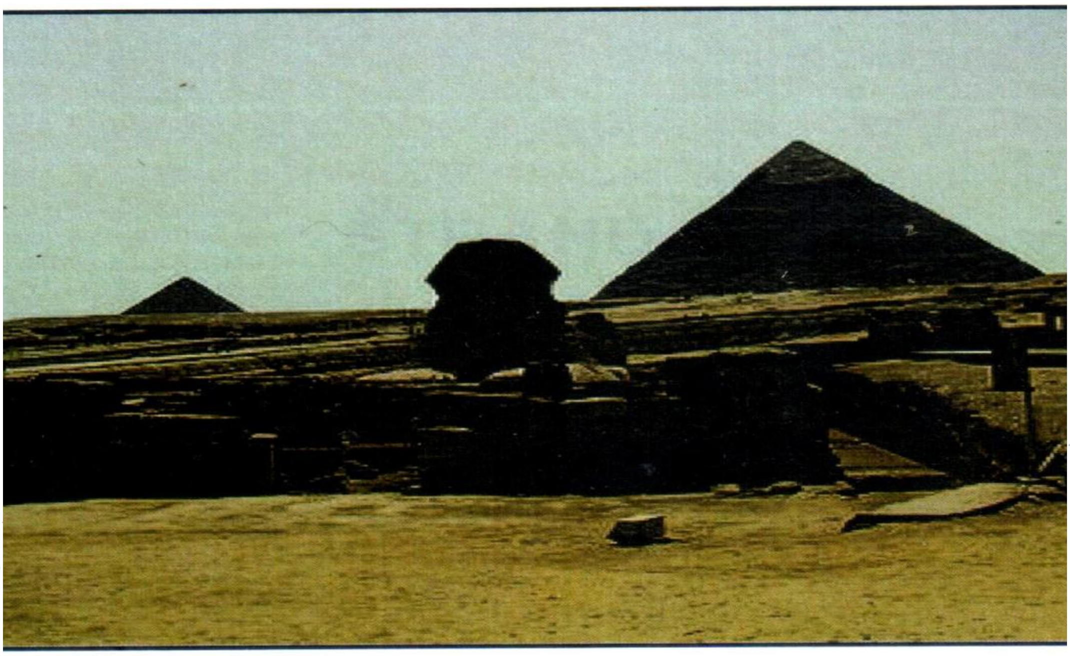
DEPARTMENT OF EDUCATION

GRADE 7

MATHEMATICS

STRAND 6



PATTERNS AND ALGEBRA

Published by:

FLEXIBLE OPEN AND DISTANCE EDUCATION
PRIVATE MAIL BAG, P.O. WAIGANI, NCD
FOR DEPARTMENT OF EDUCATION
PAPUA NEW GUINEA

GRADE 7

MATHEMATICS

STRAND 6

PATTERN AND ALGEBRA

SUB-STRAND 1:	**NUMBER PATTERNS**
SUB-STRAND 2:	**DIRECTED NUMBERS**
SUB-STRAND 3:	**INDICES**
SUB-STARND 4:	**ALGEBRA**

Acknowledgements

We acknowledge the contributions of all Secondary and Upper Primary Teachers who in one way or another helped to develop this Course.

Special thanks to the Staff of the mathematics Department of FODE who played active role in coordinating writing workshops, outsourcing lesson writing and editing processes, involving selected teachers of Madang, Central Province and NCD.

We also acknowledge the professional guidance provided by the Curriculum Development and Assessment Division throughout the processes of writing and, the services given by the members of the Mathematics Review and Academic Committees.

The development of this book was co-funded by GoPNG and World Bank.

MR. DEMAS TONGOGO
Principal- FODE

Written by: Luzviminda B. Fernandez
SCO-Mathematics Department

Flexible Open and Distance Education
Papua New Guinea

Published in 2016

ISBN: 978 - 9980 - 87 - 250 - 0

National Library Services of Papua New Guinea

Printed by the Flexible, Open and Distance Education

CONTENTS

			Page
Secretary"s Message			4
Strand Introduction			5
Study Guide			6
SUB-STRAND 1:		**NUMBER PATTERNS**	7
☐	Lesson 1:	Linear Sequence	9
☐	Lesson 2:	Triangular Numbers	13
☐	Lesson 3:	Square Numbers	18
☐	Lesson 4:	Fibonacci Numbers and Sequences	23
☐	Lesson 5:	The Next Term Patterns	27
☐	Lesson 6:	The Ordinal Number Patterns	31
		Summary	35
		Answers to Practice Exercises 1 - 6	36
SUB-STRAND 2:		**DIRECTED NUMBERS**	41
☐	Lesson 7:	The Set of Integers	43
☐	Lesson 8:	Representing Sets of Integers Diagrammatically	48
☐	Lesson 9	Addition of Integers	52
☐	Lesson 10:	Subtraction of Integers	57
☐	Lesson 11:	Multiplication of Integers	60
☐	Lesson 12:	Division of Integers	64
		Summary	65
		Answers to Practice Exercises 7-12	68
SUB-STRAND 3:		**INDICES**	75
☐	Lesson 13:	Base, Powers and Exponents	77
☐	Lesson 14	Odd and Even Powers	80
☐	Lesson 15:	Squares and Square Roots	84
☐	Lesson 16:	Cubes and Cube Roots	92
☐	Lesson 17:	Higher Powers and Roots	95
☐	Lesson 18:	Expressing Powers as Product of Repeated Factors or Vice Versa	98
		Summary	102
		Answers to Practice Exercises 13 -18	103
SUB-STRAND 4:		**ALGEBRA**	107
☐	Lesson 19:	Pronumerals	109
☐	Lesson 20:	Algebraic Expressions	112
☐	Lesson 21:	Substitution	116
☐	Lesson:22	Evaluation of Numerical and Algebraic Expressions	119
☐	Lesson 23:	Solving Equations with only One Variable	123
☐	Lesson 24:	Solving Equations with Two Variables	129
		Summary	133
		Answers to Practice Exercises 19 – 24	134
REFERENCES			137

SECRETARY‟SMESSAGE

Achieving a better future by individual students and their families, communities or the nation as a whole, depends on the kind of curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum. The learning outcomes are student-centered with demonstrations and activities that can be assessed.

It maintains the rationale, goals, aims and principles of the national curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision by Flexible, Open and Distance Education as an alternative pathway of formal education.

The course promotes Papua New Guinea values and beliefs which are found in our Constitution and Government Policies. It is developed in line with the National Education Plans and addresses an increase in the number of school leavers as a result of lack of access to secondary and higher educational institutions.

Flexible, Open and Distance Education curriculum is guided by the Department of Education‟s Mission which is fivefold:

- to facilitate and promote the integral development of every individual
- to develop and encourage an education system that satisfies the requirements of Papua New Guinea and its people
- to establish, preserve and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make the education accessible to the poor and physically, mentally and socially handicapped as well as to those who are educationally disadvantaged.

The college is enhanced through this course to provide alternative and comparable pathways for students and adults to complete their education through a one system, two pathways and same outcomes.

It is our vision that Papua New Guineans‟ harness all appropriate and affordable technologies to pursue this program.

I commend all the teachers, curriculum writers and instructional designers who have contributed towards the development of this course.

……………………………………………
DR. UKE KOMBRA, PhD
Acting Secretary for Education

STRAND 6: PATTERNS AND ALGEBRA

Dear Student,

This is the Sixth and last Strand of the Grade 7 Mathematics Course.
It is based on the CDAD Upper Primary Mathematics Syllabus and Curriculum Framework for Grade 7.

This Strand consists of four Sub-strands:

Sub-strand 1: Number Patterns
Sub-strand 2: Directed Numbers
Sub-strand 3: Indices
Sub-strand 4: Algebra

Sub-strand 1 - **Number Patterns:** You will learn to explore number patterns and relate them to algebraic statements.

Sub-strand 2 - **Directed Numbers:** You will learn to recognize and explain the use of directed numbers and use them in concrete problems.

Sub-strand 3 - **Indices:** You will learn to use positive indices greater than 1.

Sub-strand 4 - **Algebra:** You will learn to substitute numbers for pronumerals.

You will find that each lesson has reading material to study, worked examples to help you, and a Practice Exercise. The answers to practice exercises are given at the end of each sub-strand.

All the lessons are written in simple language with comic characters to guide you and many worked examples to help you. The practice exercises are graded to help you to learn the process of working out problems.

We hope you enjoy doing this Strand.

All the best!

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Strand.

Step 1: Start with SUB-STRAND 1 Lesson 1 and work through it.

Step 2: When you complete Lesson 1, do Practice Exercise 1.

Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of the SUB-STRAND 1.

Step 4: Then, revise Lesson 1 and correct your mistakes, if any.

Step 5: When you have completed all these steps, tick the check-box for the Lesson, on the Contents Page (page 3) Like this:

[√] Lesson 1: Number Patterns

Then go on to the next Lesson. Repeat the same process until you complete all of the lessons in Sub-strand 1.

> As you complete each lesson, tick the check-box for that lesson, on the Content"s page 3, like this [√]. This helps you to check on your progress.

Step 6: Revise the Sub-strand using Sub-strand 1 Summary, then do Sub-strand Test 1 in Assignment 1.

Then go on to the next Sub-strand. Repeat the process until you complete all of the four Sub-strands in Strand 2.

<u>Assignment:</u> (Four Sub-strand Tests and a Strand Test)
When you have revised each Sub-strand using the Sub-strand Summary, do the Sub-strand Test in your Assignment. The Course book tells you when to do each Sub-strand Test.

When you have completed the four Sub-strand Tests, revise well and do the Strand Test. The Assignment tells you when to do the Strand Test.

> The Sub-strand Tests and the Strand Test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment will count towards your final mark. If you score less than 50%, you will repeat that Assignment.

Remember, if you score less than 50% in three Assignments, your enrolment will be cancelled. So, work carefully and make sure that you pass all of the Assignments.

SUB-STRAND 1

NUMBER PATTERNS

Lesson 1:	**Number Patterns**
Lesson 2:	**The Triangular Numbers**
Lesson 3:	**The Square Numbers**
Lesson 4:	**The Fibonacci Sequence**
Lesson 5:	**The Next Term Patterns**
Lesson 6:	**The Ordinal Number Patterns**

SUB-STRAND 1: NUMBER PATTERNS

Introduction

Patterns in nature have fascinated man since ancient times and this interest is seen in early examples of art and architecture. From the time man first began to count he also became fascinated with patterns in numbers and recognition and study of patterns has led to many new discoveries in mathematics.

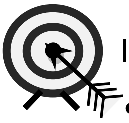
The foundation of mathematics lies in the study of patterns and this sub-strand is planned to help you explore some interesting examples in both numbers and geometry.

For example, the Fibonacci numbers can be seen in the arrangement of petals and seeds on flower heads.

In this sub-strand you will explore number patterns and relate them with algebraic statements. Sequences will be introduced. There are many applications of sequences in science and in the mathematics of finance. This sub-strand is just an introduction to a very extensive and important part of mathematics.

Lesson 1: Linear Sequence

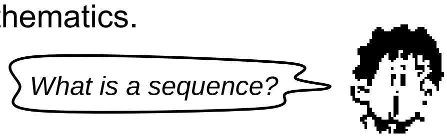
Welcome to Lesson 1 of your Strand 6 Book.



In this lesson you will:

- define the meaning of a sequence
- identify and define a term in a sequence
- find the missing number in a linear sequence.

First, you will learn the meaning of sequence in mathematics.



When you look at the following set of numbers, do you know what number comes next?

30, 40, 50, 60, 70, ...

What number comes next in the following set of numbers?

7, 10, 13, 16, ...

These set of numbers are examples of ***sequences*** of numbers.

A sequence is a set of numbers arranged in a definite order.

Here are some more examples:

1) 0, 1, 2, 3, 4, 5.
2) 2, 4, 6, 8 ...
3) 20, 16, 12 ...
4) 10, 15, 20...
5) $\frac{1}{2}, \frac{3}{4}$, 1, ...

The numbers that make up a sequence are referred to as the **terms** of the sequence.

A sequence is usually written $a_1, a_2, a_3, \ldots, a_n, \ldots$

where a_1 = First term

a_2 = Second term

a_3 = Third term

.
.
.

a_n = n^{th} term (also called the **general term** of the sequence)

.
.
.

The subscript of each term represents the **term number**.

Subscript is a character or symbol written beneath or next to and slightly below a letter or number.


If in counting the terms of a sequence, the counting comes to an end, the sequence is called a **finite sequence.** The last term of a finite sequence is represented by the symbol $\mathbf{a_n}$.

If in counting the terms of a sequence, the counting never comes to an end, the sequence is called an **infinite sequence**.

Here are examples of finite and infinite sequences.

1) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. → The period indicates that the sequence ends here.

   The set of digits is a **finite sequence**. This sequence could also be written 0, 1, 2, ... , 9. Each term is found by adding 1 to the preceding term (except the first term).

2) 0, 1, 2, 3, ... → The three dots indicate that the sequence never ends.

   The set of whole numbers is an **infinite sequence**. Each term is found by adding 1 to the preceding term (except the first term).

3) 10, 5, 0, -5, -10. → The period indicates that the sequence ends here.

   This is a **finite sequence.** Each term is found by adding -5 to the preceding term. (**preceding term** means **term that comes before**)

In the sequences discussed so far, notice that a fixed number is added to each term to find the next one. Thus we call each sequence a **linear sequence.** The fixed number is called the **common difference.**

A sequence is linear if each term after the first is found by adding the same fixed number to find the next term.

A linear sequence can be written if the first term and the common difference are known.

For example:

1) Write the first five terms of the infinite linear sequence whose first term is 7 and the common difference is 6.

Solution:
$a_1 = 7$
$a_2 = 7 + 6 = 13$
$a_3 = 13 + 6 = 19$
$a_4 = 19 + 6 = 25$
$a_5 = 25 + 6 = 31$

Therefore, the Linear Sequence is 7, 13, 19, 25, 31, ...

2) Write a six-term linear sequence whose first term is -8 and the common difference is 5.

Solution:
$a_1 = -8$

$a_2 = -8 + (5) = -3$

$a_3 = -3 + (5) = 2$

$a_4 = 2 + (5) = 7$

$a_5 = 7 + (5) = 12$

$a_6 = 12 + (5) = 17$

Therefore, the linear sequence is $-8, -3, 2, 7, 12, 17$.

In the examples discussed so far, it is possible to determine or discover each succeeding term or terms that follow by inspection.

Examples:

1) Given the sequence 20, 16, 12, ..., state the rule to determine the next three terms.

Rule: Add (-4)

Solution:
20 (1^{st} Term)

$20 + (-4) = 16$ (2^{nd} Term)

$16 + (-4) = 12$ (3^{rd} Term)

$12 + (-4) = 8$ (4^{th} Term)

$8 + (-4) = 4$ (5^{th} Term)

$4 + (-4) = 0$ (6^{th} Term)

Therefore, the next three terms are 8, 4, 0.

2) Given the sequence 10, 15, 20, ..., state the rule and determine the next three terms.

Rule: Add (5)

Solution:
10 (1^{st} Term)

$10 + (5) = 15$ (2^{nd} Term)

$15 + (5) = 20$ (3^{rd} Term)

$20 + (5) = 25$ (4^{th} Term)

$25 + (5) = 30$ (5^{th} Term)

$30 + (5) = 35$ (6^{th} Term)

Therefore, the next three terms are 25, 30, 35.

REMEMBER:

To find the next term in a sequence

- **Look at the first and second term or any two terms in the sequence.**
- **Work out the pattern rule or what number is added to get the next term.**
- **Test the rule to confirm it.**

NOW DO PRACTICE EXERCISE 1

Practice Exercise 1

---

1. State whether the following set of numbers is a linear sequence or not.

   a. 6, 12, 18, 24, ...

   b. 3, 8, 13, 18

   c. 3, $4\frac{1}{4}$, $5\frac{1}{2}$, $6\frac{1}{2}$

   d. 9, 4, -1, -6, ...

   e. 1, 3, 6, 10, 15, ...

---

2. For each of the linear sequences in Question 1, state the rule or work out the number added to each term to find the next term.

---

3. In each of the following sequences below, state the rule and determine the next three terms.

   a. 4, 8, 12, 16, 20, ___, ___, ___

   b. 1, 4, 7, 10, 13, ___, ___, ___

   c. 3, 7, 11, 15, 19, ___, ___, ___

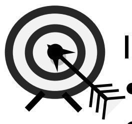
   d. 6, 11, 16, 21, 26, ___, ___, ___

   e. 34, 31, 28, 25, 22, ___, ___, ___

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 2: Triangular Numbers

You learnt the meaning of a sequence in Lesson 1. You also learnt to identify a term in a sequence and find the missing terms in a linear sequence.



In this lesson you will:

- identify triangular numbers
- find the missing term in a sequence of triangular numbers
- draw the triangle pattern for a given triangular numbers.

First, you will identify what triangular numbers are.

Study the number sequence 1, 3, 6, 10, …

The first four terms in this sequence are 1, 3, 6 and 10.

This number sequence is not a linear sequence since there is no fixed number that is added to each term to get the next term. However, the numbers that you add to each term to get the next term form a pattern.

For example,

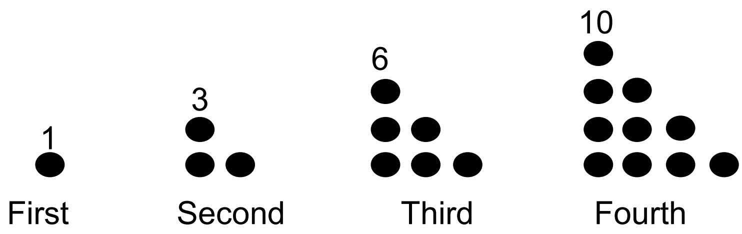
1 + 2 = 3
3 + 3 = 6
6 + 4 = 10
10 + 5 = 15
15 + 6 = 21
and so on…

Looking at the example, we can formulate the pattern. The pattern used is we add:
2 to the first term which is 1
3 to the second which is 3
4 to the third which is 6
then, 5 to the fourth which is 10, to get the fifth term which will be 15 and so on.

