lower Quartiles
			Upper Quartiles
			Variance
			The Standard Deviation
			Deriving linear equation from the Graphs

Unit: Probability	Topic: Probability of Events

Benchmark	**10.4.4.1** Calculate probabilities for simple events under different relationships, including independent events, dependent events, with replacement and without replacement procedures.

Learning Objective: By the end of this topic, students will be able to;

- explain and define the concept of probability, and
- explain and define probability events.

Essential Questions:

1. What is probability?
2. How do we calculate simple probability?
3. What is probability of events?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate and confidently use probability terms in everyday life.
Skills	Calculate Probability and probability of events.
Knowledge	Probability of events.
Mathematics Thinking	Think about how to calculate probability of events in real life.

Content Background

Simple Probability Events

The probability of any event is measure of the likelihood or chance that the event will occur. For example, we wish to know the chances of winning a competition. Knowing this probability can help us to decide the amount of preparation needed before we participate in the competition. The greater the probability of event, the greater is the likelihood that it will occur.

Consider the event where we obtain a prime number when we roll a die. Since a die has six faces with the number 1 to 6, the possible outcomes of this event are 2, 3 and 5. If we let A denote this event, i.e., the event that 'the number rolled on a die is prime', then the **favorable outcomes** of A are 2, 3 and 5. Therefore, the number of outcomes in event A is 3.

The list of all the possible outcomes is known as the **sample space**. In the case of the die, the sample space consists of the numbers 1, 2, 3, 4, 5 and 6.

To calculate the probability of any event A (in this case, event A is the probability that 'the number rolled on a die is prime'), we can use the following formula:

Probability of any event A occurring,

$$P(A) = \frac{Number\ of\ outcomes\ favourable\ to\ A}{Total\ number\ of\ possible\ outcomes}$$

$$Therefore,\quad P(A) = \frac{3}{6} = \frac{1}{2}$$

The following are some special properties of probability

- If A is an impossible event, then $P(A) = 0$.
- If A is a sure event, then $P(A) = 1$
- For any event A, $P(A) + P(\text{not } A) = 1$ $P(\text{not } A) = 1 - P(A)$

Example

An unbiased die is thrown. Find the probability that the number obtained is

a. An odd number **b.** a multiple of 3 **c.** not a multiple of 3 **d.** 9

Solution:

The sample space consists of the numbers 1, 2, 3, 4, 5 and 6.

a. Let A be the event of obtaining an odd number. Then the favourable outcomes of A are 1, 3 and 5. Thus, the number of outcomes favourable to A is 3.

$$Therefore,\ P(A) = \frac{3}{6} = \frac{1}{2}$$

b. Let B be the event of obtaining a number that is a multiple of 3. Then the favourable outcomes of B is 2

$$Therefore,\ P(B) = \frac{2}{6} = \frac{1}{3}$$

c. P(not B)$= 1 - P(B)$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

d. The number 9 is not found on the die. Thus, this is an impossible event.

Therefore, Probability that the number obtained is 9 = 0.

Unit: Probability	Topic: Independent and Dependent Events

Benchmark	**10.4.4.1** Calculate probabilities for simple events under different relationships, including independent events, dependent events, with replacement and without replacement procedures.

Learning Objective: By the end of this topic, students will be able to;

- define and explain simple probability using independent and dependent events,
- calculate simple probability using independent events, and
- calculate simple probability using dependent events.

Essential Questions:

1. What is simple probability using independent and dependent events?
2. How do we calculate simple probability using independent and dependent events?

Key Concepts(ASK-MT)	
Attitudes / Values	Show confidence in solving probability using independent and independent events.
Skills	Calculate probability of independent and dependent events.
Knowledge	Calculation of probability using independent and dependent events.
Mathematics Thinking	Think about how to solve probability using independent and independent events in real life.

Content Background

Dependent Events

Dependent events: Two events are dependent events if the occurrence of one event does affect the likelihood that the other event will occur.

Example: A drawer contains 15 socks, 7 blue and 8 white. You close your eyes and pull out a blue sock first, then a white sock, without replacing the blue sock. Are these events independent or dependent?
Whether or not you choose a blue sock first does affect the likelihood that you choose a white sock second. This is because the ratio of blue to white socks in the drawer after the first sock is pulled from the drawer changes and not put back. This is an example of **dependent** event.

Independent Events

Independent events: Two events are independent events if the occurrence of one event does not affect the likelihood that the other event will occur.

Example: A jar contains 8 red and 12 blue marbles. You randomly choose a marble, put it back, and then randomly choose another marble. This is an example of Independent event.

Probability of Independent Events
For two independent events, the probability that both events occur is the product of the probabilities of the events. i.e. If A and B are independent events, the
$P(A \text{ and } B) = P(A) \times P(B)$

Example: Tara is playing a game at a carnival where she picks a rubber duck from a pond. There are 12 ducks in the pond for which there is no prize and 4 ducks that will award a prize. What is the probability that Tara picks a prize-winning duck, replaces the duck in the pond, then picks another prize-winning duck?

Solution:

First, find the probability of each event.

$\mathrm{P(win)} = \frac{4}{16} = 0.25$ ……..The are 16 ducks in all.

$\mathrm{P(win)} = \frac{4}{16} = 0.25$ ……. Because Tara replaces the first duck, there are 4 winning ducks for the second pick.

Because the events are independent, multiply the probabilities.

$\mathrm{P(win\ and\ win)} = \mathrm{P(win)} \times \mathrm{P(win)} = 0.25 \times 0.25 = 0.0625$

Answer: The probability that Tara selects 2 winning ducks from the pond in a row is 0.0625 or 6.25 %.

Probability of Dependent Events

For two dependent events, the probability that both events occur is the product of the probability of the first event and the probability of the second event given the first.
If *A* and *B* are dependent events, then $P(A\ and\ B) = P(A) \cdot P(B\ given\ A)$.

Example: Jeffrey's mother has 10 orange juice boxes, 7 grape juice boxes, and 3 lemonade juice boxes in the cooler for Jeffrey and his friends. Jeffrey randomly takes a juice box from the cooler, then randomly chooses another juice box without replacing the first. Find the probability that both juice boxes are grape.

Solution:

Find the probability of the first event and the probability of the second event given the first. Then multiply the probabilities.

1. $\mathrm{P(grape)} = \frac{7}{20}$ ……Out of 20 juice box, 7 are grapes
2. $\mathrm{P(grape\ given\ grape)} = \frac{6}{19}$ ……of the remaining 19 juice boxes, 6 are grapes
3. $\mathrm{P(grape\ given\ grape)} = \frac{7}{20} \times \frac{6}{19} = \frac{21}{190}$ …Multiply probabilities

Answer: The probability that both juice boxes are grape is $\frac{21}{190}$.

Unit: Probability	Topic: With Replacement and Without Replacement Events

Benchmark	**10.4.4.1** Calculate probabilities for simple events under different relationships, including independent events, dependent events, with replacement and without replacement procedures.

Learning Objective: By the end of this topic, students will be able to;

- define and explain simple probability using with replacement and without events?
- calculate simple probability using with replacement events, and
- calculate simple probability using without replacement events.

Essential Questions:

1. How do we calculate simple probability using with replacement events?
2. How do we calculate simple probability using without replacement events?
3. What can be done to demonstrate these events?

Key Concepts(ASK-MT)	
Attitudes / Values	Confidently calculate simple probability using with and without replacement events.
Skills	Distinguish and Calculate simple probability using with and without replacement events.
Knowledge	Simple probability using with and without replacement events.
Mathematics Thinking	Think about how to apply simple probability with or without replacement in authentic situations.

Content Background

Sampling with replacement

I have four pens in my satchel; they are red, green, blue, and purple. I take out a pen and lay it on the desk; each pen has the same chance of being selected. In this case,
S = {R, G, B, P}, where R means 'red pen chosen' and so on. If A is the event 'red or green pen chosen', then

$$P(A) = \frac{l\,A\,l}{l\,S\,l} = \frac{2}{4} = \frac{1}{2}$$

More generally, if I have a set of N objects and choose one, with each one equally likely to be chosen, then each of the N outcomes has probability $\frac{1}{N}$, and an event consisting of m of the outcomes has probability $\frac{m}{N}$.

What if we choose more than one pen? We have to be more careful to specify the sample space. First, we have to say whether we are,

- sampling with replacement, or
- Sampling without replacement.

Sampling with replacement means that we choose a pen, note its color, put it back and shake the satchel, then choose a pen again (which may be the same pen as before or a different one), and so on until the required number of pens have been chosen. If we choose two pens with replacement, the sample space is;

{RR, RG, RB, RP, GR, GG, GB, GP, BR, BG, BB, BP, PR, PG, PB, PP}
The event 'at least one red pen' is {RR, RG, RB, RP, GR, BR, PR}, and has probability $\frac{7}{16}$.

Sampling without replacement

In general, if we choose n items from a set Ω of size N, and the sampling is done with replacement, then the sample space $\boldsymbol{S}$ consists of all ordered n −tuples of the form. $(\omega 1, \omega 2, \ldots, \omega n)$, where ωi denotes the object taken out on the i-th occasion.

Sampling without replacement means that we choose a pen but do not put it back, so that our final selection cannot include two pens of the same color. In this case, the sample space for choosing two pens is
{RG, RB, RP, GR, GB, GP, BR, BG, BP, PR, PG, PB}

And the event 'at least one red pen' is {RG, RB, RP, GR, BR, PR}, with probability

$$\frac{6}{12} = \frac{1}{2}$$

Now there is another issue, depending on whether we care about the order in which the pens are chosen. We will only consider this in the case of sampling without replacement.

Sometimes it doesn't really matter whether we choose the pens one at a time or simply take two pens out of the drawer; we are not always interested in which pen was chosen first.

If we are not interested then the sample space is {{R, G}, {R, B}, {R, P}, {G, B}, {G, P}, {B, P}}, containing six elements. (Each element is written as a set since, in a set, we don't care which element is first, only which elements are actually present. So, the sample space is a set of sets!) The event 'at least one red pen' is {{R, G}, {R, B}, {R, P}}, with probability $\frac{3}{6} = \frac{1}{2}$. We should not be surprised that this is the same as in the previous case.

Unit: Data Analysis	Topic: Measure of Central Tendency and Dispersion
Benchmark	**10.4.4.2** Calculate Calculate and analyse the measure of dispersions.

Learning Objective: By the end of this topic, students will be able to;

- calculate the mean, median and mode of a set of data, and
- calculate mean and standard deviation of each data point from the mean of a set of data.

Essential Questions:

1. How do we calculate the mean, mode and median of a set of data?
2. How do we calculate mean and standard deviation of a set of data?

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate and confidently use measures of central tendency and measures of dispersion of collected or given data.
Skills	Apply and calculate the measures of Central Tendency and Measures of Dispersion in a set of data.
Knowledge	Measures of Central and Measures of Dispersion.
Mathematics Thinking	Think about how to calculate measures of central tendency and measures of dispersion when solving authentic situations.

Content Background

Measures of central Tendency - Mean, Mode and Median

The mean is the statistical term most thought of when the word average is used. The mean of a set of scores is calculated by adding all scores and dividing this sum by the number of scores. $\bar{x}$ is the symbol used to represent the mean.

Example

Calculate the mean of the following data set:

9, 11, 9, 14, 8, 8, 9, 8, 10, 10

$$\text{Mean} = \sum x_i = \bar{x} = \frac{9+11+9+14+8+8+9+8+10+10}{10} = 8.8$$

The **mode** is the score that occurs most often, that is it is the score with the highest frequency. It is the most commonly occurring score

Example

Find the mode in this set of scores:

6, 10, 6, 6, 13, 12, 12, 7, 13, 6

The mode is 6 as it occurs more than any other scores

The **median** of a set of scores is the middle score (or average of the two middle scores) after the scores have been arranged in ascending order.(i.e. from smallest to largest)

Example

Calculate the median of the following data set:

4, 13, 10, 13, 13, 4, 2, 13, 13, 13

First order the set of scores.

2, 4, 4 ,10, 13, 13, 13, 13, 13, 13.

Since there is an even number of values in this data set (10) the median lies between the fifth and sixth place:

$$\textit{Median} = \frac{13+13}{2} = 13$$

Measures of Dispersion

Consider the two sets of data 3, 4, 5, 6, 7 and 0, 1, 5, 9, 10. For both sets the mean is 5. However it is obvious that these sets of scores are quite different in nature. The first sets of scores are closely grouped around the mean but the second set are quite scattered from the mean. Hence, as well as having a central value of scores, it is useful to have some measure of the scatter, or spread, of the scores. Such measures of spread are called measures of dispersion and we will investigate some of these next.

The range is the difference between the highest score and the lowest score. The larger the value of the range the more dispersed the scores are on the variable, the smaller the range the less dispersed the scores are on the variable.

Example: A group of 15 students count the number of sweets they each have. This is the data they collect:

a. 4, 11, 6, 7, 14
b. 5, 8, 7, 9, 12
c. 3, 13, 10, 6, 7

Fill the range of the scores

a. Range = 14 – 4
= 10

b. Range = 12–5
= 7

c. Range = 13 – 3
= 10

Quartiles

Example :

Find the interquartile range for the scores below.

a. 30, 32, 33, 35, 40, 42, 45 **b.** 9, 5, 7, 11, 10, 4, 14 , 7

Find the median by crossing off from each end.

a. ~~30, 32, 33,~~ 35, ~~40, 42, 45~~ median 35

Upper quartile is 40, 41, 42, 45 $\therefore$ Q3 = $\frac{41+42}{2}$ = 41.5

Lower quartile is 30, 32, 32, 33 $\therefore$ Q1 = $\frac{32+32}{2}$ = 32

b. Arrange in ascending order to find the median.

4, 5, 7, 7, 9, 10, 11, 14

Median = $\frac{7+9}{2}$ = 8

Upper quartile is 9, 10, 11, 14 $\therefore$ Q3 = $\frac{10+11}{2}$ = 10.5

Lower quartile is 4, 5, 7, 7 $\therefore$ Q3 = $\frac{5+7}{2}$ = 6

$\therefore$ Interquartile range = 10.5 - 6
= 4.5

Variance (σ^2) and Standard Deviation (σ)

An important measure of dispersion is the variance. The calculation of the variance requires the attributes of variables to form a numerical scale. Thus, it is an interval/ratio –level measure of dispersion.

The variance indicates how close to or far from the mean are most of the cases for a particular variable. The smaller the variable the more the cases are concentrated around the value of the mean the larger the value the variance, the more spread out from the mean is the cases.

The formula for the variance is $Variance = \frac{\sum(x_i - \bar{x})^2}{n-1}$

To calculate the variance, you must first calculate the mean. (The mean is represented in the variance formula $\bar{x}$. A bar over a variable is common way of representing the mean). Then for each score in the data set, take the value of the score and subtract the mean. Take the result of each subtraction and square it (multiply it by itself). Now add those squared values. Finally divide the sum by the number of scores in the data set minus 1. The result is the variance.

Example:

The calculation of variance using a data set of just three scores. The scores have

value 2, 4 and 6

Step 1	$\bar{x} = \frac{2+4+6}{3} = \frac{12}{3} = 4$		
Step 2	x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
	2	2 - 4= -2	-2 x -2 = 4
	4	4 – 4 = 0	0 x 0 = 0
	6	6 – 4 = 2	2 x 2 = 4
Step 3	$\sum(x_i - \bar{x})^2 = 4 + 0 + 4 = 8$		
Step 4	$\frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{8}{3-1} = \frac{8}{2} = 4$		

Standards-Based Lesson Planning

What are Standards-Based Lessons?

In a Standards-Based Lesson, the most important or key distinction is that, a student is expected to meet a defined standard for proficiency. When planning a lesson, the teacher ensures that the content and the methods of teaching the content enable students to learn both the skills and the concepts defined in the standard for that grade level and to demonstrate evidence of their learning.

Planning lessons that are built on standards and creating aligned assessments that measure student progress towards standards is the first step teacher must take to help their students reach success. A lesson plan is a step-by-step guide that provides a structure for an essential learning.

When panning a standards-based lesson, teacher instructions are very crucial for your lessons. How teachers instruct the students is what really points out an innovative teacher to an ordinary teacher. Teacher must engage and prepare motivating instructional activities that will provide the students with opportunities to demonstrate the benchmarks. For instance, teacher should at least identify 3-5 teaching strategies in a lesson; teacher lectures, ask questions, put students into groups for discussion and role play what was discussed.