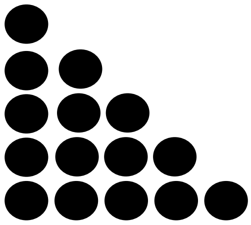
If you increase the list of numbers until you have 10 terms, the result will be: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55,… We call the number sequence a list of **triangular numbers**.

This is the case because the sequence of numbers can be represented by a triangular pattern of dots or by laying out the pattern using counters.

See the figure and count the number of dots in each triangle.



Now let us draw the triangle pattern for the triangle number 15.



Note that 15 is the 5^{th} triangular number, so the last row should be 5.

Check: 1 + 2 + 3 + 4 + 5 = 15

> Numbers that can be represented by a triangular pattern of dots are called **Triangular Numbers.**

The triangular numbers can also be given as a sum of counting numbers.

Example

What are the first fifteen numbers in the set of triangular numbers? Use the pattern below.

1 = 1 1^{st}
1 + 2 = 3 2^{nd}
1 + 2 + 3 = 6 3^{rd}
1 + 2 + 3 + 4 = 10 4^{th}
1 + 2 + 3 + 4 + 5 = 15 5^{th}
………………………… =.. … …
…………………….............=..... …
……………………………… =….. …
……………………………… =. .. 15^{th}

From your completed pattern above notice that:

The second triangular number is the sum of the first **two** counting numbers.

The third triangular number is the sum of the first **three** counting numbers.

The fourth triangular number is the sum of the first **four** counting numbers.

The fifth triangular number is the sum of the first **five** counting numbers.

In general, to find the N^{th} triangular number, you find the value of $\frac{n(n+1)}{2}$ where **n** stands for the counting numbers. Thus the sum of the 6 counting numbers is $\frac{6(6+1)}{2} = \frac{6(7)}{2} = \frac{42}{2} = 21$. Then the **sixth** triangular number is **21**.

Can you work out what the fifteenth triangular number is? the twentieth triangular number? the fifty-first triangular number?

Solution:

To find the fifteenth triangular number we have:

$$\frac{n(n+1)}{2} = \frac{15(15+1)}{2} = \frac{15(16)}{2} = \frac{240}{2} = 120$$

To find the twentieth triangular number we have:

$$\frac{n(n+1)}{2} = \frac{20\ (20+1)}{2} = \frac{20(21)}{2} = \frac{420}{2} = 210$$

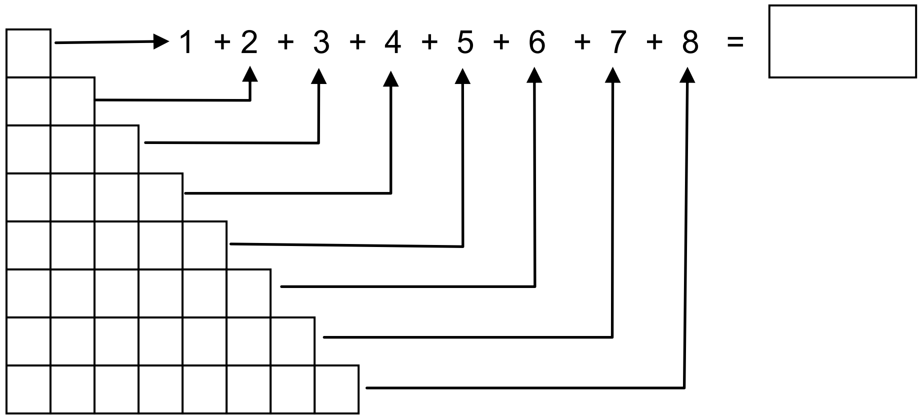
To find the fifty-first triangular number we have:

$$\frac{n(n+1)}{2} = \frac{51\ (51+1)}{2} = \frac{51(52)}{2} = \frac{2652}{2} = 1326$$

NOW DO PRACTICE EXERCISE 2

Practice Exercise 2

1. Study the pattern and answer the following questions.



a. What shape is formed? ____________________

b. How many squares are used? ____________________

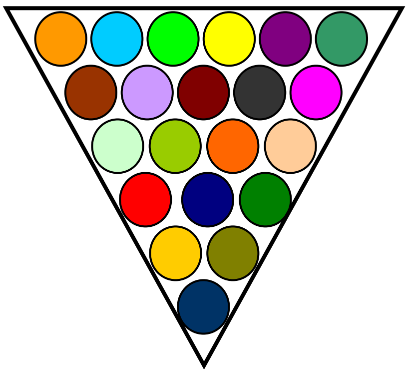
c. What is the sum of the first eight counting numbers? ______________

d. What is the 8th triangle number? ____________________

e. What do you notice about the answers to questions b, c and d?

____

2. Look at the picture.



a. What triangular number is shown here?

b. If the top six balls were removed, would the picture still show a triangular number?

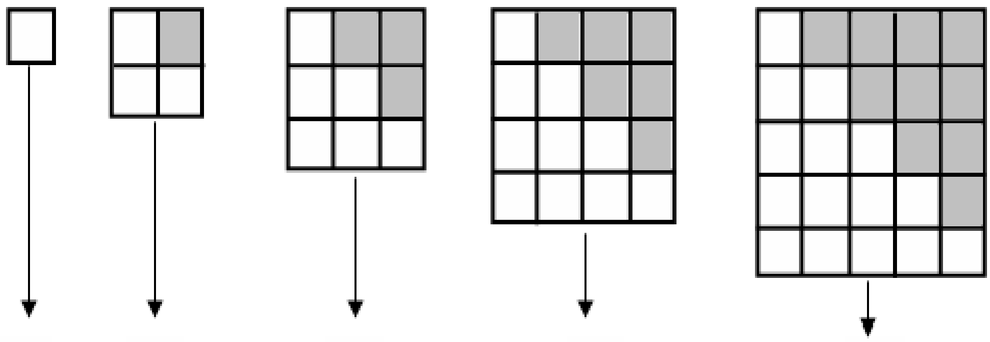
c. Write the triangular numbers that are less than 15.

3 Which of the numbers in the box below are triangular numbers?

55, 14, 78, 66, 17, 37, 21, 16, 89, 9, 10, 610, 100, 15, 707

---

4. Study the pattern.



1 3 6 10 15

The triangular numbers which are next to each other make square numbers.

Copy and complete the statements. The first one has been done for you.

a. 1^{st} Triangular Number + 2^{nd} Triangular Number = 2^{nd} Square Number

1 + 3 = 4

b. 2^{nd} Triangular Number + 3^{rd} Triangular Number = 3^{rd} Square Number

___ + ____ = ______

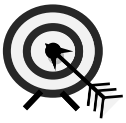
c. 3^{rd} Triangular Number + 4^{th} Triangular Number = 4^{th} Square Number

___ + ____ = ______

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1

Lesson 3: Square Numbers

You learnt about triangular numbers in the previous lesson.



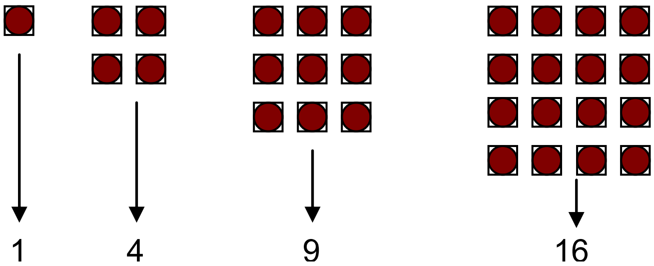
In this lesson you will:

- identify square numbers
- find the missing term in a sequence of square numbers.

Square numbers

A square number is a number that can be arranged in a square pattern.

Study the diagram below.



The square pattern forms the first four terms of a sequence of square numbers.

The first square number, 1, is shown as a square containing a single dot.

So, there is 1 x 1 = 1 dot in this square number.

The second square number, 4, is shown as a square with 2 dots on each side. It is clear that there are 2 rows each containing 2 dots.

So, there are 2 x 2 = 4 dots in this square number.

The third square number, 9, is shown as a square with 3 dots on each side. It is clear that there are 3 rows each containing 3 dots.

So, there are 3 x 3 = 9 dots in this square number.

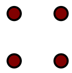
The fourth square number, 16, is shown as a square with 4 dots on each side. It is clear that there are 4rows each containing 4 dots.

So, there are 4 x 4 = 16 dots in this square number.

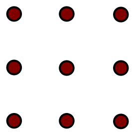
From the above discussion, we find that the following numbers are all squares.

$1^2 = 1 \times 1 = 1$

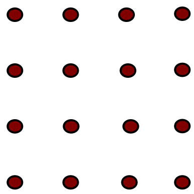
$2^2 = 2 \times 2 = 4$



$3^2 = 3 \times 3 = 9$



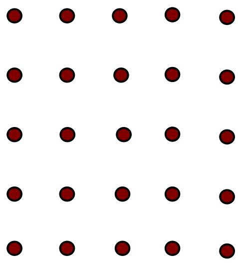
$4^2 = 4 \times 4 = 16$



Following the pattern above, we can find any square number.

Example

The fifth square number will form a square with 5 rows each containing 5 dots.
So, there are 5 x 5 = 25 dots in the square.



Likewise, the sixth square number will form a square with 6 rows each containing 6 dots. So, there are 6 x 6 = 36 dots in the square.

We read 6 x 6 as "6 squared" and it is written as 6^2.

Thus we read $6^2 = 6 \times 6$
$= 36$

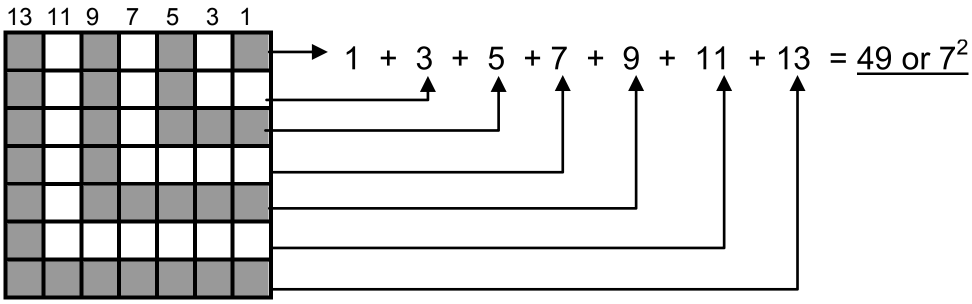
Note that 6^2 is often read as „6 to the power of 2" and 2 is called the index (or exponent).

From the discussion, we derive the generalization or rule that each number in the set of square numbers is the result of multiplying each counting number by itself.

> **Rule: A counting number times itself.**
> e.g. 1 x 1, 2 x 2, 3 x 3, 4 x 4, 5 x 5, 6 x 6, etc.

Now, here is another way of finding the square numbers.

Study the following.



The diagram shows that:

1. the sum of the first two odd numbers is 2^2.
2. the sum of the first six odd numbers is 6^2.
3. the sum of the first seven odd numbers 7^2.

We can see that this pattern will continue.

Now look at the list of the first few Square Numbers:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961, 1024, 1089, 1156, 1225, 1296, 1369, 1444, 1521, 1600, 1681, 1764, 1849, 1936, 2025.

Notice that the next number is made by squaring its position number in the pattern.

Example:

1. The second number is 2 squared (2^2 or 2×2) = 4
2. The seventh number is 7 squared (7^2 or 7×7) = 49
3. The eleventh number is 11 squared (11^2 or 11 x 11) = 121

NOW DO PRACTICE EXERCISE 3

Practice Exercise 3

1. Write all the square numbers from 1 to 300.

2. Draw diagrams to show these square numbers.

   a. 81 b. 64

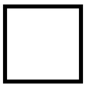
   c. 100 d. 121

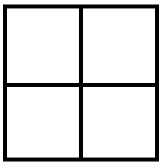
3. What must you add to 25 to get the next square number?

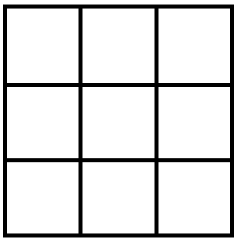
4. Circle the numbers inside the box that you think are square numbers.

38	15	25	33	16
111	64	77	144	1
121	66	81	9	122

5. Study the pattern.

a.  = 1 square = 1 x 1 ⟶ 1^{st} term

b  = 4 squares = 2 x 2 ⟶ 2^{nd} term

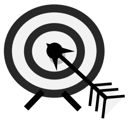
c.  = 9 squares = 3 x 3 ⟶ 3^{rd} term

Complete the pattern for the 4^{th}, 5^{th} and 6^{th} term.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 4: Fibonacci Numbers and Sequences

In Lesson 3, you learnt to identify the square numbers and find the missing term in a sequence of square numbers.

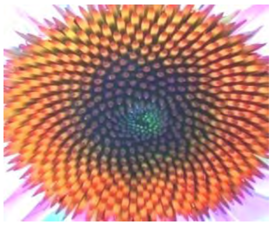


In this lesson you will:

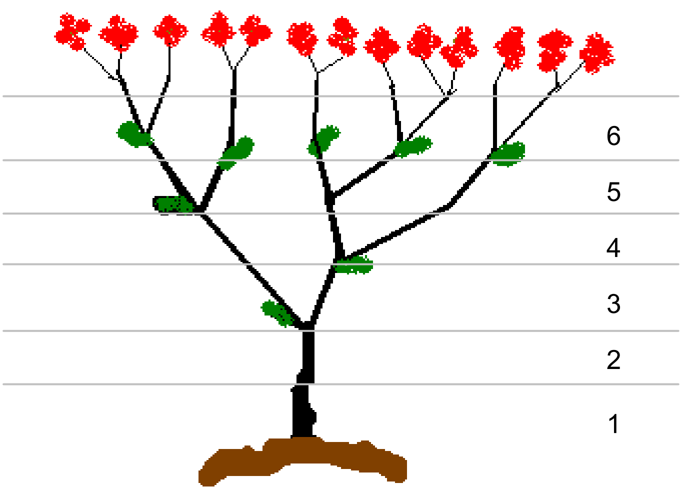
- define the Fibonacci Sequence
- find the missing terms in the Fibonacci sequence.

One of the interesting sequences of numbers is known as the Fibonacci sequence. This sequence has important applications in Botany where it serves to describe the arrangement of leaves on the stems of plants. Many plants follow a Fibonacci sequence. On many plants, the number of petals is a Fibonacci number. You can also see Fibonacci numbers in the arrangement of seeds on flower heads.

The picture shows a large sunflower with 89 and 55 spirals at the edge:



One plant in particular shows the Fibonacci numbers in the number of "growing points" that it has. If a plant puts out a new shoot, that shoot has to grow two months before it is strong enough to support branching. If it grows branches every month after that at the growing point, we get the picture shown here.



Now study the pattern.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

This sequence of numbers makes up the Fibonacci sequence of numbers.

What do you add to each term to get the next? This will apply to the third term onward.

You start with 1 and 1. The third term and those after can be worked out by adding the two previous terms.

Example

1 + 1 = 2

1 + 2 = 3

2 + 3 = 5

3 + 5 = 8

5 + 8 = 13

8 + 13 = 21

13 + 21 = 34

21 + 34 = 55

34 + 55 = ___

… + … = …..

The Fibonacci sequence of numbers is a pattern of numbers in which each term is the sum of the two terms before it, except for the first two in the pattern.

Here is a longer list of the Fibonacci sequence.

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811…

Notice that the next number is found by adding together the two numbers before it.

Example:

1. The 2 is found by adding the two numbers in front of it (1+1)
2. The 21 is found by adding the two numbers in front of it (8+13)
3. The 55 is found by adding the two numbers in front of it (21 + 34)
4. The 144 is found by adding the two numbers in front of it (55 + 89)
5. The next number in the sequence above would be 233 (89+144)

Can you figure out the **next** few numbers?

We can also find the missing term in a Fibonacci sequence using a rule.

The Fibonacci sequence can be written as a "Rule".

The Rule is $x_n = x_{n-1} + x_{n-2}$

where: (Note that this recurrence relation only works for n > or = 3 after T1 and T2 are given).Revise below?

- x_n is term number "n" and n > 3
- x_{n-1} is the previous term (n-1)
- x_{n-2} is the term before that (n-2)

The terms are numbered from 0 onwards like this:

n =	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**...**
x_n =	**0**	**1**	**1**	**2**	**3**	**5**	**8**	**13**	**21**	**34**	**55**	**89**	**144**	**233**	**377**	**...**

Example:

1. Term 6 would be calculated like this:

$$x_6 = x_{6-1} + x_{6-2} = x_5 + x_4 = 5 + 3 = 8$$

2. Term 9 would be calculated like this:

$$x_9 = x_{9-1} + x_{9-2} = x_8 – x_7 = 21 + 13 = 34$$

3. Find the 15^{th} term of the Fibonacci sequence.

$$x_{15} = x_{15-1} + x_{15-2} = x_{14} + x_{13} = 377 + 233 = 610$$

NOW DO PRACTICE EXERCISE 4

Practice Exercise 4

A. Do the following.

1. Write down the first 15 terms of the Fibonacci sequence.

2. Add the first 5 Fibonacci numbers. Compare this sum with the 7th Fibonacci number, how much less is the sum from the 7th Fibonacci number?

3. Add the first 10 Fibonacci numbers. Compare this sum with the 12th Fibonacci number, how much less is the sum from the 12th Fibonacci number?

4. Find out if the pattern in Questions 2 and 3 is true for the rest of the numbers you have written in Question 1.

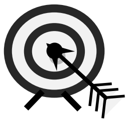
5. Circle the numbers inside the box, which do you think are Fibonacci numbers?

78	55	25	33	14
16	64	707	144	91
21	66	51	89	100

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 5: The Next Term Patterns

You learnt to define and identify Linear, Triangle, Square and Fibonacci sequences in the previous lessons.



In this lesson you will:

- work out the unknown term in a given sequence given the other terms.

Linear, Triangle, Square and Fibonacci sequences all have „next term" patterns. This means there is a rule for getting the next term from the terms that you have already known.

Now let us consider the following patterns of numbers.

1. 2, 4, 6…
2. 27, 9, 3...

The next terms in the line of these sequences of numbers could be found because each set of numbers follows a rule or a pattern.