Why is Standards-Based Lesson Planning Important?

There are many important benefits of having a clear and organized set of lesson plans. Good planning allows for more effective teaching and learning. The lesson plan is a guide and map for organizing the materials and the teacher for the purpose of helping the students achieve the standards. Lesson plans also provide a record that allows good, reflective teachers to go back, analyse their own teaching (what went well, what didn't), and then improve on it in the future.

Standards-based lesson planning is vital because the content standards and benchmarks must be comparable, rigorous, measurable and of course evidence based and be applicable in real life that we expect students to achieve. Therefore, teachers must plan effective lessons to teach students to meet these standards. As schools implement new standards, there will be much more evidence that teachers will use to support student learning to help them reach the highest levels of cognitive complexity. That is, students will be developing high-level cognitive skills.

Components of a Standards-Based Lesson Plan

An effective lesson plan has three basic components;

- aims and objectives of the course;
- teaching and learning activities;
- assessments to check student understanding of the topic.

Effective teaching demonstrates deep subject knowledge, including key concepts, current and relevant research, methodologies, tools and techniques, and meaningful applications.

Planning for under-achievers NORMA

Who are underachieving students?
Under achievers are students who fail or do not perform as expected. Underachievement may be caused by emotions (low self-esteem) and the environment (cultural influences, unsupportive family)

How can we help underachievement?
Underachievement varies between students. Not all students are in the same category of underachievement.

Given below a suggested strategies teachers may adopt to assist underachievers in the classroom.

- Examine the Problem Individually
  It is important that underachieving students are addressed individually by focusing on the student's strengths.

- Create a Teacher-Parent Collaboration
  Teachers and parents need to work together and pool their information and experience regarding the child. Teachers and parents begin by asking questions such as;

  - In what areas has the child shown exceptional ability?
  - What are the child's preferred learning styles?
  - What insights do parents and teachers have about the child's strengths and problem areas?

- Help student to plan every activity in the classroom
- Help students set realistic expectations
- Encourage and promote the student's interests and passions.
- Help children set short and long-term academic goals
- Talk with them about possible goals.
- Ensure that all students are challenged (but not frustrated) by classroom activities
- Always reinforce students

Example of Standards-Based Lesson Planning

The following sample lesson can help teachers to plan effective lessons. Teachers are encouraged to study the layout of the different components of these lessons and follow this design in their preparation and teaching of each lesson. Planning a good lesson helps the teacher in maintaining a standard teaching pattern which should not deviate students learning of the concept from the topic.

Strand: Number, Operation and Computation

Unit: Financial mathematics

Content Standard: Students will be able to represent numbers in various situations and forms, develop fluency in calculations through operations, use base ten as key for extending numbers and operations, and apply numbers in practical situations to develop number sense.

Benchmark: 10.1.1.5 Apply the skills of managing money in authentic situations

Topic : Earning, Saving and Spending Money

Lesson Title: Budgeting

Lesson Objective (s): By the end of this topic, students will be able to;

- Identify and explain the importance of budgeting.
- Draw a personal budget.

Materials: Content and activity handouts.

Key Concepts(ASK-MT)	
Attitudes / Values	Display confidence in applying the skills of managing money and appreciate its usefulness.
Skills	Managing money through earning, budgeting, spending and saving money.
Knowledge	Money management.
Mathematics Thinking	Think about how to manage financial needs.

Lesson Procedures

Teacher Activities	Student Activities
Introduction  **(5 minutes)**	
• Recap students' previous knowledge through open discussions. • Ask essential questions (i) What is important in setting personal financial goal? (ii) Why is a budget key component of a financial plan?	• Use their previous knowledge and share their ideas on the teachers' respective questions. • Think about the essential questions raised by the teacher and respond with positive attitude.

Body (20 minutes)	
Modelling	
• Model through a scenario of a personal income and expenses. • Discuss with the class on how the income was spent over a two weeks budget.	• Pay attention to the scenario presented by the teacher on income and expenses of a person. • Discuss with the teacher on how the income was budget over the two weeks.
Guided Practice	
• Give a sample question with all necessary information on Income and Expenses list and ask students to fill in the budget table. • Ask question relating to the personal budget table and discuss	• Take note of all the information provided by the teacher to complete the Budget Table. • Use the information provided in the table to answer teacher's questions.
Independent Practice	
• Ask students to identify (needs and wants) and develop a personal budget if they earn K900.00 in a fortnight.	• Individual students draw their own personal budget for a K900 fortnight.
Conclusion (15 minutes)	
• Allow students to present their solution on the board for class discussion. • Make corrections where necessary on the student's presentations and stress on the key points.	• Present and their solutions on the board and explain their answers. • Make necessary corrections if any through consolidating the key points of the lesson highlighted by the teacher.

Assessment/Lesson Evaluation

Students can;

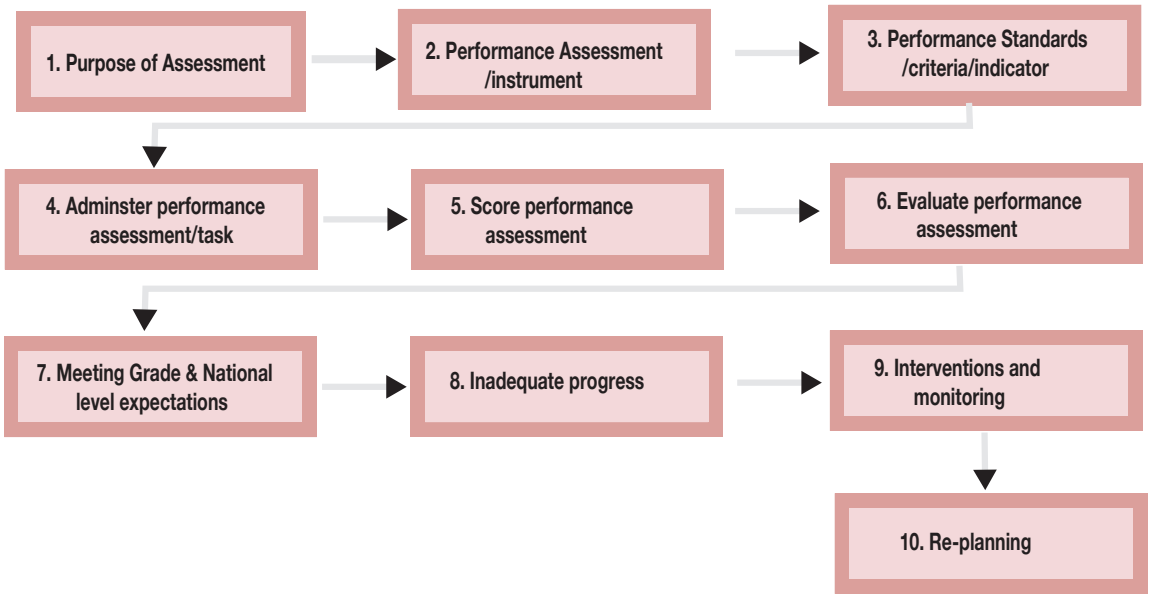
- Explain the importance of budgeting.
- Draw up a personal budget.

Assessment, Monitoring and Reporting

What is Standards-Based Assessment (SBA)?

Standards-Based Assessment is an on-going and a systematic process of assessing, evaluating, reporting and monitoring students' performance and progression towards meeting grade and national level expectations. It is the measurement of students' proficiency on a learning objective or a specific component of a content standard and progression towards the attainment of a benchmark and content standard.

Standards-Based Assessment Process



Purpose of Standards-Based Assessment

Standards-Based Assessment (SBA) serves different purposes. These include instruction and learning purposes. The primary purpose of SBA is to improve student learning so that all students can attain the expected level of proficiency or quality of learning.

Enabling purposes of SBA is to:

- Measure students' proficiency on well-defined content standards, benchmarks and learning objectives
- Ascertain students' attainment or progress towards the attainment of specific component of a content standard.
- Ascertain what each student knows and can do and what each student needs to learn to reach the expected level of proficiency.
- Enable teachers to make informed decisions and plans about how and what they would do to assist weak students to make adequate progress towards meeting the expected level of proficiency.
- Enable students to know what they can do and help them to develop and
- implement strategies to improve their learning and proficiency level.
- Communicate to parents, guardians, and relevant stakeholders the performance and progress towards the attainment of content standards or its components.
- Compare students' performances and the performances of other students.