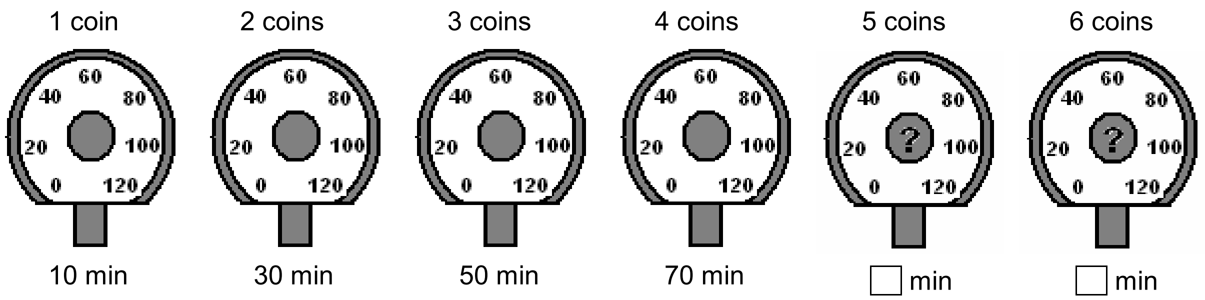
For the numbers 2, 4, 6…, the next term or number is known by using the pattern rule **„add 2"**.

For the sequence 27, 9, 3…, the next term or number is known by using the pattern rule **„divide by 3"**.

Here are other examples.

Example 1

A parking meter takes K1.00 coins. The picture below will show you what happens as Mathias puts in his coins.



Questions

1. Is there a pattern?

2. What is the rule used?

3. What parking time would you expect for a) 5 coins? b) 6 coins?

Solution:

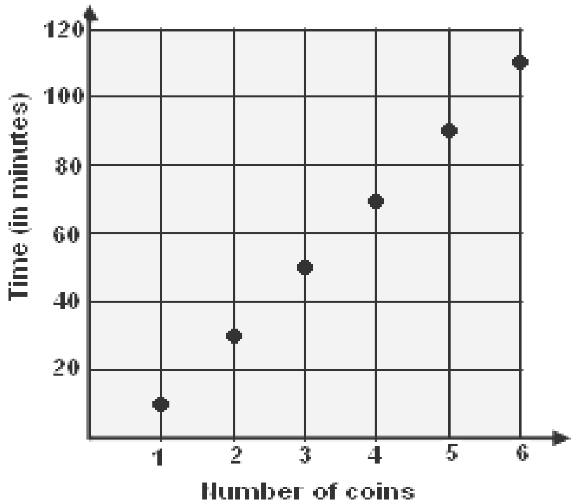
1. There is a pattern.

2. You start at 10 and then follow the rule "**add 20**"

3. a) For 5 coins you could park for 90 minutes
   b) For 6 coins you could park for 110 minutes.

We can show this pattern by drawing a graph.

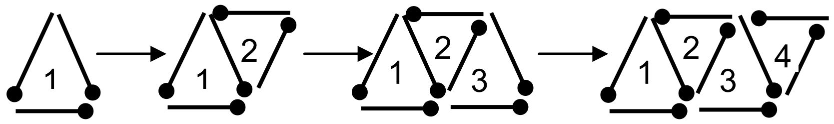
Let us draw the graph.



Example 2

Complete the number pattern for the number of matches needed to make these triangles.

No. of triangles



5

No. of matches 3 ? ? ? ?

What is the rule used to get from one number in the pattern to the next?

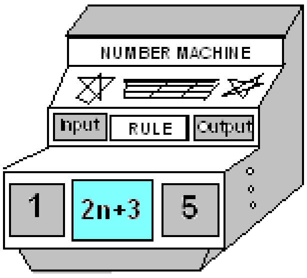
Solution:

Number pattern is 3, 5, 7, 9, 11…

The rule used is "**add 2**".

Example 3

Given the sequence 5, 7, 9, 11, 13, 15, ..., the terms of this sequence can be found as output numbers on the number machine, by using the pattern rule in the table.



Pattern Rule 2n + 3	
Input	Output
1	5
2	7
3	9
4	11
5	13
6	15
…	…

The input number gives the position of each term in the sequence.

Thus, 1^{st} term = 2 x 1 + 3 = 5

2^{nd} term = 2 x 2 + 3 = 7

3^{rd} term = 2 x 3 + 3 = 9

and so on, giving the sequence 5,7,9, 11, 13, 15,…

Example 4

Find the missing terms in the sequence 151, 123, 95, …, 39, …, -17.

Solution: The rule is "**subtract 28**"

95 – 28 = 67
39 – 28 = 11

Thus, the missing terms in the sequence are 67 and 11.

NOW DO PRACTICE EXERCISE 5

Practice Exercise 5

1. Write the missing number from each pattern.

   a. 3, 10, …………., 24, 31
   b. 4, 9, ………, 19, 24
   c. 6, …………, 24, 48, 96
   d. 5, 8, ……, 17, 23
   e. 2, ……….18, 54, 162

2. Using the first number and the rule given, write down the next two numbers in each pattern.

   a. 5; add 4
   b. 12; add 9
   c. 3; multiply by 3
   d. 3; multiply the number by itself
   e. 100; divide by 2

3. Copy and complete these tables and write down the first 6 terms of the sequence formed by the output numbers in each case.

Pattern Rule $2n + 5$	
Input	Output
1	
2	
3	
4	
5	
6	
…	

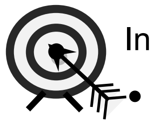
Pattern Rule $3n - 2$	
Input	Output
1	
2	
3	
4	
5	
6	
…	

Pattern Rule $n^2 + 1$	
Input	Output
1	
2	
3	
4	
5	
6	
…	

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 6: The Ordinal Number Patterns

In Lesson 5, you learnt to work out the unknown terms in a given sequence given the other terms.



In this lesson you will:

- create a number pattern using your own rule in working out number patterns

When objects are placed in order, we use ordinal numbers to tell their position. Ordinal numbers are similar to the numbers that you learnt before (cardinal numbers).

If ten students ran a race, we would say that the student that ran the fastest was in first place, the next student was in second place, and so on.

The first ten ordinal numbers are:

First = 1^{st}
Second = 2^{nd}
Third = 3^{rd}
Fourth = 4^{th}
Fifth = 5^{th}
Sixth = 6^{th}
Seventh = 7^{th}
Eighth = 8^{th}
Ninth = 9^{th}
Tenth = 10^{th}

In the sequence 5, 10, 15, 20,25,…each has an ordinal number. 25 is the 5^{th} term. Its ordinal number is 5^{th}, or sometimes just written 5. The symbol for ordinal number is **N**.
So, for the term 25, N = 5.

We can write out a table connecting the terms in the sequence to their ordinal numbers.

Look at the table below.

N	1	2	3	4	5
Term	5	10	15	20	25

The pattern rule that connects the ordinal number and the term is

$5 = 5 \times 1$, $10 = 5 \times 2$, $15 = 5 \times 3$, $20 = 5 \times 4$, $25 = 5 \times 5$

or in general terms, N^{th} term = 5 x N

We can use this general expression to work out other terms in the sequence.

For example, the 20^{th} term and 40^{th} term in the linear sequence above would be

20^{th} Term = 5 x 20 = 100.

40^{th} Term = 5 x 40 = 400

Here are other examples.

1. The table below connects the terms in the sequence to their ordinal numbers. Find the pattern rule that connects the ordinal number and the term.

N	1	2	3	4	5	6	7
Term	1	4	9	16	25		

Solution:

Let us describe the sequence of numbers starting from the 1^{st} Term which is 1.

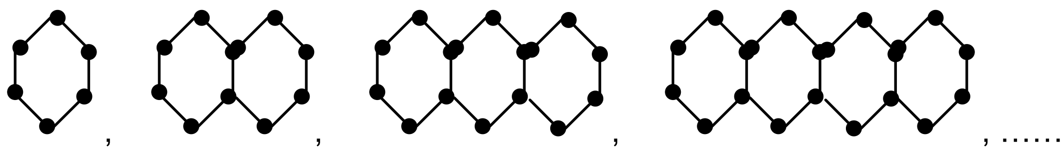
N	1	2	3	4	5	6	7	N
Pattern Rule	1x 1	2 x 2	3 x 3	4 x 4	5 x 5	6 x 6	7 x 7	N x N
Term	1	4	9	16	25			N^2

The pattern rule that connects the ordinal number and the term is N X N = N^2

Thus, to find the sixth and seventh terms in the table would be:

6^{th} Term = N x N = 6 x 6 = 36
7^{th} Term = N x N = 7 x 7 = 49

2. Matchsticks have been used to make the pattern of hexagons below.



This table shows the number of match sticks needed for each part of the pattern.

No. of hexagons (h)	1	2	3	4	5	6	100
No. of matchsticks (N)	6	11	16	21			

What is the rule for this pattern?

Solution:

The pattern rule that links the number of matches used to the number of hexagons can be described in two methods. Go to the next page to understand the two methods,

Method 1

Let us describe the pattern starting from 1 match.

No. of hexagons (h)	1	2	3
No. of matchsticks (N)	1 + 5 = 6	1 + 5 + 5 = 11	1 + 5 + 5 + 5 = 16
or	1 + (5 x 1) = 6	1 + (5 x 2) = 11	1 + (5 x 3) = 16
or	1 + (1 x 5) = 6	1 + (2 x 5) = 11	1 + (3 x 5) = 16

We could describe the pattern rule as: The number of matchsticks is "one plus 5 times the number of hexagons, or N = 1 + 5 x h

Method 2

Let us describe the pattern starting from 1 hexagon (6 matches)

No. of hexagons (h)	1	2	3
No. of matchsticks (N)	6	6 + 5 = 11	6 + 5 + 5 = 16
or	6	6 + (5 x 1) = 11	6 + (5 x 2) = 16

We describe the pattern rule as: The number of matchsticks is "6 plus 5 times one less than the number of hexagons, or N = 6 + 5 x (h - 1)

Now using these two rules, we can find the next term in the table.

For 5 hexagons: Method 1 would lead us to 1 + (5 x 5) = 26
Method 2 would lead us to 6 + (5 x 4) = 26

For 6 hexagons: Method 1 would lead us to 1 + (5 x 6) = 31
Method 2 would lead us to 6 + (5 x 5) = 31

For 100 hexagons: Method 1 would lead us to 1 + (5 x 100) = 501
Method 2 would lead us to 6 + (5 x 99) = 501

- **If we can find a rule, we can work out any term in the pattern.**
- **In the rule, the letters used are called variables because they can stand for any chosen value. They can vary.**

NOW DO PRACTICE EXERCISE 6

Practice Exercise 6

1. Consider the sequence 2, 4, 6, 8, 10, 12,…

   The first term is 2 x 1

   The second term is 2 x 2

   The third term is 2 x 3 and so on

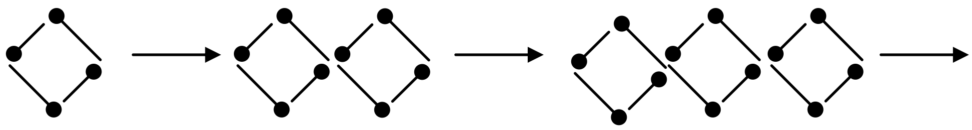
   (a) What would be the 10^{th} term?

   (b) What would be the 20^{th} term?

   (c) What would be the 100^{th} term?

   (d) What would be the N^{th} term?

2. Matches have been used to make the following figures.



   a) Find the rule that links the number of matches used to the number of squares in each figure.

   b) Complete the table below.

Number of squares	1	2	3	4	5	6	7	8
Number of matchsticks								

3. For each of the following sequences, write down the next term and the 10^{th} term.

   a) (1 + 1), (2 + 1), (3 + 1), (4 + 1), (5 + 1), …

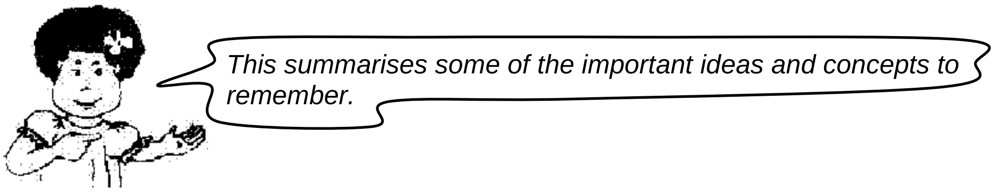
   b) (3 + 1), (3 + 2), (3 + 3), (3 + 4), (3 + 5), …

   c) (1 x 1), (2 x 2), (3 x 3), (4 x 4), (5 x 5), …

   d) (10 - 1), (10 - 2), (10 - 3), (10 - 4), (10 - 5), …

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUBSTRAND 1.

SUB-STRAND 1: SUMMARY



- A **sequence** is a set of numbers arranged in a definite order. The numbers that make up a sequence are called the **terms** of the sequence.
- If in counting the terms of a sequence, the counting comes to an end, the sequence is called a **finite sequence.**
  If in counting the terms of a sequence, the counting never ends, the sequence is called an **infinite sequence.**
- A sequence is **linear** if each term after the first is found by adding the same fixed number to find the next term.
- **Triangular numbers** are numbers that can be represented by a triangular pattern of dots.
- **Square numbers** are numbers that can be arranged in a square pattern.
- The **Fibonacci sequence** of numbers is a pattern of numbers in which each term is the sum of the two terms before it, except for the first two in the pattern.
- The **Nth Term** of a sequence is called the **general term** and any term of the sequence can be calculated by using the rule.
- **Ordinal numbers** are numbers that give or tell the position of an object when they are placed in order. Ordinal numbers tells the position of each term in a sequence or pattern.

REVISE LESSON 1-6 THEN DO SUB-STRAND TEST 1 IN ASSIGNMENT 6.

ANSWERS TO PRACTICE EXERCISES 1- 6

Practice Exercise 1

1. a. linear
   b. linear
   c. not linear
   d. linear
   e. not linear

2. a. add 6 b. add 5 d. subtract 5

3. a. add 4; 24, 28, 32
   b. add 3; 16, 19, 22
   c. add 4; 23, 27, 31
   d. add 5; 31, 36, 41
   e. subtract 3; 19, 16, 13

Practice Exercise 2

1. a. triangle
   b. 36
   c. 36
   d. 36
   e. they are the same and 36 is a triangular number

2. a. 21
   b. yes
   c. 1, 3, 6, 10

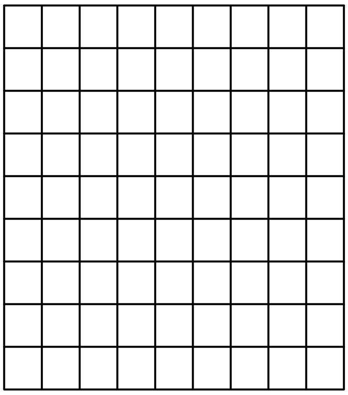
3. 10, 15, 21, 55, 66,78

4. b. 3 + 6 = 9
   c. 6 + 10 = 16

Practice Exercise 3

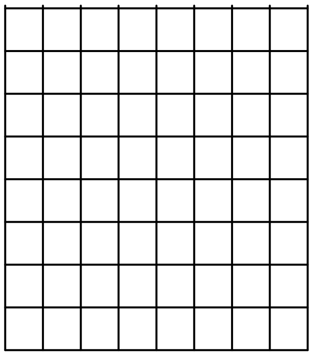
1. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225. 256, 289

2. a.



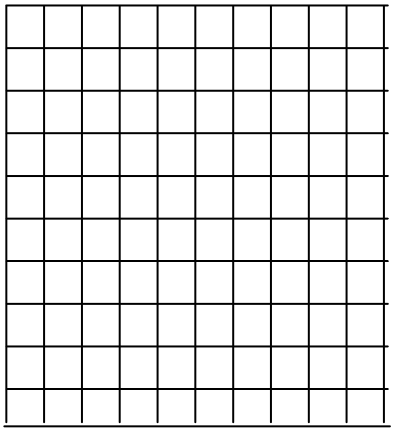
9 x 9 = 81

b.



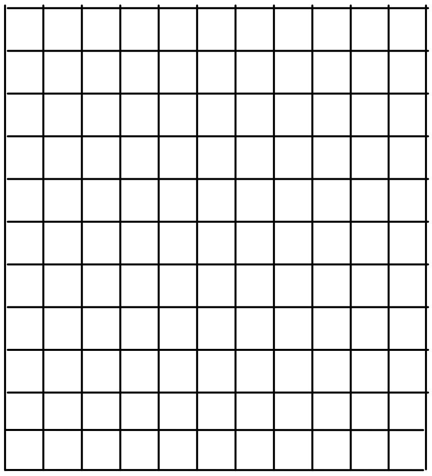
8 x 8 = 64

c.



10 x 10 = 100

d.



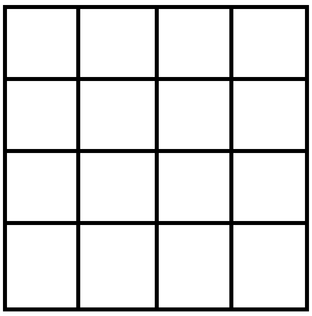
11 x 11 = 121

3. 11

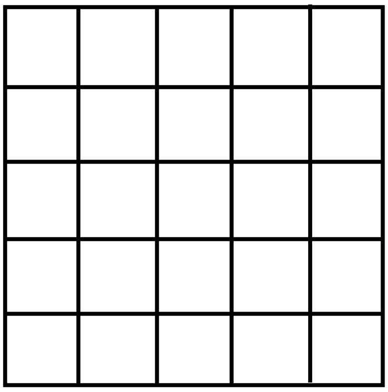
4.

38	15	(25)	33	(16)
111	(64)	77	(144)	(1)
(121)	66	(81)	(9)	122

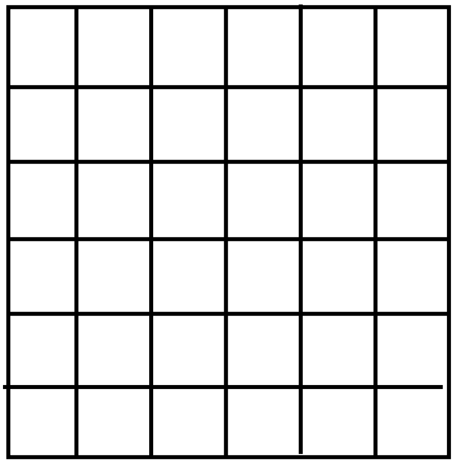
5.



12 squares = 4 x 4 = 4th term



25 squares = 5 x 5 = 5^{th} term



36 square = 6 x 6 = 6^{th} term

Practice Exercise 4

1. 1,1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610
2. Sum is 12. Yes
3. Sum is 143. Yes
4. The pattern is true to the rest of the numbers in Question 1
5.

78	55	25	33	14
16	64	707	144	91
21	66	51	89	100

Practice Exercise 5

1. a. 17
   b. 14
   c. 12
   d. 12
   e. 6

2. a. 9, 13
   b. 21,30
   c. 9, 27
   d. 9, 81
   e. 50, 25

3.

Pattern Rule 2n + 5	
Input	Output
1	**7**
2	**9**
3	**11**
4	**13**
5	**15**
6	**17**
…	**…**

Pattern Rule 3n -2	
Input	Output
1	**1**
2	**4**
3	**7**
4	**10**
5	**13**
6	**16**
…	**…**

Pattern Rule $n^2 + 1$	
Input	Output
1	**2**
2	**5**
3	**10**
4	**17**
5	**26**
6	**37**
…	**…**

Practice Exercise 6

1. a. 2 x 10
   b. 2 x 20
   c. 2 x 100
   d. 2 x N

2. a. Number of matches = (Number of squares) x 4
   b.

Number of squares	1	2	3	4	5	6	7	8
Number of matchsticks	**4**	**8**	**12**	**16**	**20**	**24**	**28**	**32**

3. a. (6 + 1), (10 +1)
   b. (3 + 6), (3 + 10)
   c. (6 x 6), (10 x 10)
   d. (10 – 6), (10 -10)

END OF SUB-STRAND 1

SUB-STRAND 2

DIRECTED NUMBERS

Lesson 7:	**The Set of Integers**
Lesson 8:	**Diagramming Sets of Integers**
Lesson 9:	**Addition of Integers**
Lesson 10:	**Subtraction of Integers**
Lesson 11:	**Multiplication of Integers**
Lesson 12:	**Division of Integers**

SUB-STRAND 2: DIRECTED NUMBERS

Introduction

In our previous lessons, we dealt with whole numbers. Whole numbers are use when we wish to express quantities such as 5 pawpaws, 12 kilometres, 6 hours, or K35. There is no confusion since 5 pawpaws is exactly 5 pawpaws and K35 means exactly K35.

Suppose John says there is a difference of K250 in his savings since the year started. What does he mean? Is his savings K250 more or K250 less? Similarly, if Job says he lives 4 blocks from the church, does it mean that his house is 4 blocks east of the church or 4 blocks west?

In the equation, n + 8 = 3, there is no whole number that will make the equality true; 3 – 8 is not meaningful in the system of whole numbers.

The situations cited suggest the need to expand the whole number system so that ideas can be conveyed more clearly and more meaningfully and how any equation like the one given above can be solved.

We use numbers in our everyday life. Also, think of numbers when considering opposites. Situations of opposing actions are very common in our everyday life. We often use words to indicate opposite actions. Some examples are winning and losing in a game, depositing and withdrawing money, turning left versus turning right. Likewise, east is the opposite of west, north is the opposite of south, opening and closing the door, the rise and fall of prices are also examples. Can you give other examples?

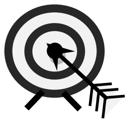
In Mathematics, the symbols (+) and (–) are used to indicate opposite directions. In mathematical language, 5 blocks east is expressed as +5, 5 blocks west as -5; 6 kilos heavier as +6, 6 kilos lighter as -6, K2 increase in price as +2, and K2 decrease in price as -2. In these examples, there is a number that refers to a definite quantity and to a word that tells you the direction. Numbers that represent both quantity (magnitude) and direction are called **directed numbers.**

In this sub-strand, you will recognize and explain the use of directed numbers and use them in concrete problems.

.

Lesson 7: The Set of Integers

In the previous lessons, you dealt with whole numbers and explored number patterns.

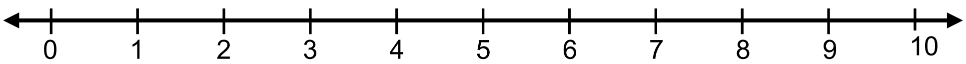


In this lesson, you will

- identify integers and their order on a number line
- arrange directed numbers in increasing and decreasing order
- use directed numbers to indicate real life quantities with opposite senses.

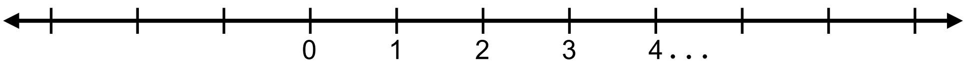
The Set of Integers

Early in your study of arithmetic you learnt to represent the whole numbers on the number line as in the illustration below.

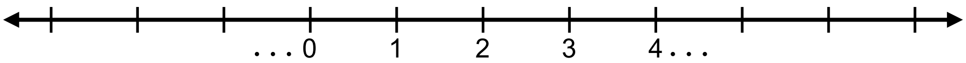


You also learned that a line extends infinitely in both directions. Have you ever wondered that there must be numbers on the left of 0 since there are numbers on the right? And given that there are numbers to the right of 0, what numbers can be placed to its left? The answer is, the opposites of those to its right, and the two sets together with 0 form the **set of integers.**

To understand the new set of numbers, let us go back to the number line above and mark the whole numbers on it as usual.



Now, equally placed points can also be marked on the left of 0. We can also label these points as 1, 2, 3,... if we like, but in doing this we would not know whether we are referring to a point to the right or to its left. To avoid confusion, a minus sign (-) is placed before each of the numbers to the left of 0. We now have a more complete number line.



To express the difference in direction or sense of the numbers to the right of zero from those on the left, each number is given a "+" sign. This means the numbers to the right of zero are written +1, +2, +3,… These numbers are called **positive integers** while those to the left of zero are called **negative integers.** Zero is written without a sign.

The set of integers consists of the positive numbers, the negative numbers and zero.

Sometimes the set of integers is also called the **set of directed numbers** because it indicates opposing directions or positions with respect to a reference point. It can also be made to indicate oppositeness of nature. For example, if the positive numbers are used to represent forward movements, the backward movement must be represented by the negative numbers.

Here are more examples

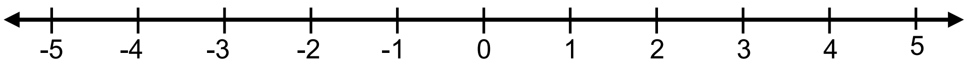
1. Going 15 kilometres upstream may be written as +15 and 15 kilometres downstream may be written as -15.

2. A gain of K25 may be represented by +25 and a loss of K25 may be represented by -25

3. a rise in temperature of 5 degrees may be written as +5 and a drop in temperature of 3 degrees by -3.

Can you think of similar applications of the set of integers?

It is also called the **set of signed numbers** because it uses the plus (+) and minus (-) signs. A positive integer does not need to have a (+) sign before it; it is understood to be positive.



Points that are equally distant from zero, the **origin**, but are located on the opposite sides of it, are referred to as **opposites**. For example: -1 and 1, -2 and 2, -3 and 3, -4 and 4 and so on. That is -1 is the opposite of 1, -2 is the opposite of 2, -3 is the opposite of 3, -4 is the opposite of 4, and so on.

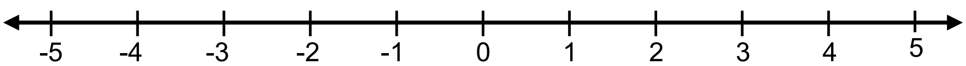
What is the opposite of the opposite of a number?

The opposite of the opposite of a number is the number itself.

Order on the Number Line

The order relations greater than (>) and less than (<) can be explained clearly on the number line by "to the right of" and "to the left of", respectively.

See the number line below.



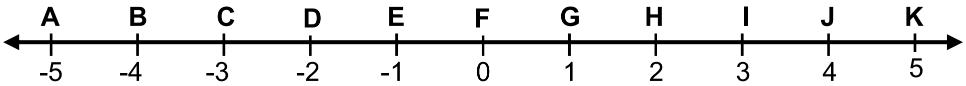
A number to the right of another on the number line is the greater number; as for example, 3 > 0, 4 > 1, 5 >2, 0 > -4, -1 > -3 and -2 > -5.

A number to the left of another on the number line is the lesser number; as for example, 1 < 2, -5 < 0, -2 < -1, -4 < 4 and 0 < 3.

How do we compare two negative numbers? Which is greater (-1) or (-3)? From the number line you will notice that (-1) is to the right of (-3), so (-1) > (-3).

All positive numbers are to the right of zero. All positive numbers are greater than zero.
All negative numbers are to the left of zero. All negative numbers are less than zero.
All positive numbers are to the right of negative numbers, therefore any positive number is greater than any negative number.
Of two positive or two negative numbers, the one located to the right of the other in the number line has the greater value.

The number line shows not only the orders of numbers, but also the distance between a point and zero. Each space is one unit.

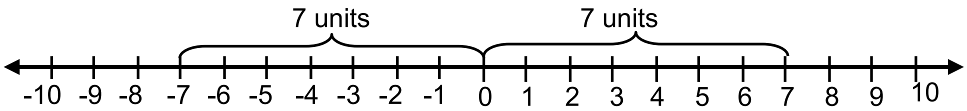


Given the distance 2 units from 0 to a certain point, find the point or points in the number line. The answers are C and G because C is 2 units away from zero and G is also 2 units away from zero. What are the numbers corresponding to the points C and G?

What is the distance between 0 and 4?
What is the distance between 0 and (-4)?
What is the distance between 0 and 6?
What is the distance between 0 and (-6)?

Absolute Value

The **absolute value** of a number is the distance of that number from the origin 0 on a number line. The symbol for absolute value is two vertical bars | |. Thus, |-7| is read as “the absolute value of negative seven” and |+7| is read as “the absolute value of positive seven”. The absolute value of –7 is 7 and the absolute value of +7 is 7.



The distance between -7 and 0 and between 0 and 7 is 7.

The absolute value of any number is greater than or equal to zero and has no sign.

The distance of a point from zero gives the absolute value of the numbers, that is, the absolute value of +2 is 2 and the absolute value of -2 is 2.
The absolute value of the number is the value of the number regardless of its sign.
Using the absolute value notation, |4| = 4, |-4| = 4

Here are more examples.

1. |75| = 75 2. | 0 | = 0 3. |-101| = 101

NOW DO PRACTICE EXERCISE 7

Practice Exercise 7

1. Represent by the appropriate integer

   a. A withdrawal of K100 __________
   b. A debt of K5000 __________
   c. A gain of K25 __________
   d. 300 m above sea __________
   e. Earnings of K5000 in a business __________
   f. A temperature of 10º below zero __________
   g. A profit of K450 __________
   h. A salary increase of K100 __________
   i. Going 5 kilometres upstream __________

2. Write the phrase that is the opposite of the following.

   a. 10 kilometres east __________
   b. 7 steps to the right __________
   c. 5 kilos overweight __________
   d. A price increase of 5 __________
   e. 12º east longitude __________

3. Replace each □ with< or > to make a true statement.

a.	8 □ 5	f.	-5 □ -7
b.	-5 □ 8	g.	6 □ -8
c.	-5 □ -8	h.	0 □ -2
d.	-3 □ -1	i.	-3 □ -7
e.	-I □ -5	j.	-10 □ 0

4. Arrange each set of integers from smallest to the largest.

   a. {-3, 4, 7, -10, 0} __________
   b. {-5, -1, -8, 20, -15} __________
   c. {-2, -18, 51, -21, 16} __________
   d. {10, 3, 5, -3, -31} __________
   e. {-6,-16, 6, -26,16} __________

5. Write the indicated integer if it exists.

a. the integer 5 units to the right of -2 __________
b. the integer 4 units to the right of 3. __________
c. The integer 7 units to the left of 4. __________
d. The integer 10 units to the right of -5 __________
e. The integer that is neither positive nor negative. __________

---

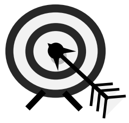
6. Which of the following sentences are true?

a. $|3| > |1|$
b. $|-6| < |7|$
c. $|2| < |-4|$
d. $|-3| \leq |-4|$
e. $|-3| < |3|$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 8: Representing Sets of Integers Diagrammatically

In the previous lesson, you learnt the meaning of integers and their order on a number line. You also learnt to arrange directed numbers in ascending and descending order, the meaning of opposites and use directed numbers to indicate oppositeness in nature.



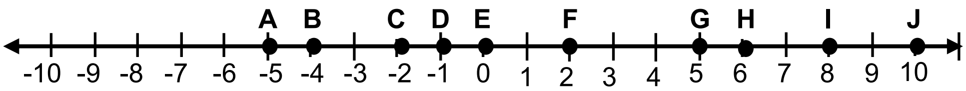
In this lesson, you will:

- diagram or graph the set of integers on the number line
- identify which set of integers is represented from a given graph or diagram

---

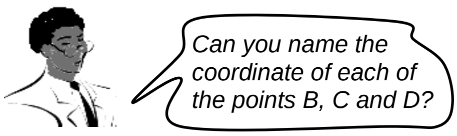
The set of integers can be presented graphically on the number line.

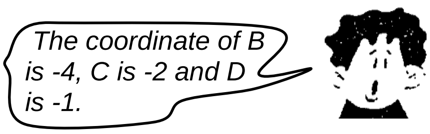
Look at the number line below.



Each point on the number line that is associated with an integer is called a **diagram** or **graph** of that integer, while the integer is called the **coordinate** of that point.

We have used capital letters to name some points on the number line shown. Point **A** is associated with -5 and we say that -5 is the coordinate of point **A**. The graph of -5 is shown by the heavy dot for the point associated with that integer. From the number line we can also say that point **F** is the graph of +2.



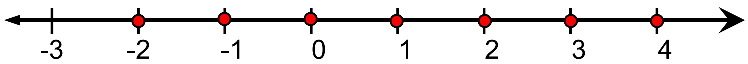


Now we are going to represent set of integers on a number line.

Look at the following examples.

1. List the numbers described and graph them on a number line.

   a. The integers greater than -3

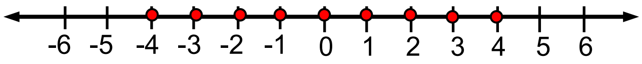


The integers greater than -3 are -2, -1, 0, 1, 2, 3, and so on or {-2, -1, 0, 1, 2, 3, …}.

To show the infinite continuing pattern, we use 3 dots in the set. On the number line, a heavy arrowhead is used.

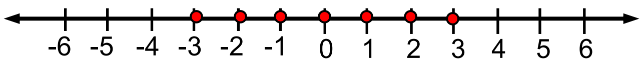
b. {-5 < integers< +5 }

This set is the intersection of the set of integers less than +5 and the set of integers greater than -5.



Set = {-4, -3, -2, -1, 0, 1, 2, 3 4}

b. {integers greater than or equal to -3 but less than +4 }



Set = {-3, -2, -1, 0, 1, 2, 3}

d. (n + 4) such that n is an integer greater than 1.

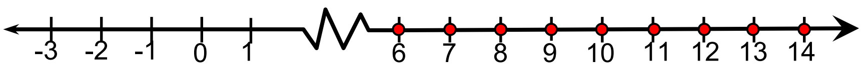
Solution:

1. The integers greater than 1 are +2, +3, + 4, +5, +6 + 7 and so on.

2. Substitute each integer in n + 4

   Set = {+6, +7, +8, +9, +10, +11, +12, +13,…}

3. Show the graph on the number line.



To show the infinite continuing pattern, we use 3 dots in the set. On the number line, a heavy arrowhead is used.

NOW DO PRACTICE EXERCISE 8

Practice Exercise 8

1. Write the elements of each set of integers described below.

   a. greater than -7
   b. less than -11
   c. greater than +6
   d. less than +35
   e. greater than or equal to -4
   f. less than or equal to -4
   g. greater than – 5 but less than +5
   h. greater than 0 but less than +8
   i. greater than or equal to 0
   j. less than -10 but greater than -18

---

2. List the elements of each set of integers described, then, graph them on the number line.

   a. {intergers between -5 and +7}

   b. {intergers between +3 and +11}

   c. {intergers greater than -2}

   d. {intergers less than -8}

   e. {intergers less than or equal to +9}

   f. {integers greater than or equal to -11}

g. $\{-10 \leq \text{integers} < +10\}$

h. $\{-5 < \text{integers} \leq +3\}$

i. $\{-1 \leq \text{integers} \leq +1\}$

---

3. Find **n** for each set described below. Show the graph of each set on the number line.

a. $\{2n \mid n \text{ is an integer less than } 0\}$

b. $\{3n \mid n \text{ is an integer greater than } -2\}$

c. $\{n^2 \mid n \text{ is an integer between } 0 \text{ and } 4\}$

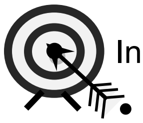
d. $\{n - 10 \mid n \text{ is an integer between } 10 \text{ and } +15\}$

e. $\{n^2 \mid n \text{ is an integer between } -1 \text{ and } +4\}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 9: Addition of Integers

In Lesson 8, you learnt how to identify the elements of a given set of integers, diagramming them on a number line and to distinguish which set of integers is represented from a given diagram.



In this lesson you will:

- add integers using the number line.
- give the rules in adding integers with like and unlike signs.
- perform addition of integers with like and unlike signs using the rule.

Addition of Integers

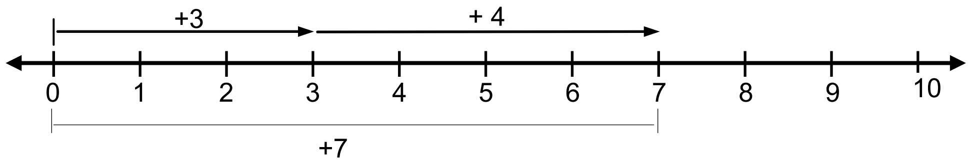
Consider a series of games, each game consisting of two rounds which John played on a machine. Here is a record of the points scored:

Game	**Points each Round**		**Net Score**
1	Round 1:	won 3 points	3 + 4 = 7
	Round 2:	won 4 points	
2	Round 1:	won 6points	6 + (- 3) = 3
	Round 2:	lost 3 points	
3	Round 1:	lost 5 points	(-5) + 2 = (-3)
	Round 2:	won 2 points	
4	Round 1:	lost 4 points	(-4) + (-3) = (-7)
	Round 2:	lost 3 points	
5	Round 1:	broke even	0 + 3 = 3
	Round 2:	won 3 points	
6	Round 1:	lost 4 points	4 + 0 = 4
	Round 2:	broke even	

The addition of two integers can be illustrated by using the number line. The operation can be thought of as combining two separate movements: the first starting from zero and the second starting from the first. The sum is the last position with respect to 0 as the starting point. A positive number moves to the right and a negative number moves to the left.