Principles of Standards-Based Assessment

The principle of SBA is for assessment to be;

- emphasising on tasks that should encourage deeper learning,
- be an integral component of a course, unit or topic and not something to add on afterwards,
- a good assessment requires clarity of purpose, goals, standards and criteria of practices that should use a range of measures allowing students to demonstrate what they know and can do,
- based on an understanding of how students learn,
- improving performance that involves feedback and reflection,
- on-going rather than episodic,
- given the required attention to outcomes and processes, and
- be closely aligned and linked to learning objectives, benchmarks and content standards.

Standards-Based Assessment Types

In standards-Based Assessment, there are three broad assessments types.

1. Formative Assessment

Formative assessment includes 'assessment for and as learning' and is conducted during the teaching and learning of activities of a topic.

Purposes of assessment for Learning

- On-going assessment that allows teachers to monitor students on a day-to-day basis.
- Provide continuous feedback and evidence to the teachers that should enable them to identify gaps and issues with their teaching, and improve their classroom teaching practice.
- Helps students to continuously evaluate, reflect on, and improve their learning.

Purposes of assessment as Learning

- Occurs when students reflect on and monitor their progress to inform their future learning goals.
- Helps students to continuously evaluate, reflect, and improve their own learning.
- Helps students to understand the purpose of their learning and clarify learning goals.

2. Summative Assessment

Summative assessment focuses on 'assessment of learning' and is conducted after or at the conclusion of teaching and learning of activities or a topic.

Purposes of assessment of Learning

- Help teachers to determine what each student has achieved and how much progress he/she has made towards meeting national and grade-level expectations.
- Help teachers to determine what each student has achieved at the end of a learning sequence or a unit.
- Enable teachers to ascertain each student's development against the unit or topic objectives and to set future directions for learning.
- Help students to evaluate, reflect on, and prepare for next stage of learning.

3. Authentic Assessment

- Is performed in a real life context that approximates as much as possible, the use of a skill or concept in the real world.
- Is based on the development of a meaningful product, performance or process.
- Students develop and demonstrate the application of their knowledge, skills, values and attitudes in real life situations which promote and support the development of deeper levels of understanding.

Authentic assessment refers to assessment that:

- Looks at students actively engaged in completing a task that represents the achievement of a learning objective or standard.
- Takes place in real life situations.
- Asks students to apply their knowledge, skills, values and attitudes in real life situations.
- Students are given the criteria against which they are being assessed.

Performance Assessment

Performance assessment is a form of testing that requires students to perform a task rather than select an answer from a ready-made list. For example, a student may be asked to explain historical events, generate scientific hypotheses, solve math problems, converse in a foreign language, or conduct research on an assigned topic. Teachers, then judge the quality of the student's work based on an agreed-upon set of criteria. It is an assessment which requires students to demonstrate that they have mastered specific skills and competencies by performing or producing something.

Types of performance assessment;

i. Products

This refers to concrete tangible items that students create through either the visual, written or auditory media such as;

- Creating a health/physical activity poster,
- Video a class game or performance and write a broadcast commentary,
- Write a speech to be given at a school council meeting advocating for increased time for health and physical education in the curriculum,
- Write the skill cues for a series of skill photo's,
- Create a brochure to be handed out to parents during education week,
- Develop an interview for a favourite sports person,
- Write a review of a dance performance,
- Essays, and
- Projects

ii. Process Focused Tasks

It shows the thinking processes and learning strategies students use as they work such as;

- Survival scenarios
- Problem solving initiative/adventure/ activities
- Decision making such as scenario's related to health issues
- Event tasks such as creating a game, choreographing a dance/gymnastics routine, creating an obstacle course
- Game play analysis
- Peer assessment of skills or performances
- Self-assessment activities
- Goal setting, deciding a strategy and monitoring progress towards achievement

iii. Portfolio
This refers to a collection of student work and additional information gathered over a period of time that demonstrates learning progress.

iv. Performances
It deals with observable affective or psycho-motor behaviours put into action such as;
- Skills check during game play
- Role plays
- Officiating a game
- Debates
- Performing dance/gymnastics routines
- Teaching a skill/game/dance to peers

Performance Standards

Performance Standards are concrete statements of how well students must learn what is set out in the content standards, often called the "be able to do" of "what students should know and be able to do." Performance standards are the indicators of quality that specify how competent a students' demonstration or performance must be. They include explanations of how well students must demonstrate the content, explaining how good is good enough.

Performance standards:

- measure students' performance and proficiency (using performance indicators) in the use of a specific knowledge, skill, value, or attitude in real life or related situations
- provide the basis (performance indicators) for evaluating, reporting and monitoring students' level of proficiency in use of a specific knowledge, skills, value, or attitude
- are used to plan for individual instruction to help students not yet meeting expectations (desired level of mastery and proficiency) to make adequate progress towards the full attainment of benchmarks and content standards
- are used as the basis for measuring students' progress towards meeting grade-level benchmarks and content standards

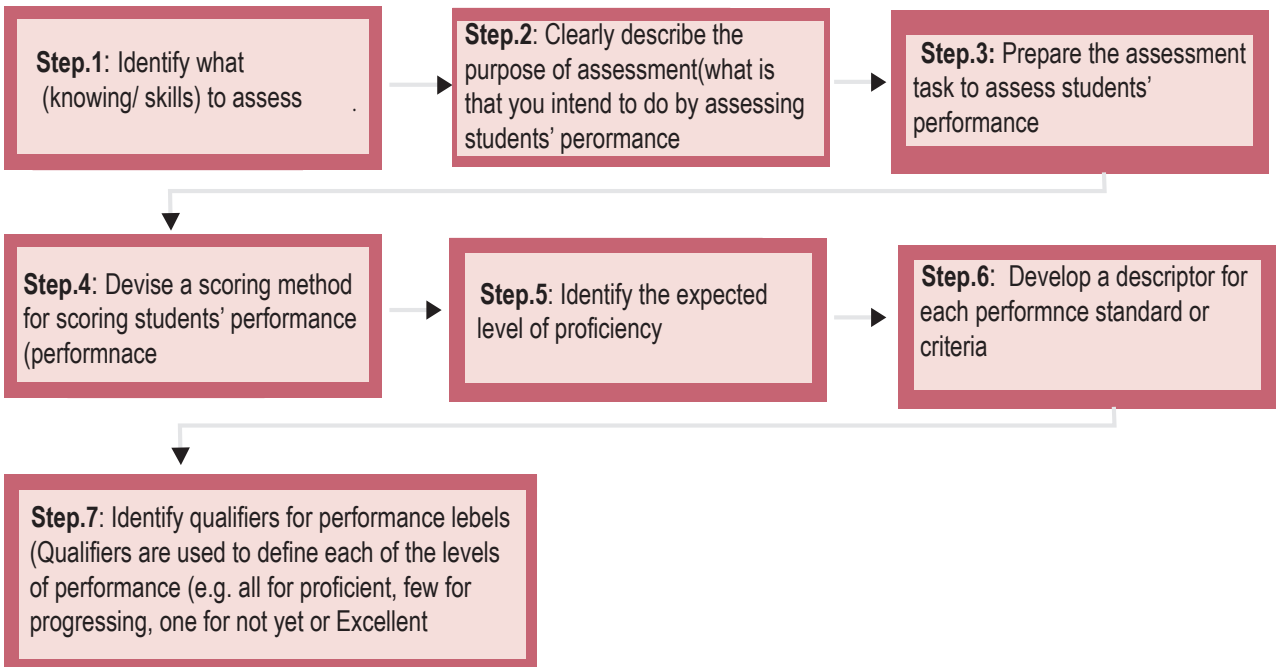
Assessment Strategies

It is important for teachers to know that, assessment is administered in different ways. Assessment does not mean a test only. There are many different ways to find out about student's strengths and weaknesses. Relying on only one method of assessing will not reflect student's achievement.

Provided in the appendices is a list of suggested strategies you can use to assess student's performances. These strategies are applicable in all the standards-based assessment types.

How to Develop effective SBA tools

Teachers are required to use the steps outlined below when planning assessment. These steps will guide you to develop effective assessments to improve student's learning as well as evaluating their progress towards meeting national and grade –level expectations.