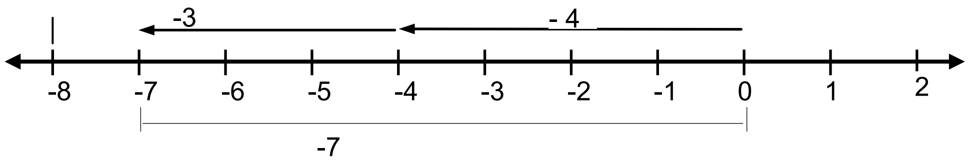
We can use arrows on the number line. Arrows pointing to the right are for positive integers while arrows pointing to the left are for negative integers.

Example 1 What is the sum of +3 and +4?



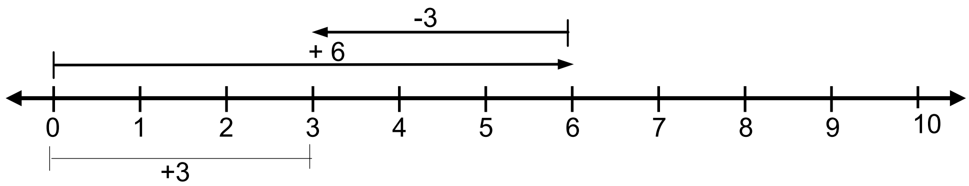
If you move 3 units to the right from zero, the point whose mark is 3 is reached. If you move 4 units to the right of this point (3), the point whose mark is 7 is reached. The sum of these movements is the same as a single movement of 7 units to the right from zero. Therefore, (+3) + (+4) = +7

Example 2: What is the sum of -4 and -3?



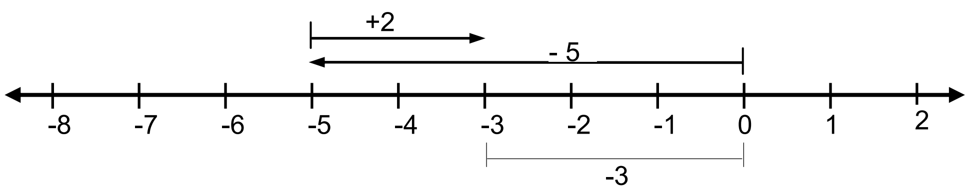
If you move 4 units to the left from zero, the point whose mark is (-4) is reached. If you move 3 units to the left of this point (-4), the point whose mark is (-7) is reached. The sum of these movements is the same as a single movement of (-7) units to the left from zero. Therefore, (-4) + (-3) = -7

Example 3:



If you move 6 units to the right from zero, the point whose mark is 6 is reached. From this point, moving 3 units to the left will get to the point whose mark is 3. The sum of these two movements is the same as a single unit movement of 3 units to the right of zero. Therefore, 6 + (-3) = +3.

Example 4:



If you move 5 units to the left from zero, the point whose mark is (-5) is reached. From this point, moving 2 units to the right will get the point whose mark is -3. The sum of these two movements is the same as a single unit movement of 3 units to the left of zero. Therefore, (-5) + 2 = -3.

You must have observed from the above examples that the direction of the movement depended on the sign of the integer while the distance travelled was based on the numerical value of the integer.

Using the concept of absolute value, we can now generalize how to add two integers.

1. To add two numbers having the same sign, find the sum of their absolute values and write their common sign.

Examples a. (+4) + (+5) = +9

b. (-4) + (-5) = -9

2. To add two numbers with different signs, get the difference of their absolute values and write the sign of the number having the greater value.

Examples a. (+17) + (-11) = +6

b.. (+8) + (-13) = -5

Notice that each addend in the examples is enclosed in parentheses so that the sign of the integer will be distinguish from the sign of operations. If parentheses are not used, then the sign of the numbers is raised like the following: $^{+}2 + {}^{+}4 + {}^{+}5$. However, since +2 = 2, +4 = 4 and +5 = 5 we may just write 2 + 4 + 5 since all the addends are positive integers.

REMEMBER:

If two positive integers are added, their sum is positive.

If two negative integers are added the sum is negative.

If a positive and a negative integer are added, the sum is obtained by subtracting their absolute values.

The sign of the sum follows that of the addend with greater absolute value.

Note: Addends are the numbers that we add.

NOW DO PRACTICE EXERCISE 9

Practice Exercise 9

1. Using the number line, show the sum of the following set of integers.

   a. (+2) + (+7)

   b. (+15) + (-8)

   c. (-7) + (-4)

   d. (- 6) + (+13)

2. Write whether the indicated sum is positive, negative, or zero.

   a. (-5) + (-30)
   b. (-7) + 5
   c. (-17) + 17
   d. (-7) + 7
   e. (-6) + 15
   f. (-23) + (-8)
   g. 70 + 5
   h. (-40) + 5
   i. (-27) + (-35)
   j. (-3) + 5

3. Add the following integers.

   a. (-1) + (-15) + (-25)
   b. 20 + (-18) + 25
   c. (-25) + (-47) + 56
   d. (-10) + 15 + 25
   e. 15 + (-6) + (-37)

---

4. Find the missing number in each of the following.

   a. 3 + (-9) = n
   b. (-13) + 8 = n
   c. 21 + (-31) = n
   d. (-5) + (-16) = n
   e. (-59) + n = 14

---

5. Solve this problem.

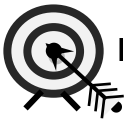
   Benua had K2000 savings deposit in a bank. He withdrew K500 on the first month, K300 on the second month, and deposited K1200 on the third month.

   How much money did he have in the bank after the third month?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 10: Subtraction of Integers

Your early work on the operation of subtraction or taking away was limited to situations where the **minuend** (the number from which you subtract) was less than or equal to the **subtrahend** (the number by which you subtract).



In this lesson you will:

- perform subtraction of integers using the number line.
- define the opposite of a number

First you have to know what subtraction means and find out how subtraction is performed with the set of integers.

Answer the following and compare the results in each item.

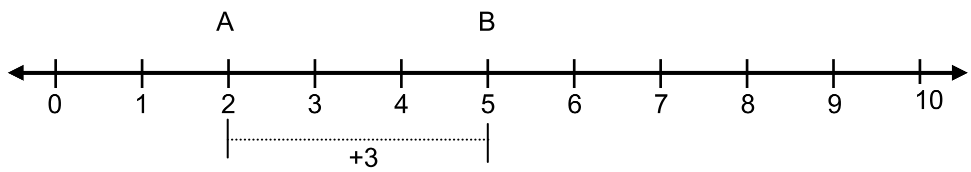
1. Add the opposite of 6 to 8.
   Subtract 6 from 8.
2. Add (-20) to 100.
   Subtract 20 from 100.
3. Add (-1) to 3.
   Subtract 1 from 3.
4. 14 – 8 = _____
   14 + (-8) = ______

Did you notice that adding the opposite of a number gives the same result as subtracting the same number? This means the positive subtrahend will become the negative addend and the negative subtrahend will become the positive addend.

Subtracting an integer is the same as adding the opposite of that integer.

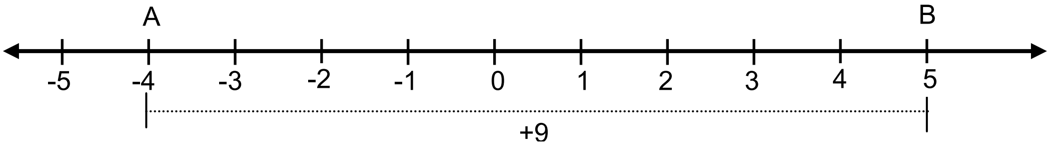
The meaning of subtraction can be illustrated on the number line by answering the question "How many more steps are needed to go from where I am (the subtrahend A) to get to where I want to be (the minuend B) as shown in the following examples:

1. Subtract 2 from 5



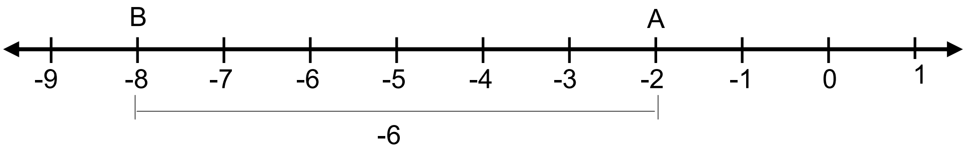
and so, 5 – 2 = 3.

2. Subtract (-4) from 5



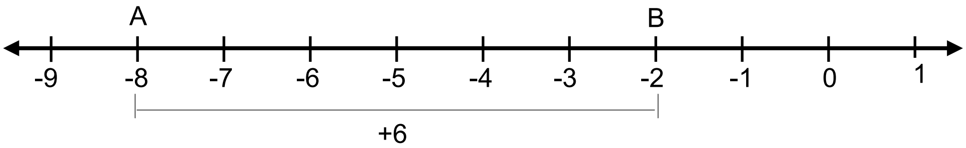
and so, 5 – (-4) = +9

3. Subtract (-2) from (-8)



And so, (-8) – (-2) = (-6)

4. How much greater is (-2) than (-8)?



Thus, (-2) – (-8) = 6.

Looking at the examples, can you now answer the following question?

Do these equations suggest a way of defining subtraction?

If A and B are integers, to subtract B from A, add the opposite of B to A.
In symbols we write, A – B = A + (-B)

To subtract integers, find the additive inverse of the subtrahend and proceed to addition.

NOW DO PRACTICE EXERCISE 10

Practice Exercise 10

1. Answer the following problems.

   a. Subtract (-6) from 14.
   b. Subtract (-10) from (-30).
   c. From 50, subtract (-15).
   d. From 24, take away from (-4).
   e. (-15) is how much greater than (-22)?

2. Find the simplest number for each of the following.

   a. $13 - (-5)$
   b. $13 - (-6)$
   c. $12 - 7$
   d. $12 - (-7)$
   e. $(-12\ ) - 7$
   f. $0 - (-21)$
   g. $0 - 21$
   h. $(-21) - (-20)$
   i. $22 - (-7)$
   j. $(-14) - 6$

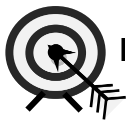
3. Write the correct number in the box to make the statement true.

   a. $6 - 11 = \square$
   b. $\square - 23 = 9.$
   c $\square - 20 = 1$
   d. $(-4) - \square = 5$
   e. $(-35) - \square = 0$
   f. $\square - (-8) = 20$
   g. $\square - 23 = 14$
   h. $\square - (-9) = 15$
   i. $(-15) - \square = 0$
   j. $\square - (-1) = 8$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 11: Multiplication of Integers

In your previous lessons, you learnt how to add and subtract integers.



In this lesson you will:

- multiply integers

As you will recall, multiplication is repeated addition. For example,

3 x 5 = 5 + 5 + 5 =15

While there should be no doubt in your mind about 3 multiplied by 5 being equal to 15, let us still investigate how 3 x 5 = 5 + 5 + 5 works by using the number line.

Since we want to multiply 5 by 3, 5 is to be counted 3 times, thus

5 + 5 + 5 = 15 or 3 x 5 = 15

Look at the figure below.

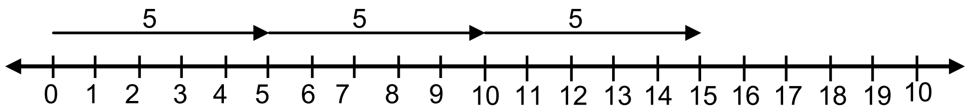


Figure 5.1

Suppose we are to find the product of a positive integer and a negative integer. For example, 3 x (-5).

To be consistent with the concept that multiplication is repeated addition,

3 x (-5) = (-5) + (-5) + (-5) = (-15).

Figure 5.2 illustrates the process.

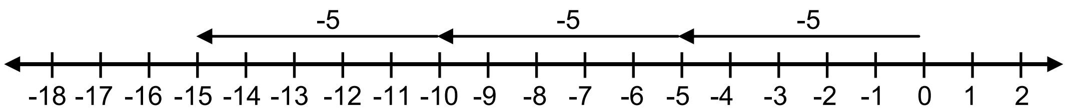


Figure 5.2

Thus, 3 x (-5) = (-15).

You learnt in the previous lessons that positive integers are like whole numbers.

And so, (+6)(+2) = (-6)(-2) = 12

You also know that multiplication can be interpreted as repeated addition.

Thus (+3)(+2) = (2) + (2) + (2) = 6

(+3)(-2) = (-2) + (-2) + (-2) = -6

Now examine the pattern in the following products:

(-8) x 4 = (-32)	(-8) x 0 = 0
(-8) x 3 = (-24)	(-8) x (-1) = 8
(-8) x 2 = (-16)	(-8) x (-2) = ____
(-8) x 1 = (-8)	(-8) x (-3) = ____
(-8) x 0 = 0	(-8) x (-4) = ____

Observe that each time the second factor decreases by 1, the product increases by 8.

Thus, the product of (-8) and (-1) is 8 more than 0. That is (-8) x (-1) = 8

Also, the product of (-8) and (-2) is 8 more than 8. That is (-8) x (-2) = 16

The product of (-8) and (-3) is 8 more than 16. That is (-8) x (-3) = 24

This illustrates that the product of two negative integers is positive.

The following will also help you decide on how to find the product of a positive and a negative or the product of two negative integers. Examine the patterns involved.

2 x 5 = 10	2 x 3 = 6	2 x 1 = 2
2 x 4 = 8	2 x 2 = 4	2 x 0 = 0

What should come next to fit the pattern? Here it is:

2 x (-1) = -2	2 x (-3) = -6	2 x (-5) = -10
2 x (-2) = -4	2 x (-4) = -8	and so on…

Let us build another pattern.

(-2) x (5) = -10	(-2) x (3) = -6	(-2) x (1) = - 1
(-2) x (4) = -8	(-2) x (2) = -4	(-2) x (0) = 0

Do you know what to write next? Here it is:

(-2) x (-1) = 2	(-2) x (-3) = 6	(-2) x -5) = 10
(-2) x (-2) = 4	(-2) x (-4) = 8	

From this pattern building and from the previous multiplication problems discussed, you can now answer the following:

1. What is the product of two integers with like signs?
2. What is the product of two integers with unlike signs?

The product of two positive or two negative integers is a positive integer.

The product of a negative and a positive integer is a negative integer.

Suppose you are to find the product of three or more integers, will the product be negative or positive? In such a case, the number of negative factors determines the sign of the product.

Study the following examples.

Example 1 (+1) x (-2) x (+3) x (-4) = +24

How many negative numbers are there?

What is the sign of the product?

Example 2 (-2) x (+4) x (-1) x (+2) x (-3) = -48

How many negative numbers are there?

What is the sign of the product?

Can you give your own examples where there are 4 negative factors? 6 negative factors? Determine the sign of the product in each case.

Suppose there are 5 negative factors? 7 negative factors? What can you say about the sign of the product?

How do you determine the sign of the product of three or more integers?

When multiplying three or more integers, we can say that:

- **If the number of negative factors is odd, the product is negative.**
- **If the number of negative factors is even, the product is positive.**

NOW DO PRACTICE EXERCISE 11

Practice Exercise 11

---

1. Find the product of the following.

   a. (-4)(-3)
   b. (-4)(-5)
   c. (-20)(-20)
   d. (-16)(-4)
   e. (-11)(-20)
   f. (-8)(11)
   g. $(-4)^2$
   h. $(-3)^3$

---

2. Give the value of **n** in the following.

   a. (3)(n) = 24
   b. (-3)(n) =24
   c. (4)(n) = -24
   d. (-2)(n) = (-24)
   e. (5)(n) = (-40)
   f. (-6)(n) = 42
   g. (-20)(-11) = n
   h. (-40)(n) = 240
   i. (-15)(3)(-3) = n
   j. (-1)(n) = 33

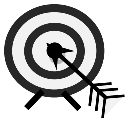
3. Perform the following multiplication.

   a. (+7)(-6)(+3)
   b. (+11)(-10)(-20)
   c. (+7)(-6)(+10)(-3)
   d. (+8)(-15)(-11)(+7)(0)
   e. -(6)(-3)(-3)(-2)(-7)

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 12: Division of Integers

In your previous lessons, you learnt how to add, subtract and multiply integers.



In this lesson you will

- divide with integers
- formulate rules on division of integers.

We learnt that subtraction can undo what addition does, while division will undo what multiplication does. We saw that subtraction is the inverse of addition and division is the inverse of multiplication in the case of whole numbers. We saw that this relation between addition and subtraction also holds for integers. This also holds for multiplication and division of integers. This relation will make it easy for us to divide one integer by another.

For every multiplication statement, there is a corresponding division statement.

If ***a, b*** and ***c*** are three numbers and ***a* x *b* = *c***, this means ***a*** and ***b*** are factors of ***c*** or ***c* = *ab****.* So, when the product called ***dividend*** in division and one factor, called the ***divisor***, are given then ***dividend ÷ divisor = missing factor,*** or ***divisor x missing factor = dividend.***

Examples

Multiplication Statement	Division Statement
8 x 3 = 24	24 ÷ 3 = 8 and 24 ÷ 8 = 3
7 x 4 = 28	28 ÷ 4 = 7 and 28 ÷ 4 = 7

Similarly, in the set of integers every multiplication statement where there is no zero factor gives two division statements.

Example;

Multiplication Statement	Division Statement
(-8) x (+6) = +48	+48 ÷ +8 = +6 and 48 ÷ +6 = +8
(+8) x (-6) = -48	-48 ÷ (-6) = +8 and -48 ÷ (+8) = -6
(-8) x (-6) = 48	48 ÷ (-6) = -8 and 48 ÷ (-8) = -6

Take a second look at the division statements carefully.

24 ÷ 3 = 8	28 ÷ 4 = 7	-48 ÷ (+8) = -6
24 ÷ 8 = 3	+48 ÷ +8 = +6	48 ÷ (-6) = -8
28 ÷ 4 = 7	-48 ÷ (-6) = +8	

What do you notice?

What is the quotient of two integers with like signs?

What is the quotient of two integers with unlike signs?

You can now give the rules for division of integers.

The quotient of two integers with the same sign is a positive number
The quotient of two integers with unlike signs is a negative number.

Examples

1. (+60) ÷ (+5) = +12
2. (-120) ÷ (-15) = + 8
3. (+42) ÷ (-7) = -6
4. (-63) ÷ (+9) = - 7

NOW DO PRACTICE EXERCISE 12

Practice Exercise 12

1. Find the value of **n.**

a. 6 ÷ n = 3

b 6 ÷ n = (-3)

c. (-30) ÷ n = (-6)

d 20 ÷ (-4) = n

e. (-20) ÷ (-4) = n

f. (-140) ÷ n = (-20)

g. (-132) ÷ (-11) = n

h. (84) ÷ (-7) = n

i. (-100) ÷ n = (-5)

2. Divide the following.

a. $\frac{18}{-2}$

b. $\frac{-12}{-4}$

c. $\frac{-10}{-2}$

d. $\frac{-18}{6}$

e. $\frac{0}{-25}$

f. $\frac{180}{-90}$

g. $\frac{-150}{-50}$

h. $\frac{-35}{-7}$

i. $\frac{-32}{8}$

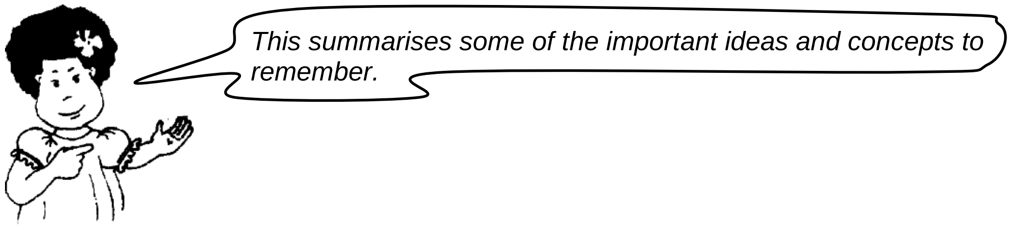
j. $\frac{20}{-5}$

k. $\frac{27}{(-3) + 0}$

l. $\frac{7 + 2}{9 + (-6)}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END FO SUB-STRAND 2.

SUB-STRAND 2: SUMMARY



- The **set of integers** consists of the set of **positive integers** (or natural numbers), the set of **negative numbers** and **zero.** All positive numbers are greater than 0; all negative numbers are less than 0. **Zero** is neither negative nor positive.
- The **set of integers** are also called the **set of directed numbers** because it indicates oppositeness of directions. It is also called the **set of signed numbers** because it uses the (+) and (-) signs. A positive integer does not need to have any "+" sign before it; it is understood to be positive.
- The **number line** is commonly used to illustrate the set of integers. This also helps us to determine the order of the integers. Any number to the left of a certain number is less than the number and any number to the right of a certain number is greater than the number.
- The distance of any integer from 0 is called the **absolute value** of the integer. Thus the absolute value of a number is always positive.
- The **opposite** of a given number is also called the additive inverse of the number. The sum of a given number and its opposite is equal to 0.
- To add two numbers having the same sign, find the sum of their absolute values and prefix or write their common sign.
- To add two numbers with different signs, get the difference of their absolute values and prefix or write the sign of the number having the greater value.
- To subtract integers, find the additive inverse of the subtrahend and add this to the minuend.
- The product of two integers with like signs is a positive integer while the product of two integers with unlike sign is a negative integer.
- The quotient of two numbers that have the same sign is a positive number.
- The quotient of two numbers that have unlike signs is a negative number.

REVISE LESSONS 7-12 THEN DO SUB-STRAND TEST 2 IN ASSIGNMENT 6.

ANSWERS TO PRACTICE EXERCISES 7 – 12

Practice Exercise 7

1. a. -100
   b. -5000
   c. +25
   d. +300
   e. +5000
   f. -10
   g. +450
   h. +100
   i. +5

2. a. 10 kilometres west
   b. 7 steps to the left
   c. 5 kilos underweight
   d. a price decrease of 5
   e. 12º west longitude

3.

a.	>	f.	>
b.	<	g.	>
c.	>	h.	>
d.	<	i.	>
e.	>	j.	<

4. a. {-10, -3, 0, 4, 7}
   b. {-15, -8, -5, -1, 20}
   c. {-21, -18, -2, 16, 51}
   d. {--31, -3, 3, 5, 10}
   e. {--26, -16, -6, 6, 16}

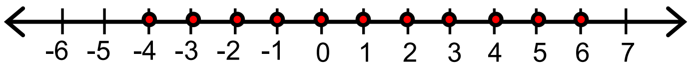
5. a. 3
   b. 7
   c. -2
   d. 5
   e. 0

6. a. b, c, d

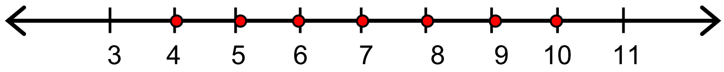
Practice Exercise 8

1. a. {-6, -5, -4, -3, -2, …}
   b. {-12, -13, -14, -15, …}
   c. {+7, +8, +9, …}
   d {34, 33, 32, 31, …}
   e {-4, -3, -2, -1, 0, …}
   f. {-4, -5, -6, -7, …}
   g. {-4, -3, -2, -1,0 1, 2, 3, 4}
   h. {1, 2, 3, 4, 5, 6, 7}
   i. {0, 1, 2, 3, …}
   j. {-17,-16, -15, -14, -13, -12, -11}

2. a. Set = { -4, -3, -2-, -1, 0, 1, 2, 3, 4, 5, 6}

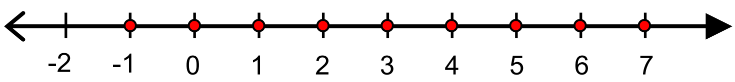


b. Set = { 4, 5, 6, 7, 8, 9, 10}

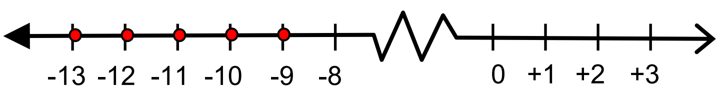


7

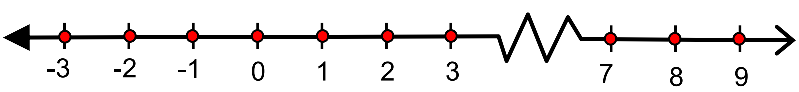
c. Set = { -1, 0, +1, +2, +3, +4, …}



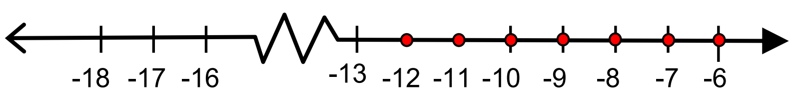
d. Set = { -9, -10, -11, -12, …}



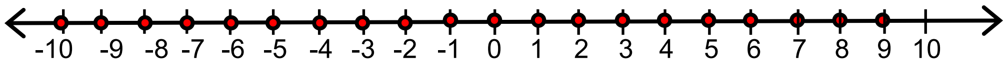
e. Set = { 9, 8, 7, 6, …}



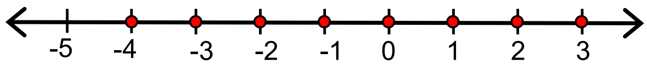
f. Set = { -11, -10, -9, …}



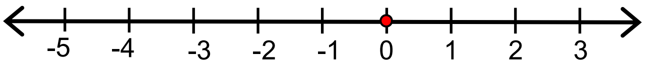
g. Set = { -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}



h. Set = { -4, -3,-2, -1, 0, 1, 2, 3}



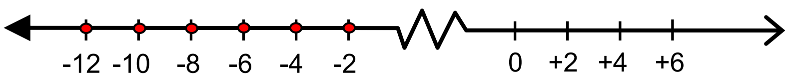
i. Set = { 0}



3. a. The integers less than 0 are -1, -2, -3, -4 and so on.

Substitute each integer in **2n**.

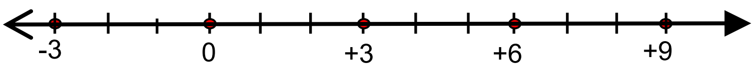
{ -2, -4, -6, -8, …}



b. The integers greater than -2 are -1, 0, 1, 2, 3, 4, 5 and so on.

Substitute each integer in **3n**.

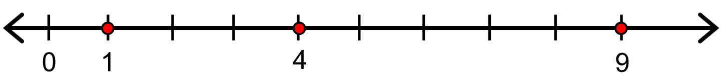
{ -3 0, 3, 6, 9…}



c. The integers between 0 and 4 are 1, 2 and 3.

Substitute each integer in $\mathbf{n^2}$.

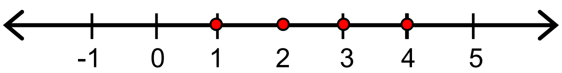
{ 1, 4, 9}



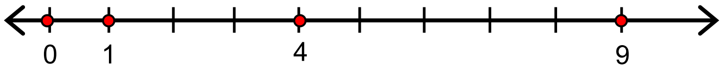
d. The integers between 10 and +15 are 11, 12, 13, and 14.

Substitute each integer in **n – 10.**

{ 1, 2, 3,4}

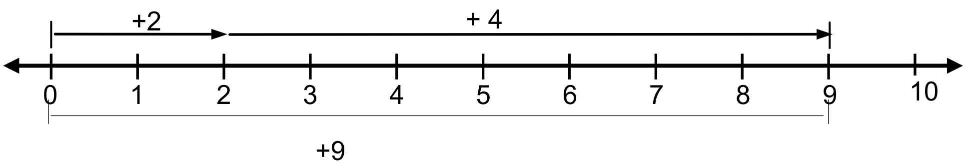


e. The integers between -1 and +4 are 0, 1, 2 and 3.
Substitute each integer in $\mathbf{n^2}$.
{ 0, 1, 4, 9}

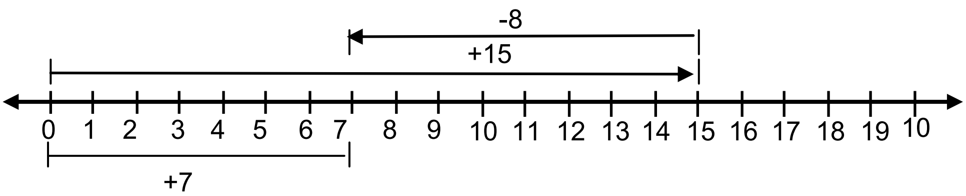


Practice Exercise 9

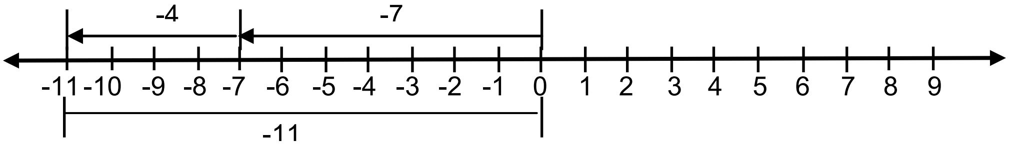
1. a. (+2) + (+7) = +9



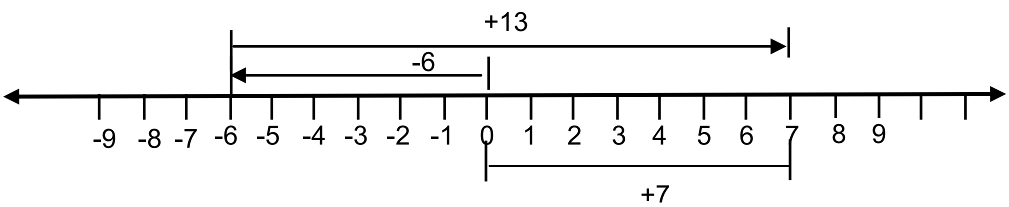
b. (+15) + (-8) = +7



c. (-7) + (-4) = -11



d. (-6) + (+13) = +7



2. a. negative
b. negative
c. zero
d. zero
e. positive
f. negative
g. positive
h. negative
i. negative
j. positive

3. a. - 41
   b. 27
   c. -16
   d. 30
   e. -28

---

4. a. -6
   b. -5
   c. -10
   d. -21
   e. 73

5. K2400

Practice Exercise 10

1. a. 20
   b. -20
   c. +65
   d. 28
   e. 7

2. a. +18 f. +21
   b. +19 g. -21
   c. +5 h. -1
   d. +19 i. +29
   e. -19 j. -8

3. a. -5 f. 12
   b. 42 g. 37
   c. 21 h. 6
   d. -9 i. -15
   e. -35 j. 7

Practice Exercise 11

1. a. +12 e. +220
   b. +20 f. -88
   c. +400 g. 16
   d. +64 h. -27

2. a. +8 f. -7
   b. -8 g. 220
   c. -6 h. -6
   d. +12 i. +135
   e. -8 j. -33

3. a. -126
   b. 2200
   c. -1260
   d. 0
   e. -756

Practice Exercise 12

1. a. +2 f. 7
   b. -2 g. 12
   c. 5 h. -12
   d. -5 i. 20
   e. +5

2. a. +6 f. -2 k. -9
   b. 3 g. 3 l. 3
   c. 5 h. 5
   d. -3 i. -4
   e. 0 j. -4

END OF SUB-STRAND 2

SUB-STRAND 3

INDICES

Lesson 13:	**Base, Powers and Exponents**
Lesson 14:	**Odd and Even Powers**
Lesson 15:	**Squares and Square Roots**
Lesson 16:	**Cubes and Cube Roots**
Lesson 17:	**Higher Roots**
Lesson 18:	**Expressing a Power as a product of Repeated Factors or Vice Versa**

SUB-STRAND 3: INDICES

Introduction

Many computing devices have been developed to overcome the chore of calculation in our everyday life. Even though we have some of them at our disposal, we still need to understand computing skills. Skills for calculating mentally are useful since a calculator is not always handy and writing our answers would not be convenient.

In Sub-strand 3 and 4, we shall learn methods that will not only extend our mental computation skills but will also simplify our work with problems.

See diagram below.

square of n = n^2
square of 5 = 5 x 5 = 5^2 = 25

Perfect Squares:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100

$$\sqrt{n}$$

$$\sqrt{100} = 10$$

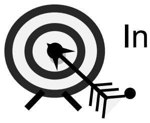
because 10 x 10 = 100

Indices, or powers, are an important part of Mathematics as they are necessary to indicate that a number is multiplied by itself for a given number of times.

In this sub-strand, you will learn more about indices. You will use positive indices greater than the power of 1.

Lesson 13: Base, Powers and Exponents

You learnt to add, subtract, multiply and divide integers in Sub-strand 2.



In this lesson you will:

- define and identify the base, the exponents and the power in a given expression.

Now that we have learnt to multiply signed numbers, it is possible to consider products in which the same number is repeated as a factor.

For example; $3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$

The 5 indicates that 3 is used as a factor 5 times

In the numerical expression or symbol 3^5, the 3 is called the **base.** The 5 is called the **exponent** and is written above and to the right of the base 3. The entire symbol 3^5 is called the **fifth power of three** and is commonly read **"three to the fifth power."**

See the figure below.

Exponent

$$3^5 = 243$$

Base — Fifth power of 3

Here are some more examples.

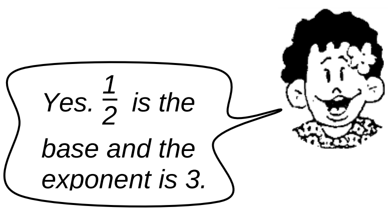
1. The numerical expression 6^3 means 6 x 6 x 6.The exponential form 6^3 is read "6 cubed," the cube of 6," or "the third power of 6." Since 6^3 = 216, then 216 is the third power of 6, the exponent is 3, and 6 is the base.

2. The numerical expression 2^5 means 2 x 2 x 2 x 2 x 2. The exponential form 2^5 is read "2 to the fifth power" or "the fifth power of 2." Since 2^5 = 32, then 32 is the fifth power of 2, the exponent is 5 and 2 is the base.

3. The numerical expression 10^4 means10 x 10 x 10 x 10. The exponential form 10^4 is read "ten to fourth power," or "the fourth power of 10." Since 10^4 = 10 000, then 10 000 is the fourth power of 10, the exponent is 4 and the base is 10.

4. The expression $\left(\frac{1}{2}\right)^3$ is read "$\frac{1}{2}$ cubed,"the cube of $\frac{1}{2}$," or "the third power of $\frac{1}{2}$".

   Can you name the base and exponent of $\left(\frac{1}{2}\right)^3$?



Here again are some more examples.

1. 8^3 means 8 x 8 x 8, the exponent is 3 and the base is 8.

Since 8^3 = 8 x 8 x 8 = 512

Exponent

$8^3 = 512$

Base — Third power of 8

2. $\left(\frac{2}{3}\right)^3$ means $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$, the exponent is 3 and the base is $\frac{2}{3}$.

$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$ ← Third power of $\frac{2}{3}$

A word of caution: We often think that expressions such as $(-6)^2$ and -6^2 are the same. They are **not** the same**.** The exponent applies only to the symbol immediately preceding it.

$(-6)^2 = (-6)(-6) = 36$ the exponent applies to ( ).

$-6^2 = -(6)(6) = -36$ the exponent applies to the 6.

Therefore: $(-6)^2 \neq -6^2$ („≠" means „not equal to")

- **An exponent is a number written at the upper right hand of a symbol. It tells how many times the symbol will be used as a factor.**
- **The factor being repeated is the base.**
- **The power is the product of equal factors.**

NOW DO PRACTICE EXERCISE 13

Practice Exercise 13

1. Name the base and the exponent.

a. 5^2 Base ____________ exponent____________

b. $(-8)^3$ Base ____________ exponent____________

c. 10^2 Base ____________ exponent____________

d. $\left(\frac{5}{8}\right)^4$ Base ____________ exponent____________

e. -5^6 Base ____________ exponent____________

2. Find the indicated power.

a. 5^3

b. $(-3)^3$

c. 10^2

d. $\left(\frac{1}{3}\right)^4$

e. $(-1)^6$

f. $(15 - 13)^4$

g. $(-2)^3$

h. $(3 + 4)^2$

i. -7^2

j. $-(-3)^5$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 14: Odd and Even Powers

In Lesson 13, you learnt to identify the base, the exponent and the power of a given numerical expression.