Samples of Assessment Types

Sample 1: Formative Assessment

STRAND 1: Number Operations & Computation

Content Standard 1: Students will be able to represent numbers in various situations and forms, develop fluency in calculations through operations, use base ten as key for extending numbers and operations, and apply numbers in practical situations to develop number sense.

Benchmark: 10.1.1.5 Apply the skills of managing money in authentic situations.

Unit: Financial Mathematics

Topic: Consumer Arithmetic

Lesson Title: Budgeting

Learning Objectives: By the end of this topic, students will be able to;

- Identify and explain the importance of budgeting.
- Draw a personal budget.

Materials: Content and activity handouts.

What is to be assessed? - (ASK-MT)

Key Concepts(ASK-MT)	
Attitudes / Values	Display confidence in applying the skills of managing money and appreciate its usefulness.
Skills	Managing money through earning, budgeting, spending and saving money.
Knowledge	Budgeting
Mathematics Thinking	Think about how to manage financial needs.

Purpose of the assessment
To measure students' proficiency on the achievement of the benchmark and learning objectives.

Expected level of proficiency
Apply the skills of managing money in authentic situations.

Performance Task
- Identify and explain the importance of budgeting and draw a personal budget.

Assessment Strategy
This assessment can be conducted in one lesson as an assessed lesson exercise.

Assessment Scoring

Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess the child's learning through the assessment tool for a lesson exercise.

Performance standards/ Criteria	A	B	C	D	Score
	Advance 10	Proficient 9-5	Progressing 3-4	Not Yet 2	__/10 Marks
Identify and explain the importance of budgeting and draw a personal budget.	Identify and explain the importance of budgeting and draw a personal budget with advanced understanding.	Identify and explain the importance of budgeting and draw a personal budget with proficiency.	Satisfactory identify and explain the importance of budgeting and draw a personal budget.	Could not identify and explain the importance of budgeting or draw a personal budget	

Sample 2: Summative Assessment

STRAND 3: PATTERNS AND ALGEBRA

Content Standard 3: Students will be able to interpret various types of patterns and functional relationships, use symbolic forms to represent, model, and analyse mathematical situations and collect, organise, and represent data to answer questions.

Benchmark: 10.3.3.5- 10.3.3.7: (refer to the benchmarks in unit: Polynomials of Strand 3)

Unit: Polynomials

Topic : (refer to the topics in unit: Polynomials of Strand 3)

Lesson Title: (refer to the lesson titles in unit: Polynomials of Strand 3)

Learning Objectives: (refer to unit: Polynomials of Strand 3)

What is to be assessed? - (ASK-MT)

Key Concepts(ASK-MT)	
Attitudes / Values	Appreciate the ability to sketch and describe features of polynomials.
Skills	Sketch graphs of Polynomials with different degrees.
Knowledge	Sketching of polynomials with different degrees
Mathematics Thinking	Think about the processes involved to solve and sketch polynomials.

Purpose of the assessment

To measure students' proficiency on the achievement of the benchmarks and learning objectives in this unit. (This assessment is to be conducted after teaching the unit)

Expected level of proficiency

All students are expected to;

- sketch a range of curves and describe the features of these curves from their equation.

Performance Task

Students will do an assignment out of 20 marks. You can use other assessment tools (assignment, projects, etc.) assess student's proficiency on these benchmarks.

Task: Students will be given two week to complete this assignment. They are to:

On the same plane Sketch and describe the features of $f(x)=x^2$, $f(x)=2x^2$,and $f(x)=5x^2$

One the same plane sketch and describe $f(x)= (x-2)^2 (x+1)$ and $f(x)= (x-1)^2 (x+3)$.

Assessment Strategy

An assignment will be used to measure students' proficiency.

Assessment Scoring

Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess the child's learning through the assessment tool for an assignment.

Criteria	Model/Exemplar (20 points)	Proficient (13-19 points)	Developing (6-12 points)	Beginning (2-5 points)	Score
Quality and Effort	Maximum effort was put forth to complete the assignment in a professional manner. Assignment demonstrates a high degree of quality and attention to detail.	Some effort was made to complete the assignment to a level that was sufficient for grading, but does not meet a professional level of quality or appearance.	Minimal effort was made to complete the assignment but still meets the minimal standard.	Little or no effort was made to produce a quality assignment.	
Mathematical Calculations	All calculations are very clear, organized, and neatly completed with no inaccuracies.	All calculations are clear, organized, and neatly completed with 1-2 inaccuracies.	Most calculations are clear, organized, and neatly completed with 3-4 in accuracies.	Calculations are unclear and disorganized and 5 or more inaccuracies may be present.	
Diagrams	Correct sketches of the polynomial functions and represented all information correctly; appropriately scaled and labeled	Correct sketches of the polynomial functions and represented all information correctly; not appropriately scaled and labeled.	Some polynomial functions are not neatly produced; not appropriately scaled and labeled,	All diagrams are not neatly produced; not appropriately scaled and labeled,	

Sample 3: Authentic Assessment

STRAND 1: NUMBER, OPERATIONS & COMPUTATION

Content Standard 1: Students will be able to represent numbers in various situations and forms, develop fluency in calculations through operations, use base ten as key for extending numbers and operations, and apply numbers in practical situations to develop number sense.

Benchmark: 110.1.1.6 – 10.1.1.7 (refer to the benchmarks in unit: Financial Mathematics, Strand 1)

Unit: Financial Mathematics

Topic: (refer to the topics in unit: Financial Mathematics, Strand 1)

Lesson Title: (refer to the topics in unit: Financial Mathematics, Strand 1)

Learning Objectives: (refer to the topics in unit: Financial Mathematics, Strand 1)

What is to be assessed? - (ASK-MT)

Key Concepts(ASK-MT)	
Attitudes / Values	Confidently draw a model for simple interest using graph.
Skills	Draw a model for simple interest using graph
Knowledge	Model for simple interest using graph
Mathematics Thinking	Think about how to draw a model for simple interest using graph.

Purpose of the assessment

To measure students proficiency on the achievement of the benchmarks and learning objectives in this unit. This assessment is to be conducted after teaching this unit.

Expected level of proficiency

All students are expected to;

• Demonstrate the concepts of simple interest using model.

Performance Task

Students will do an assignment out of 20 marks. You can use other assessment tools (assignment, projects, etc.) assess student's proficiency on these benchmarks.

Task

Your task is to model for Simple Interest using a graph that compares interest and the rate for fixed amount of money over a number years. Students will be given two week to complete this assignment.

Task Details

- Draw a graph showing the amount of interest earned over a period of 10 years if K10,000.00 is invested at a rate of 8% *pa.*
- Must be predictive model of the format $I=prn$. This model could be used to predict outcomes into the future.
- Find the time in period in which the earned interest is K550.00

Assessment Strategy
An assignment will be used to measure students' proficiency.

Assessment Scoring
Rubrics must be developed to articulate the real proficiency of the child. This is an analytical rubrics used to assess the child's learning through the assessment tool for assignment.

Criteria	Model/Exemplar	Proficient	Developing	Beginning	Score
	(20 points)	(13-19 points)	(6-12 points)	(2-5 points)	
Quality and Effort	Maximum effort was put forth to complete the assignment in a professional manner. Assignment demonstrates a high degree of quality and attention to detail.	Some effort was made to complete the assignment to a level that was sufficient for grading, but does not meet a professional level of quality or appearance.	Minimal effort was made to complete the assignment but still meets the minimal standard	Little or no effort was made to produce a quality assignment. Assignment obviously does not meet minimal standards.	
Mathematical Calculations	All calculations are very clear, organized, and neatly completed with no in accuracies.	All calculations are clear, organized, and neatly completed with 1-2 inaccuracies.	Most calculations are clear, organized, and neatly completed with 3-4 in accuracies.	Calculations are unclear and disorganized and 5 or more in accuracies may be present.	
Model for Simple Interest	Accurate simple Interest Predictive Model and can be used to predict outcomes into the future, and represented all information correctly; appropriately scaled and labeled.	Simple Interest Predictive Model and can be used to predict outcomes into the future, and few information not represented correctly; labeled but not appropriately scaled.	Some information on Simple Interest Predictive Model seen, bit difficult to predict outcomes into the future, labeled but not appropriately scaled	Graph not neatly produced; not appropriately scaled and labeled,	

STEAM Assessment

Sample 4; (Integrated Strands in relation to the project from integrated subjects)

Unit: (Integrated Units from all Subjects in this project)

Content Standard: (Integrated Content Standard from all Subjects in project)

Benchmark: (Integrated Benchmarks from all Subjects in this project)

Topic: (Integrated Topics from all Subjects in this project)

Lesson topic: (Integrated Topics from all Subjects in concern)

Instructional Objective (s): Students will be able to;

- Create a STEAM project "building a prototype model of a catapult launching system" to enhance their understand of this concept

What is to be assessed? (KSAVs

VASK-MT	
Values/Attitudes	Appreciate the beauty of the application of mathematics during the designing process of the project.
Skills	Calculating size and space Time management and efficiency, Linear measurement and scaling techniques, Calculating mechanical advantage
Knowledge	Size and space Time management and efficiency, Linear measurement and scaling techniques
Mathematical Thinking	Think about how to integrate and apply the mathematical knowledge in the project

Integrated subjects concepts used designing the projects.

Purpose of the assessment

To measure students proficiency on the achievement of the benchmarks and learning objectives for integrated subjects in the project. (STEAM Project)

Expected level of proficiency

All students are expected to;

- "Build a prototype model of a catapult launching system" through integrating concepts learned in other subjects.

Performance Task

Student will carry out a project worth 30 marks that should contribute to the School Learning Improvement Program (SLIP). This project will assess students proficiency on the mentioned benchmarks. In order for this assessment type to attain its intended purpose the following must be done carefully;

Task: Students will be given a month to complete this project.

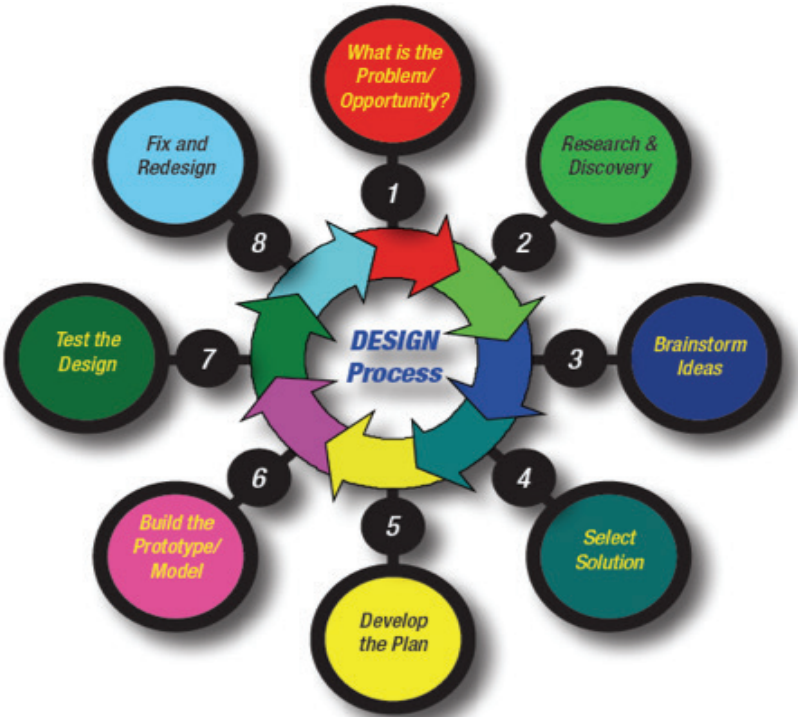
(1) all grade 10 Mathematics teachers discuss the STEAM project with their HOD
(2) the Mathematics HOD brings this project to the attention of the Head Teacher hence it will involve the learning of all grade 10 classes in the school.
(3) once approved by the Head Teacher, the Mathematics HOD now convenes a meeting with all other subject HOD to integrate this project into their learning. HOD for Mathematics will have developed criteria already and will discuss around that.
(4) the HOD for other subjects meet with their respective subject teachers to gauge their views and write up criteria's with reference to the theme of the project, "STEM Design and Engineering Challenge" bringing out the essence of their subjects in this project.
(5) the Head Teacher then convenes a meeting with all teachers as they are now aware of the project. HOD for respective subjects give feedback from their meetings. Issues concerning this project must be ironed out and all subjects now carry out this assessment, starting with Mathematics.

The grade 10 Mathematics teacher will now do the following;

(i) Group the students into groups of 6 to design (drawing and manual) a tangible technology that will enhance the notion of "building a prototype model of a catapult launching system"
(ii) The teacher then assesses their designs and the best designs now compete with the other best designs from other grade 10 classes.
(iii) All the best designers now create models of their designs with assistance from their class members. At this stage the other subjects now carry forward this assessed projects theme, 'building a prototype model of a catapult launching system" however in the context of their subjects. STEAM is an integrated approach of teaching. All subjects must incorporate the theme put forward by Mathematics. They develop criteria that should address this theme. For instance; Technology and Industrial Arts (TIA) will develop criteria that will engage the students to construct the models. Science teachers will develop criteria to test students' knowledge of the Science process of Engineering Design thinking when they create the models around the theme of "prototype model of a catapult launching system". The English subject teachers will set criteria and guidelines for students on how to write reports so they write to tell others what they have learned and experienced. They must also be given guidelines to writing report. Students get to write report of how they designed this technology. The Mathematics teacher will provide criteria for the students in terms of the measurements, angles and operations used to work out the size and shape of the technology.

Task: Students will be given 6 weeks to complete this project. They are to;

- Design and build a prototype model of a catapult launching system that is easy to use and easy to transport.
- Follow the Design Process to prepare their prototype model in time.
- Write and prepare a short presentation to explain the catapult that was built and the process of building it.



Design Specification:

The catapult should be designed to launch a golf ball at least fifteen feet, to a 18 cm x 18cm target.

- The catapult should include a system for determining range, reliability, and accuracy.
- The catapult should be mobile, yet stable. Outriggers or other support systems need to be included to maintain stability when the launcher is used.
- The catapult should be no larger than 30 cm long x 30 cm deep x 90cm tall.
- The catapult should feature a locking pin or trigger that activates the catapult to launch.
- Your team should prepare to deliver a presentation about the merits of your catapult model and design.

Assessment Strategy

Design Project will be used to measure student's proficiency.

The students will be reinforced in the following STEAM concepts.

Science

- Applications of simple machines, including wheels and axles, levers, and pulleys
- Balance and equilibrium
- Energy transformations, such as rotary motion to linear motion
- Mechanical advantage

Technology and Engineering

- Prototyping and modelling
- Invention and innovation
- Structural integrity/strength
- Brainstorming and problem solving
- Trial and error engineering concepts

ARTS

- Perspective drawing (3-D)
- Critical Thinking Process
- Applying the Principles of Graphic design
  - balance
  - proximity
  - repetition
  - colour
  - negative/positive space
- Applying creative process

Math

- Calculating size and space
- Time management and efficiency
- Linear measurement and scaling techniques
- Calculating mechanical advantage

Project Rubric

Category	Advanced	Satisfactory	Partial Credit	Unacceptable
	9 -10 points	7- 8 points	1 - 6 points	0 points
Quality/ Workmanship	Maximum effort was put forth to complete the project in a professional manner. Project demonstrates a high degree of quality and attention to detail. Workmanship is excellent.	Some effort was made to complete the project to a level that was sufficient for grading, but does not meet a professional level of quality or appearance. Workmanship is of acceptable quality.	Minimal effort was made to complete the project and the quality and workmanship is sub-par, but still meets the minimal standard.	Little or no effort was made to produce a quality project. Project obviously does not meet minimal standards.
Creativity/ Design	Project reflects many fundamental elements of design and creativity. Project demonstrates an advanced understanding of creative thinking and attention to aesthetics and presentation.	Project reflects some of the elements of design and creativity, but lacks attention to aesthetics and presentation.	Project was completed, but does not reflect the acceptable levels of design and creativity. Effort was minimal and project is mediocre at best.	Project was not completed on time or reflects little or no effort to complete assignment at an acceptable level.
Functionality	Project meets or exceeds the design requirements of purpose and functionality. All elements of the design have been met and the project does what it was designed to do.	Project meets some of the design requirements of purpose and functionality. Not all elements of the design have been met, but the project does what it was designed to do.	Project is somewhat functional, but reflects minimal effort. It is intermittent and doesn't always do what it was designed to do.	Project does not work and demonstrates a lack of effort or understanding of the basic elements of functionality and purpose.
Design Process	Project reflects a clear understanding and application of design process including evidence of research, brainstorming, design and problem solving, prototyping and testing.	Project reflects some understanding and application of accepted design loop principles and sequence including evidence of research, brainstorming, design and problem solving, prototyping and testing.	Project reflects minimal understanding and application of design process.	Project does not show evidence that design process was used. Project does not meet accepted levels of design criteria.
Criteria/ Constraints	Project was completed with all constraints and criteria met or exceeded. Reflects attention to detail and quality.	Project was completed with some of the constraints and criteria met. Reflects some attention to detail, but quality is minimal.	Project was completed with a few of the constraints and criteria met. Reflects minimal effort and lacks detail or quality.	Project was not completed and does not reflect the adherence to the constraints or criteria.