In this lesson you will:

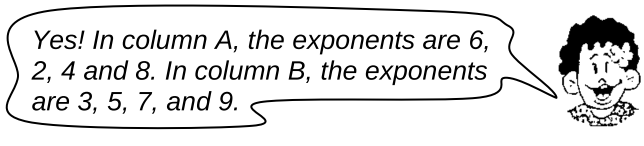
- identify odd and even powers
- find the odd powers of negative numbers
- find the power of zero.

We learnt that in the numerical expression or symbol 3^5, the 3 is called the **base.** The 5 is called the **exponent** and is written above and to the right of the base 3. The entire symbol 3^5 is called the **fifth power of three** and is commonly read **"three to the fifth power."**

Now, let us look at the following expressions in each column of the table below.

A	B
5^6	3^3
10^2	4^5
$\left(\frac{1}{3}\right)^4$	$\left(\frac{1}{3}\right)^3$
-7^8	-2^7
$(-1)^6$	$(2+ 5)^3$
$(3 + 4)^2$	$(-2)^9$

Can you identify the exponent in each expression given in column A and column B?



What can you say about the exponent or index of the base in each expression in column A?

You will notice that the exponents 2, 4, 6, 8 of the base in each expression in Column A are all even numbers and the exponents 3, 5, 7 and 9 of the base in each expression on Column B are all odd numbers.

Do you recall the meaning of these numbers in your Lower Primary Mathematics?

Let us go to the next page in case you forgot their meaning.

Even numbers are numbers which are exactly divisible by 2.

Odd numbers are numbers which are not divisible by 2.

Now let us go back to the table again.

A	B
5^6	3^3
10^2	4^5
$\left(\frac{1}{3}\right)^4$	$\left(\frac{1}{3}\right)^3$
-7^8	-2^7
$(-1)^6$	$(2+5)^3$
$(3+4)^2$	$(-2)^9$

Since all the bases of the expressions in Column A have exponents which are exactly divisible by 2, we say they are all **even powers** of the bases.

Since all the bases of the expressions in Column B have exponents which are not exactly divisible by 2, we say they are all **odd powers** of the bases.

- **If a base has an exponent that is exactly divisible by 2, we say that it is an even power of the base.**
- **If a base has an exponent that is not exactly divisible by 2, we say that it is an odd power of the base.**

Here are examples of powers.

1. $2^4 = 2 \times 2 \times 2 \times 2 = 16$
2. $1^4 = 1 \times 1 \times 1 \times 1 = 1$
3. $(-4)^2 = (-4) \times (-4) = 16$
4. $(-3)^2 = (-3) \times (-3) = 9$
5. $-3^2 = -(3 \times 3) = -9$
6. $2^3 = 2 \times 2 \times 2 = 8$
7. $(-2)^3 = (-2)\times(-2)\times(-2) = -8$
8. $(-3)^5 = (-3)\times(-3)\times(-3)\times(-3)\times(-3) = -243$

After discussing all the given examples, we arrive at the following generalization.

- An even power of a negative number is positive.
- Any power of 1 is 1.
- An even power of a positive base is positive.
- An odd power of a negative number is negative.
- An odd power of a positive number is positive.

Powers of Zero

Zero raised to any power is zero.

Examples of powers of zero

1. $0^2 = 0 \times 0 = 0$
2. $0^6 = 0 \times 0 \times 0 \times 0 \times 0 \times 0 = 0$
3. $0^7 = 0 \times 0 \times 0 \times 0 \times 0 \times 0 \times 0 = 0$

NOTE: In the examples given, care must be taken not to write 0 as an exponent. Cases where 0 appears as an exponent will be discussed in Grade 8 Mathematics.

NOW DO PRACTICE EXERCISE 14

Practice Exercise 14

1. Find the value of the following expressions.

   a. 4^3
   b. $(-3)^4$
   c. $2^3 + 5^2$
   d. 6^4
   e. $(-15)^2$
   f. 0^9
   g. $3^3 - 5^2$
   h. -9^3
   i. $(-1)^7$
   j. $(-2)^6$

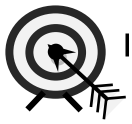
2. Evaluate the following.

   a. $(1.2)^2$
   b. $(0.3)^3$
   c. $(-5)^3 – 2^2$
   d. $(-6)^2 + 0^{25}$
   e. $\left(\frac{3}{4}\right)^3$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 15: Squares and Square Roots

In Lesson 14, you learnt to identify odd and even powers and to determine the odd powers of negative numbers and zero.



In this lesson you will:

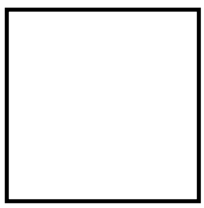
- identify squared numbers up to 100
- evaluate squares
- find the square roots of numbers up to 100.

When we know the length of a side of a square region, we can easily find its area by multiplying the number by itself.

This process of multiplying a number by itself is called **squaring a number.**

$A = s \times s = s^2$

$= 2 \times 2 = 2^2 = 4$



s = 2

Examples

1. The square of 4 = $4^2 = 4 \times 4 = 16$
2. The square of 7 = $7^2 = 7 \times 7 = 49$
3. The square of $\frac{2}{5} = (\frac{2}{5})^2 = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$
4. The square of 0.4 = $(0.4)^2 = 0.4 \times 0.4 = 0.16$
5. The square of 15 = $15^2 = 15 \times 15 = 225$

These examples show that when a number **(N)** is multiplied by itself, that is, **N x N** the product is called the square of the number (**N^2**).

When a number is multiplied by itself, the product is called the square of the number.

Using the inverse operation of finding the area of a square above is finding the length of a side, given the area. The inverse operation involved is finding the **square root** of a positive number.

Example: A square lot has an area of 289 square metres. How long is the side of the lot?

$$A = s \times s = s^2$$

$$A = s \times s = 289$$

Such a number is called the **square root** of 289 and we write it as $\sqrt{289}$. In this case, **s** can be replaced by 17 since 17 x 17 = 289.

What other number will satisfy the condition s x s = 289?

The square root of a number is the number which when multiplied by itself gives the given number.

Study the following examples:

Since $(+5)^2 = 25$, then +5 is a square root of 25.
Since $(-5)^2 = 25$, then -5 is a square root of 25.

Since $(+9)^2 = 81$, then +9 is a square root of 81.
Since $(-9)^2 = 81$, then -9 is a square root of 81.

Since $(+20)^2 = 400$, then +20 is a square root of 400.
Since $(-20)^2 = 400$, then -20 is a square root of 400.

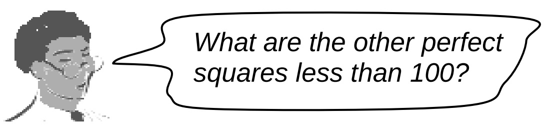
These examples show that 49, 81and 400 have positive and negative square roots. If the positive square root of a number **n** is being considered, the symbol $\sqrt{n}$ is used. This is called the **principal square root** of a number.

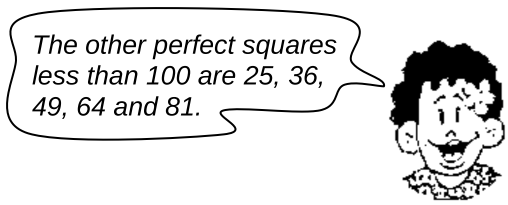
To indicate the negative square root of a number n, we use $-\sqrt{n}$. The symbol $\sqrt{\ }$ is called the **radical sign** and the number under the radical sign is called the **radicand**.

If the square root of a number is a whole number, the given number is called a **perfect square.**

Here are some examples.

The numbers 1, 4, and 9 are perfect squares. And we know that $\sqrt{1} = 1$, $\sqrt{4} = 2$ and $\sqrt{9} = 3$.



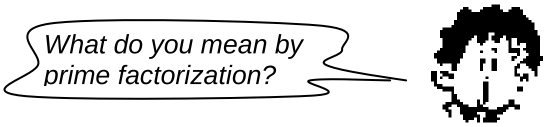


Now we will learn to find the square roots of a number.

Estimating Square Roots

Finding the square root of a number is called **extracting** the square root of a number. While the calculator may be used for this purpose, it is still useful to know how to find the square root of a number through manual computation.

One method of finding the square root of a number up to 100 is the use of **Prime Factorization method.**



Prime Factorization is a method of writing a number as a product of its prime factors.

The set of prime numbers less than 100 is shown below.

{2, 3, 5, 7, 11, 13, 17, 23, 29, 31, 37, 41, 43,
47, 53, 59, 61, 67, 71, 73, 79, 83, 87, 97}

Each of these numbers has only two factors, one and itself.

Let us recall the prime factorization and square roots of some perfect square.

Number	Prime Factorization	Square Root
4	$2 \times 2 = 2^2$	$\sqrt{2^2} = 2$
9	$3 \times 3 = 3^2$	$\sqrt{3^2} = 3$
16	$4 \times 4 = 2 \times 2 \times 2 \times 2 = 2^2 \times 2^2$	$\sqrt{2^2 \times 2^2} = 2 \times 2 = 4$
25	$5 \times 5 = 5^2$	$\sqrt{5^2} = 5$
36	$2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$	$\sqrt{2^2 \times 3^2} = 2 \times 3 = 6$
49	$7 \times 7 = 7^2$	$\sqrt{7^2} = 7$
64	$8 \times 8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^2 \times 2^2 \times 2^2$	$\sqrt{2^2 \times 2^2 \times 2^2} = 2 \times 2 \times 2 = 8$

Notice that each prime factor of a perfect square occurs with an even number of times.

Example 1

Let us find the prime factorization of 75.

75 → 5 | 75
5 | 15
3

or

75
5 x 15
5 x 3 x 5

Hence, $75 = 5 \times 5 \times 3 = 5^2 \times 3$

To get the exact square root of 75, we have

$$
\begin{aligned}
75 &= \sqrt{5x5x3} \\
&= \sqrt{5^2 x3} \\
&= 5\sqrt{3}
\end{aligned}
$$

$5\sqrt{3}$ is read as "5 times the square root of 3".

Example 2 What is the square root of 120?

120 → 2 | 120
2 | 60
2 | 30
5 | 15
3

or

120
2 x 60
2 x 6 x 10
2 x 2 x 3 x 2 x 5
2 x 2 x 2 x 3 x 5

Since $120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^2 \times 2 \times 3 \times 5$

Therefore, $\sqrt{120} = \sqrt{2^2 \times 2 \times 3 \times 5} = 2\sqrt{2 \times 3 \times 5} = 2\sqrt{30}$

$2\sqrt{30}$ means "2 times the square root of 30".

Example 3 Find the square root of 250.

Solution: By prime factorization, $250 = 5 \times 5 \times 5 \times 2 = 5^2 \times 5 \times 2$

Therefore,
$$
\begin{aligned}
\sqrt{250} &= \sqrt{5^2 \times 5 \times 2} \\
&= 5\sqrt{5 \times 2} \\
&= 5\sqrt{10}
\end{aligned}
$$

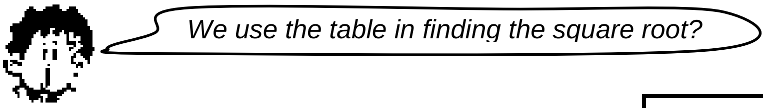
It means 5 times the square root of 10.

Approximations of square roots are listed in the table below. The table also shows the squares of whole numbers from 1 to 100.

TABLE OF SQUARES AND SQUARE ROOTS

N	N^2	$\sqrt{N}$	N	N^2	$\sqrt{N}$
1	1	1	26	676	5.099
2	4	1.414	27	729	5..196
3	9	1.732	28	784	5.292
4	16	2.000	29	841	5.385
5	25	2.236	30	900	5.477
6	36	2.446	31	961	5.568
7	49	2.646	32	1024	5.567
8	64	2.828	33	1089	5.745
9	81	3.000	34	1156	5.831
10	100	3.162	35	1225	5.916
11	121	3.317	36	1296	6.000
12	144	3.464	37	1369	6.083
13	169	3.606	38	1444	6.164
14	196	3.742	39	1521	6.245
15	225	3.873	40	1600	6.325
16	256	4.000	41	1681	6.403
17	289	4.123	42	1764	6.481
18	324	4.243	43	1849	6.557
19	361	4.359	44	1936	6.633
20	400	4.472	45	2025	6.708
21	441	4.583	46	2116	6.782
22	484	4.690	47	2209	6.856
23	529	4.796	48	2304	6.928
24	576	4.899	49	2401	7.000
25	625	5.000	50	2500	7.071

	N^2	$\sqrt{N}$	N	N^2	$\sqrt{N}$
51	2601	7.141	76	5776	8.718
52	2704	7.211	77	5929	8.775
53	2809	7.280	78	6084	8.832
54	2916	7.348	79	6241	8.888
55	3025	7.416	80	6400	8.944
56	3136	7.483	81	6561	9.000
57	3249	7.550	82	6724	9.055
58	3364	7.616	83	6889	9.110
59	3481	7.681	84	7056	9.165
60	3600	7.746	85	7225	9.220
61	3721	7.810	86	7396	9.274
62	3844	7.874	87	7569	9.327
63	3969	7.937	88	7744	9.381
64	4096	8.000	89	7921	9.434
65	4225	8.062	90	8100	9.487
66	4356	8.124	91	8281	9.539
67	4489	8.185	92	8464	9.592
68	4624	8.246	93	8649	9.644
69	4761	8.307	94	8836	9.695
70	4900	8.367	95	9025	9.747
71	5041	8.426	96	9216	9.798
72	5184	8.485	97	9409	9.849
73	5329	8.544	98	9604	9.899
74	5476	8.602	99	9801	9.950
75	5625	8.660	100	10 000	10.000



Example 1 Use the table to approximate $\sqrt{19}$

Solution: Find 19 in the column for N. Across in the column for square root ($\sqrt{N}$) is 4.359.

Hence, $\sqrt{19}$ = 4.359

N	N^2	$\sqrt{N}$
16	256	4.000
17	289	4.123
18	324	4.243
19	361	4.359
20	400	4.472

Example 2 Use the table to find the square root of 54.6

Solution: 54.6 is between 54 and 55.

$7.348 < \sqrt{54.6} < 7.416$

But 54.6 is nearer to 55 than 54,

therefore, $\sqrt{54.6} \approx 7.416$

N	N^2	$\sqrt{N}$
51	2601	7.141
52	2704	7.211
53	2809	7.280
54	2916	7.348
55	3025	7.416

Example 3 Find $\sqrt{260}$.

Solution: 260 is no longer in the N column. It is not in the squares column, but between 256 and 289. we can estimate $\sqrt{260}$ between 16 and 17. Is $\sqrt{260}$ nearer to 16 or 17?

N	N^2	$\sqrt{N}$
16	256	4.000
17	289	4.123
18	324	4.243
19	361	4.359
20	400	4.472

For a number bigger than 100, we may approximate the square by following example 3 above or by using our knowledge of prime factorization.

Example 4 Find $\sqrt{260}$

Solution: $\sqrt{260} = \sqrt{2^2 \times 5 \times 13}$

$= 2\sqrt{65}$

From the table, $\sqrt{65}$ = 8.062

Therefore, $\sqrt{260} = 2\sqrt{65}$

$= 2 \times 8.062$

$= 16.124$

N	N^2	$\sqrt{N}$
61	3721	7.810
62	3844	7.874
63	3969	7.937
64	4096	8.000
65	4225	8.062

NOW DO PRACTICE EXERCISE 14

Practice Exercise 15

1. Find the squares of each number.

a.	9	f.	40
b.	23	g.	100
c.	18	h.	56
d.	13	i.	21
e.	15	j.	12

2. Find the square root using prime factorization.

a. 27
b. 50
c. 200
d. 55
e. 90

3. Give the exact square root of the following.

a. $\sqrt{14 x 14}$
b. $\sqrt{2 x 2 x 3}$
c. $\sqrt{3 x 3 x 3 x 7}$
d. $\sqrt{2^2 x 3^2 x 5^2}$
e. $\sqrt{13^2 x 3}$

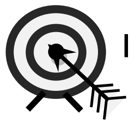
4. Using the table, find the squares and square roots of the following numbers.

a.	15	b.	66
c.	71	d.	85
e.	16	f.	28
g.	33	h.	49
i.	92	j.	7

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 6.

Lesson 16: Cubes and Cube Roots

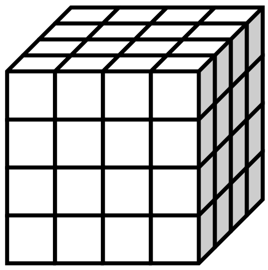
In Lesson 15, you learnt squares and square roots.



In this lesson, you will

- identify cubic numbers
- evaluate cubes

Consider the shape below.



This shape is a cube made up of 64 blocks, with 4 by 4 blocks in each layer.

There are 4 layers.

So there are 4 x 4 x 4 blocks in the cube.

Since the shape in the figure is a cube, the product 4 x 4 x 4 is called “4 cubed”.

4 x 4 x 4 can be expressed and shortened to 4^3.

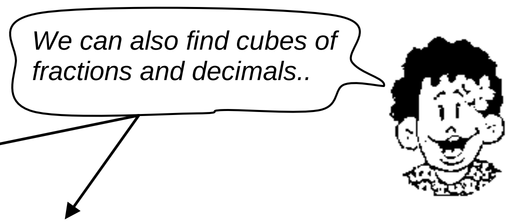
4^3, is read as „**four raised to the third power**‟ or „**four to the power of 3**‟ where 4 is the base and 3 is the index or exponent.

To find the value of 4^3, we multiply the base with 4 by itself three times.

Therefore, $4^3 = 4 \times 4 \times 4 = 64$.

Here are other examples.

1. Cube of 7 $= 7^3 = 7 \times 7 \times 7 = 343$
2. Cube of 10 $=10^3 = 10 \times 10 \times 10 = 1000$
3. Cube of 5 $= 5^3 = 5 \times 5 \times 5 = 125$
4. Cube of 9 $= 9^3 = 9 \times 9 \times 9 = 729$
5. Cube of $\frac{1}{3} = \left(\frac{1}{3}\right)^3 = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$
6. Cube of 0.01 $= (0.01)^3 = 0.01 \times 0.01 \times 0.01 = 0.000001$



These examples lead us to the following generalization.