Time Management	Project completed and turned in on time. Student worked diligently when project time was available. Student was on task most of the time.	Project was completed, but had notable errors. Student utilized project time somewhat efficiently, but spent time socializing. Student was on task 70% - 80% of the time.	Project was not turned in on time and/or complete. The student was on task less than 60% of the time.	Project was not turned in on time and was not completed. Student wasted project time and at times was disruptive to others.
Resource Management	Always takes responsibility for use and care of all building components and resources. Always returns building components and materials to proper storage compartments.	Consistently takes responsibility for use and care of building components and resources. Somewhat consistent in returning building components to proper storage compartments.	Sometimes takes responsibility for use and care of building components and resources. Inconsistent in returning building components to proper storage compartments.	Does not take responsibility for the proper use and care of building components and resources. Is careless and does not practice proper storage and safety practices.
Teamwork	Notable teamwork shown with a determination to participate/contribute to team success. Completed required individual tasks that contributed to the success of the team.	Teamwork was noted, but was sometimes off task or working on non-related tasks. Contributed to the success of the team, but could have been more engaged to complete tasks sooner.	Notable time off-task with minimal effort given for team success, or did the project alone without relying on others to do their share of the project.	Was not a team player. Either took over project completely, or did not engage in team direction or plans.
Writing/ Reflection	Writing/reflection is very well organized and explained. Student includes all details in design process. Document has almost no grammatical errors.	Writing/reflection is somewhat organized and explained. Student includes most details in design process. Document has very few grammatical errors.	Writing/reflection is not organized and explained. Student includes only a few details in design process. Document has many grammatical errors.	Writing/reflection is incomplete or not turned in. Student includes no details in design process. Document has many grammatical errors.
Presentation	Presentation was well organized and presented in a logical sequence. Presentation reflects a full knowledge of the topic with clear answers and explanations to questions asked.	Presentation was fairly organized and most information presented in a logical sequence. Answers to questions were vague or lacked clarity or accuracy.	Presentation was unorganised and lacked a logical sequence. Presentation reflected little attention to detail. Answers to questions were inaccurate and confusing.	Presentation was not acceptable and reflects a lack of organization or knowledge of the topic. Presentation shows little effort to meet expectations.

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Appendices

APPENDIX 1: BLOOM'S TAXONOMY

LEVEL OF UNDERSTANDING	KEY VERBS
CREATING **Can the student create a new product or point of view?**	Construct, design, and develop, generate, hypothesize, invent, plan, produce, compose, create, make, perform, plan,produce, assemble, formulate,
EVALUATING **Can the student justify a stand or decision?**	Appraise, argue, assess, choose, conclude, critique, decide, defend, evaluate, judge, justify, predict, prioritize, provoke, rank, rate, select, support, monitor
ANALYZING **Can the student distinguish between the different parts?**	Analysing, characterize, classify, compare, contrast, debate, criticise, deconstruct, deduce, differentiate, discriminate, distinguish, examine, organize, outline, relate, research, separate, experiment, question, test,
APPLYING **Can the student use the information in a new way**	Apply, change, choose, compute, dramatize, implement, interview, prepare, produce, role play, select, show, transfer, use, demonstrate, illustrate, interpret, operate, sketch, solve, write,
UNDERSTANDING **Can the student comprehend ideas or concepts?**	Classify, compare, exemplify, conclude, demonstrate, discuss, explain, identify, illustrate, interpret, paraphrase, predict, report, translate, describe, classify,
REMEMBERING **Can the student recall or remember the information?**	Define, describe, draw, find, identify, label, list, match, name,quote, recall, recite, tell, write, duplicate, memorise, recall,repeat, reproduce, state,

APPENDIX 2: 21ST CENTURY SKILLS

WAYS OF THINKING	**Creativity and innovation** • Think creatively • Work creatively with others • Implement innovations **Critical thinking, problem solving and decision making** • Reason effectively and evaluate evidence • Solve problems • Articulate findings **Learning to learn and meta-cognition** • Self-motivation • Positive appreciation of learning • Adaptability and flexibility
WAYS OF WORKING	**Communication** • Competency in written and oral language • Open minded and preparedness to listen • Sensitivity to cultural differences **Collaboration and teamwork** • Interact effectively with others • Work effectively in diverse teams • Prioritise, plan and manage projects
TOOLS FOR WORKING	**Information literacy** • Access and evaluate information • Use and manage information • Apply technology effectively **ICT literacy** • Open to new ideas, information, tools and ways of thinking • Use ICT accurately, creatively, ethically and legally • Be aware of cultural and social differences • Apply technology appropriately and effectively
LIVING IN THE WORLD	**Citizenship – global and local** • Awareness and understanding of rights and responsibilities as a global citizen • Preparedness to participate in community activities • Respect the values and privacy of others **Personal and social responsibility** • Communicate constructively in different social situations • Understand different viewpoints and perspectives **Life and career** • Adapt to change • Manage goals and time • Be a self-directed learner • Interact effectively with others

APPENDIX 3: TEACHING AND LEARNING STRATEGIES

STRATEGY	TEACHER	STUDENTS
CASE STUDY Used to extend students' understanding of real life issues	Provide students with case studies related to the topic of the lesson and allow them to analyse and evaluate.	Study the case study and identify the problem addressed. They analyse the problem and suggest solutions supported by conceptual justifications and make presentations. This enriches the students' existing knowledge of the topic.
DEBATE A method used to increase students' interest, involvement and participation	Provide the topic or question of debate on current issues affecting a bigger population, clearly outlining the expectations of the debate. Explain the steps involved in debating and set a criteria/standard to be achieved.	Conduct researches to gather supporting evidence about the selected topic and summarising the points. They are engaged in collaborative learning by delegating and sharing tasks to group members. When debating, they improve their communication skills.
DISCUSSION The purpose of discussion is to educate students about the process of group thinking and collective decision.	The teacher opens a discussion on certain topic by asking essential questions. During the discussion, the teacher reinforces and emphasises on important points from students responses. Teacher guide the direction to motivate students to explore the topic in greater depth and the topic in more detail. Use how and why follow-up questions to guide the discussion toward the objective of helping students understand the subject and summarise main ideas.	Students ponder over the question and answer by providing ideas, experiences and examples. Students participate in the discussion by exchanging ideas with others.
GAMES AND SIMULATIONS Encourages motivation and creates a spirit of competition and challenge to enhance learning	Being creative and select appropriate games for the topic of the lesson. Give clear instructions and guidelines. The game selected must be fun and build a competitive spirit to score more than their peers to win small prices.	Go into groups and organize. Follow the instructions and play to win
OBSERVATION Method used to allow students to work independently to discover why and how things happen as the way they are. It builds curiosity.	Give instructions and monitor every activity students do	Students possess instinct of curiosity and are curious to see the things for themselves and particularly those things which exist around them. A thing observed and a fact discovered by the child for himself becomes a part of mental life of the child. It is certainly more valuable to him than the same fact or facts learnt from the teacher or a book. Students Observe and ask essential questions Record Interpret

PEER TEACHING & LEARNING (power point presentations, pair learning) Students teach each other using different ways to learn from each other. It encourages; team work, develops confidence, feel free to ask questions, improves communication skills and most importantly develop the spirit of inquiry.	Distribute topics to groups to research and teach others in the classroom. Go through the basics of how to present their peer teaching.	Go into their established working groups. Develop a plan for the topic. Each group member is allocated a task to work on. Research and collect information about the topic allocated to the group. Outline the important points from the research and present their findings in class.
PERFORMANCE-RELATED TASKS (dramatization, song/ lyrics, wall magazines) Encourages creativity and take on the overarching ideas of the topic and are able to recall them at a later date	Students are given the opportunity to perform the using the main ideas of a topic. Provide the guidelines, expectations and the set criteria	Go into their established working groups. Being creative and create dramas, songs/ lyrics or wall magazines in line with the topic.
PROJECT (individual/group) Helps students complete tasks individually or collectively	Teacher outline the steps and procedures of how to do and the criteria	Students are involved in investigations and finding solutions to problems to real life experiences. They carry out researches to analyse the causes and effects of problems to provide achievable solutions. Students carefully utilise the problem-solving approach to complete projects.
USE MEDIA & TECHNOLOGY to teach and generate engagement depending on the age of the students	Show a full movie, an animated one, a few episodes form documentaries, you tube movies and others depending on the lesson. Provide questions for students to answer before viewing	Viewing can provoke questions, debates, critical thinking, emotion and reaction. After viewing, students engage in critical thinking and debate

APPENDIX 4: ASSESSMENT STRATEGIES

STRATEGY	DESCRIPTION
ANALOGIES	Students create an analogy between something they are familiar with and the new information they have learned. When asking students to explain the analogy, it will show the depth of their understanding of a topic.
CLASSROOM PRESENTATIONS	A classroom presentation is an assessment strategy that requires students to verbalize their knowledge, select and present samples of finished work, and organize their thoughts about a topic in order to present a summary of their learning. It may provide the basis for assessment upon completion of a student's project or essay.
CONFERENCES	A conference is a formal or informal meeting between the teacher and a student for the purpose of exchanging information or sharing ideas. A conference might be held to explore the student's thinking and suggest next steps; assess the student's level of understanding of a particular concept or procedure; and review, clarify, and extend what the student has already completed
DISCUSSIONS	Having a class discussion on a unit of study provides teachers with valuable information about what the students know about the subject. Focus the discussions on higher level thinking skills and allow students to reflect their learning before the discussion commences.
ESSAYS	An essay is a writing sample in which a student constructs a response to a question, topic, or brief statement, and supplies supporting details or arguments. The essay allows the teacher to assess the student's understanding and/or ability to analyse and synthesize information.
EXHIBITIONS/ DEMONSTRATIONS	An exhibition/demonstration is a performance in a public setting, during which a student explains and applies a process, procedure, etc., in concrete ways to show individual achievement of specific skills and knowledge.
INTERVIEWS	An interview is a face-to-face conversation in which teacher and student use inquiry to share their knowledge and understanding of a topic or problem, and can be used by the teacher to explore the student's thinking; assess the student's level of understanding of a concept or procedure and gather information, obtain clarification, determine positions, and probe for motivations.
LEARNING LOGS	A learning log is an ongoing, visible record kept by a student and recording what he or she is doing or thinking while working on a particular task or assignment. It can be used to assess student progress and growth over time.
OBSERVATION	Observation is a process of systematically viewing and recording students while they work, for the purpose of making programming and instruction decisions. Observation can take place at any time and in any setting. It provides information on students' strengths and weaknesses, learning styles, interests, and attitudes.
PEER ASSESSMENT	Assessment by peers is a powerful way to gather information about students and their understanding. Students can use set criteria to assess the work of their classmates.
PERFORMANCE TASKS	During a performance task, students create, produce, perform, or present works on "real world" issues. The performance task may be used to assess a skill or proficiency, and provides useful information on the process as well as the product.

PORTFOLIOS	A portfolio is a collection of samples of a student's work, and is focused, selective, reflective, and collaborative. It offers a visual demonstration of a student's achievement, capabilities, strengths, weaknesses, knowledge, and specific skills, over time and in a variety of contexts.
QUESTIONS AND ANSWERS (ORAL)	In the question–and-answer strategy, the teacher poses a question and the student answers verbally, rather than in writing. This strategy helps the teacher to determine whether students understand what is being, or has been, presented, and helps students to extend their thinking, generate ideas, or solve problems.
QUIZZES, TESTS, EXAMINATIONS	A quiz, test, or examination requires students to respond to prompts in order to demonstrate their knowledge (orally or in writing) or their skills (e.g., through performance). Quizzes are usually short; examinations are usually longer. Quizzes, tests, or examinations can be adapted for exceptional students and for re-teaching and retesting.
QUESTIONNAIRES	Questionnaires can be used for a variety of purposes. When used as a formative assessment strategy, they provide teachers with information on student learning that they can use to plan further instruction.
RESPONSE JOURNALS	A response journal is a student's personal record containing written, reflective responses to material he or she is reading, viewing, listening to, or discussing. The response journal can be used as an assessment tool in all subject areas.
SELECTED RESPONSES	Strictly speaking a part of quizzes, tests, and examinations, selected responses require students to identify the one correct answer. The strategy can take the form of multiple-choice or true/ false formats. Selected response is a commonly used formal procedure for gathering objective evidence about student learning, specifically in memory, recall, and comprehension.
STUDENT SELF-ASSESSMENTS	Self-assessment is a process by which the student gathers information about, and reflects on, his or her own learning. It is the student's own assessment of personal progress in terms of knowledge, skills, processes, or attitudes. Self-assessment leads students to a greater awareness and understanding of themselves as learners.

APPENDIX 5: Standard-based Lesson Plan Template

Strand: .....

Unit: .....

Content Standard: .....

Benchmark: .....

Topic : .....

Lesson Title: .....

Lesson Objective (s): By the end of the lesson, students will be able to;

- .....
- .....

Materials:

Key Concepts(ASK-MT)	
Attitudes / Values	
Skills	
Knowledge	
Mathematics Thinking	

Lesson Procedure

Teacher Activity	Student Activity
Introduction  (time in minutes)	
Body  (time in minutes)	
Modelling	
Guided Practice	
Independent Practice	
Conclusion  (time in minutes)	

Assessment/Lesson Evaluation

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APPENDIX 6: Standard based Lesson Plan template - Integrating STEAM

Strand: .....

Unit: .....

Content Standard: .....

Benchmark: .....

Topic : .....

Lesson Title: .....

Lesson Objective (s): By the end of the lesson, students will be able to;

- .....

Essential Questions

- .....

Materials:

Key Concepts(ASK-MT)	
Attitudes / Values	
Skills	
Knowledge	
Mathematics Thinking	
STEAM Knowledge and Skills	
Skills	
Knowledge	
STEAM Performance Indicator:	

Lesson Procedure

Teacher Activity	Student Activity
Introduction (time in minutes)	
Body (time in minutes)	
Modeling	
Guided Practice	
Independent Practice	
Conclusion (time in minutes)	

Assessment/Lesson Evaluation

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APPENDIX 7: Time Allocation

Grades 9 and 10	No. lesson/ wk	Min/week	Grades 11 and 12	No. lessons/ wk	Min/week
English	6	6 x 40 = 240	Applied English	6	6 x 40 = 240
Mathematics	5	5 x 40 = 200	L & L	6	6 x 40 = 240
Science	5	5 x 40 = 200	Advance Math	8	5 x 80 = 400
Social Science	5	5 x 40 = 200	Gen Math	6	8 x 40 = 320
PD	5	5 x 40 = 200	Physics	6	6 x 40 = 240
Business Studies	5	5 x 40 = 200	Biology	6	6 x 40 = 240
Design & Technology	5	5 x 40 = 200	Chemistry	6	6 x 40 = 240
Arts	5	5 x 40 = 200	Applied Science	6	6 x 40 = 240
CCVE	3	3 x 40 = 120	Geology	6	6 x 40 = 240
RI	1	1 x 60 = 60	Geography	6	6 x 40 = 240
Agriculture	5	5 x 40 = 200	History	6	6 x 40 = 240
TOTALS	50	2020min/ wk	Legal Studies	6	6 x 40 = 240
			HPE	6	6 x 40 = 240
			PE	6	6 x 40 = 240
			RE	1	1 x 60 = 60
			Business Studies	6	6 x 40 = 240
			Accounting	6	6 x 40 = 240
			Economics	6	6 x 40 = 240
			Design & Tech	6	6 x 40 = 240
			Computer Studies	6	6 x 40 = 240
			ICT	6	6 x 40 = 240
			CCVE	2	3 x 40 = 120
			ANRM	6	6 x 40 = 240
			TOTALS	128 lessons/ wk	5,460 min/wk