When a number is multiplied by itself three times, the product is called the cube of the number.

Now we will look at the inverse operation of finding the cube of a number.

For example, 4 is the cube root or third root of 64 (written $\sqrt[3]{64}$) because 4 multiplied by itself three times gives 64 (4 x 4 x 4 = 64).

Taking the cube root of a number is the opposite of finding the cube of a number.

The cube root of a number is indicated by the symbol $\sqrt[3]{\ }$. The $\sqrt{\ }$ symbol is called a radical sign. The small raised number 3, or index, of the root is the number of times that the root appears in the multiplication.

Examples

1. $\sqrt[3]{125}$ = 5, because 5 x 5 x 5 = 125
2. $\sqrt[3]{216}$ = 6, because 6 x 6 x 6 = 216
3. What is the cube root of 27?

   Solution: The cube root of 27 in symbols is $\sqrt[3]{27}$.

   Therefore, $\sqrt[3]{27}$ = 3, because 3 x 3 x 3 = 27
4. $\sqrt[3]{0.000027}$ = 0.03, because 0.03 x 0.03 x 0.03 = 0.000027
5. $\sqrt[3]{-64} = \sqrt[3]{(-4)^3}$ = -4, because -4 x -4 x -4 or $(-4)^3$ = -64
6. $\sqrt[3]{-27} = \sqrt[3]{(-3)^3}$ = -3, because -3 x -3 x -3 or $(-3)^3$ = -27
7. $\sqrt[3]{0} = \sqrt[3]{0^3}$ = 0, because 0^3 = 0

The examples given lead us to the following generalisations.

- **The cube root of a number is the number which when multiplied by itself three times gives the number.**
- **The cube root of a negative number is negative, while the cube root of a positive number is positive.**
- **The cube root of zero is zero.**

NOW DO PRACTICE EXERCISE 16

Practice Exercise 16

1. Work out the cubes of the following.

a.	8	f.	10
b.	12	g.	1
c.	6	h.	15
d.	-3	i.	-2
e.	9	j.	0.4

2. Evaluate the cube roots.

   a. $\sqrt[3]{343}$

   b. $\sqrt[3]{1728}$

   c. $\sqrt[3]{512}$

   d. $\sqrt[3]{0.125}$

3. In a retail outlet, the cost **c** is related to **n** the number of items sold by the equation $c = \sqrt[3]{n}$.

   a. Find c when n = 27

   b. Find c when $n = 5^3$

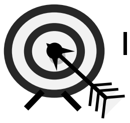
   c. $\sqrt[3]{0}$ Find n when c = 0

   d. Find n when c = 10.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 6.

Lesson 17: Higher Powers and Roots

In Lessons 15 and 16, you learnt the meanings of square roots and cube roots and their inverse operations.



In this lesson, you will

- evaluate expressions with higher roots.

We have studied squares, cubes, square roots and cube roots in the previous lessons. Now we are going to look at powers other than squares and cubes. We will also look at the inverse operation of raising a number to a power other than 2 and 3.

Study the following examples with higher roots.

1. In the expression 2^5

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

2 is called the fifth root of 32, written $\sqrt[5]{32} = 2$

32 is called the fifth power of 2, written 2^5 (fifth power of 2)

2. In the expression 3^6

$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$

3 is called the sixth root of 729, written $\sqrt[6]{729} = 3$

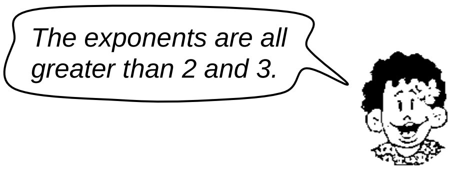
729 is called the sixth power of 3, written 3^6 (sixth power of 3)

3. In the expression $(-1)^5$

$(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$

-1 is called the fifth root of -1, written $\sqrt[5]{-1} = -1$

-1 is called the fifth power of -1, written -1^5 (fifth power of -1)



The expressions 2^5, 3^6, $(-1)^5$ are examples of **higher powers** because they all have exponents greater than 2 and 3.

The fourth root, fifth root, sixth root and all other roots other than square root and cube roots are called **higher roots**.

Finding the root of a number is the inverse of raising that number to a power because it „**undos**" the operation raising to that power.

Example 4

Note that 2 is where we begin and 2 is where we end

$2^4 = 16$ $\sqrt[4]{16} = 2$

Raising to the fourth power Finding the fourth root

Inverse operations

Here some more examples of finding the root of a number.

1. $\sqrt[5]{32} = (\sqrt[5]{2^5}) = (2) = 2$
2. $\sqrt[4]{81} = \left(\sqrt[4]{81}\right) = \left(\sqrt[4]{3^4}\right) = (3) = 3$
3. $\sqrt[6]{64} = \sqrt[6]{2^6} = 2$
4. $\sqrt[5]{3125} = \sqrt[5]{5^6} = 5$
5. $\sqrt[7]{1} = 1$

NOW DO PRACTICE EXERCISE 17

Practice Exercise 17

1. Evaluate:

   a. 4^5

   b. 10^4

   c. 2^7

   d. 3^5

   e. 9^6

---

2. Find the indicated roots of the following.

   a. $\sqrt[7]{128}$

   b. $\sqrt[5]{1024}$

   c. $\left(\sqrt[4]{81}\right)$

   d. $\sqrt[5]{243}$

   e. $\sqrt[8]{256}$

   f. $\sqrt[7]{1}$

   g. $\sqrt[9]{-1}$.

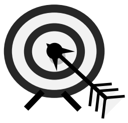
   h. $\sqrt[6]{2^6}$

   i. $\sqrt[7]{0}$

CORRECT YOUR WORK ANSWERS ARE AT THE END OF SUB-STRAND 6.

Lesson 18: Expressing a Power as a Product of Repeated Factors or Vice Versa

In Lesson 17, you learnt how to evaluate higher powers and higher roots.



In this lesson you will:

- express a power as a repeated product of each factor
- express products in index notation

When multiplying a number or expression by itself several times such as the expression 3 x 3 x 3 x 3 x 3 x 3, it is convenient to use a short method to avoid writing long strings of factors.

We call this method the **index or exponential notation**.

For example, we write 3 x 3 x 3 x 3 x 3 x 3 as 3^6.

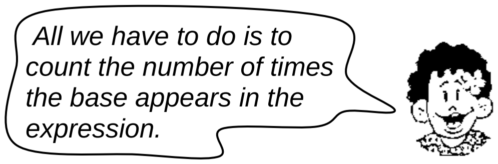
We say $\mathbf{3^6}$ is the index or exponential notation of the expression 3 x 3 x 3 x 3 x 3 x 3.

3 is the **base**.

6 is the **exponent** or the **index** of the expression and equals the number of times the base **3** occurs or is taken as a factor in the product.

Here are some more examples of writing expressions to a power.

1. 5 x 5 x 5 x 5 x 5 = 5^5
2. 4 x 4 x 4 x 4 x 4 x 4 x 4 x 4 x 4 = 4^9
3. 15 x 15 x 15 x 15 x15 x 15 = 15^6
4. 10 x 10 x10 x 10 x 10 x 10 x 10 x 10 = 10^8
5. 7 x 7 x 7 x 7 x 7 x 7 x 7 x 7 x 7 x 7 x 7 x 7 = 7^{12}
6. 1 x 1 x 1 x 1 x 1 x 1 x 1 = 1^7



Now that you know how to write expressions in index notation, we are going to look at the inverse operation of this method. This method is called **expanded notation.**

Examples:

Rewrite the following expressions as products of repeated factor.

a. 8^4 = 8 x 8 x 8 x 8

b. 12^5 = 12 x 12 x 12 x 12 x 12

c. 3^{12} = 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3

d. 10^7 = 10 x 10 x 10 x 10 x 10 x 10 x 10

e. 7^9 = 7 x 7 x 7 x 7 x7 x 7 x 7 x7 x 7

The examples lead us to this generalization.

When an expression in the index notation is written as a product of repeated factors, it is said to be in its expanded form.

Now we will look at examples of evaluating expressions in index notation using products of repeated factors. Your knowledge and skill learnt in previous lessons will help you do this.

Examples

1. Evaluate 6^5 using product of repeated factors.

   Solution: 6^5 = 6 x 6 x 6 x 6 x 6 = 46 656

2. Find the value of 2^{12}.

   Solution: 2^{12} = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 = 4096

3. Work out the value of 3^{10}.

   Solution: 3^{10} = 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 = 59049

4. Evaluate 1^{13}.

   Solution: 1^{13} = 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 = 1

Generally, if the base is 1, the expression equals 1 no matter what the power is. This leads us to the generalization

One raised to any power is 1.

.

Example 5 Find the value of **n** in each of the following.

a. $128 = 2^n$

b. $12,617 = 23^n$

c. $17 = 17^n$

Solutions:

a. $128 = 2^n$

$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

Therefore, n = 6

Rewrite as a product of repeated factors then to index notation.

b. $12,617 = 23^n$

$12\ 617 = 23 \times 23 = 23^2$

Therefore, n = 6

c. $17 = 17^n$

$17 = 1 \times 17 = 17^1$

Therefore, n = 1

Any number raised to the power of 1 is the number.

Example 6 Evaluate the following expressions.

a. $2^4 - 4^2$

Solution: $2^5 - 4^2 = (2 \times 2 \times 2 \text{x} 2 \text{ x} 2) – (4 \times 4)$

$= 32 – 16$

$= 16$

b. $5^7 \div 5^3$

Solution: $5^7 \div 5^3 = (5 \times 5 \times 5\text{x } 5 \times 5 \times 5 \times 5) \div (5 \times 5 \times 5)$

$= 78125 \div 125$

$= 625$

NOW DO PRACTICE EXERCISE 18

Practice Exercise 18

1. Express the following products in index notation.

   a. 5 x 5 x 5 x 5 x 5
   b. 8 x 8 x 8 x 8 x 8 x 8
   c. 2 x 2 x 2 x 2 x 2
   d. 4 x 4 x 4 x 4 x 4 x 4 x 4 x 4
   e. 11 x 11 x 11 x 11 x 11 x 11 x 11 x 11
   f. 9 x 9 x 9 x 9
   g. 1 x 1 x 1 x 1 x 1
   h. 7 x 7 x 7 x 7 x 7 x 7 x 7 x 7 x 7
   i. 15 x 15 x 15 x 15
   j. 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10

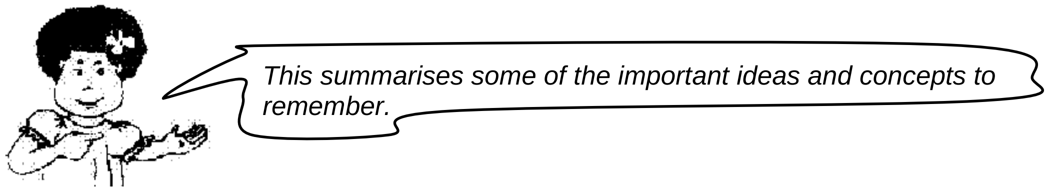
---

2. Evaluate the following expressions.

   a. 0^{30}
   b. 25^4
   c. 100^5
   d. 9^5
   e. $7^4 + 8^2$
   f. $3^4 - 2^5$
   g. $2^8 \div 2^3$
   h. $(1.3)^4 \div (1.3)^2$
   i. $\left(\frac{1}{5}\right)^3 \div \left(\frac{1}{5}\right)^2$
   j. $3^4 - 2^3 + 4^2$

CORRECT YOUR WORK ANSWERS ARE AT THE END OF SUB-STRAND 6.

SUB-STRAND 3: SUMMARY



- An **exponent** is a small number written at the upper right hand of a symbol indicating the number of times the symbol is taken as a factor.
- **Base** is the repeated factor.
- **Power** is the product of equal factors.
- If a base has an exponent that is exactly divisible by 2, we say it is an **even power** of the base.
- If a base has an exponent that is not exactly divisible by 2, we say that it is an **odd power** of the base.
- **Zero** raised to any power is zero.
- When a number is multiplied by itself, the product is called the **square** of the number.
- The **square root** of a number is the number which when multiplied by itself gives the number.
- A **Perfect square** is a number **whose square root is a whole number.**
- **Prime factorization** is the process of writing a number as a product of its prime factors.
- **Prime number** is a number that has only itself and 1 as its pair of factors.
- When a number is multiplied by itself three times, the product is called the **cube** of the number.
- The **cube root** of the number is the number which when multiplied by itself three times gives the number.
- The cube root of negative number is negative, while the cube root of a positive number is positive.
- The cube root of zero is zero.
- The cube root of 1 is 1.
- Roots other than the square roots and cube roots are called **higher roots**, likewise, powers other than squares and cubes are called **higher powers**.
- The **Index notation** is the process of expressing a product of repeated factors in a short way using exponent.

REVISE LESSONS 13 – 18 THEN DO SUB –STRAND TEST 3 IN ASSIGNMENT 6.

ANSWERS TO PRACTICE EXERCISES 13 - 18

Practice Exercise 13

1. a. Base: 5; exponent: 2
   b. Base: -8; exponent: 3
   c. Base: 10; exponent: 2
   d. Base: $\frac{5}{8}$; exponent: 4
   e. Base: -5; exponent: 6

.

2. a. 125 f. 16
   b. -27 g. -8
   c. 100 h. 49
   d. $\frac{1}{81}$ i. 49
   e. 1 j. 243

Practice Exercise 14

1. a. 64 f. 0
   b. 81 g. 2
   c. 33 h. -729
   d. 1296 i. -1
   e. 225 j. 64

2. a. 1.44
   b. 0.027
   c. -121
   d. 36
   e. $\frac{27}{256}$

Practice Exercise 15

1. a. 81 f. 1600
   b. 529 g. 1000
   c. 324 h. 3136
   d. 169 i. 441
   e. 225 j. 144

2. a. $\sqrt{3x3x3} = 3\sqrt{3}$
   b. $\sqrt{5x5x2} = 5\sqrt{2}$
   c. $\sqrt{5x5x2x2x2} = 5x2x\sqrt{2} = 10\sqrt{2}$
   d. $\sqrt{5x11}$
   e. $\sqrt{3x3x2x5} = \sqrt{3^2x2x5} = 3\sqrt{2x5} = 3\sqrt{10}$

3. a. 14
   b. $2\sqrt{3}$
   c. $3\sqrt{21}$
   d. 2 x 3 x 5 = 30
   e. $13\sqrt{3}$

4. a. 225; 3.873
   b. 4356; 8.124
   c. 5041; 8.426
   d. 7225; 9.22
   e. 256; 4
   f. 784; 5.292
   g. 1089; 5.745
   h. 2401; 7
   i. 8464; 9.592
   j. 49; 2.646

Practice Exercise 16

1. a. 512
   b. 1728
   c. 216
   d. -27
   e. 729
   f. 1000
   g. 1
   h. 3375
   i. -8
   j. 0.064

2. a. 7
   b. 12
   c. 8
   d. 0.5

3. a. 3
   b. 125
   c. 0
   d. 1000

Practice Exercise 17

1. a. 1024
   b. 10 000
   c. 128
   d. 243
   e. 531441

2. a. 2 f. 1
   b. 4 g. -1
   c. +3 h. 2
   d. 3 i. 0
   e. 2

Practice Exercise 18

1. a. 5^5 f. 9^4
   b. 8^6 g. 1^5
   c. 2^5 h. 7^9
   d. 4^8 i. 15^4
   e. 11^8 j. 10^9

2. a. 0 f. 49
   b. 390 625 g. 32
   c. 10 000 000 000 h. 1.69
   d. 59 049 i. $\frac{1}{5}$
   e. 2337 j. 89

END OF SUB-STRAND 3

SUB-STRAND 4

ALGEBRA

Lesson 19:	**Pronumerals**
Lesson 20:	**Algebraic Expressions**
Lesson 21:	**Substitution**
Lesson 22:	**Evaluation of Numerical and Algebraic Expressions**
Lesson 23:	**Solving Equations with only One Variable**
Lesson 24:	**Solving Equations with Two Variables**

SUB-STRAND 4: ALGEBRA

Introduction

The word Algebra is derived from the Arabic word "al-jabr" which means "a re-union or joining together of parts." This word changed later when the Moors brought the word "algebrista", meaning "bonesetter" (someone who joins or puts together bones) to Spain in the Middle Ages. Signs over barber shops in Spain saying *Algebrista y Sangradoe* indicated that the shop offered a bonesetter and blood letter.

At that time, and for centuries, barbers performed minor medical procedures to supplement their income. The traditional red-and-white-striped barber pole symbolized blood and bandages. Maybe that is why the word *algebra,* which comes from *algebrista,* has a reputation for being painful at times.

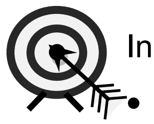
Algebra is the language of symbols. It deals with numbers and operations but unlike arithmetic, it uses letters to represent numbers that correspond to specific values. When a number is represented by a letter it is called a ***literal number***. Since a letter may represent a number it is sometimes called a ***pronumeral*.**

The ability to use symbols to represent mathematical phrases is one important skill in mathematics that we learn. We can do this by using numbers, the symbols of operations such as + , - , x or ÷, and even the grouping symbols like the parentheses ( ), brackets [ ], or braces { }, and pronumerals or variables.

In this sub-strand, you will substitute numbers for pronumerals. You will learn to evaluate a pronumeral in a number sentence and substitute pronumerals with numbers in a simple equation.

Lesson 19: Pronumerals

Welcome to the first lesson of your Sub-strand 4.



In this lesson you will:

- define pronumerals or variables and constants
- use letters to represent numbers.

First, you will learn the meaning of the words pronumeral, variable and constant.

These are new words. Could you explain what they mean?

Study the examples.

The expressions x + 5, y – 12, a – 4, 6b, □ - 10, and 8 + ___, convey certain ideas. Just like the symbols □ and ____, the letters x, a, and b, refer to unknown values. Since the values may vary, these expressions are called **variables or pronumerals**. In Strand 2, the letters **n** or **x** used in the exercises on percents and equal ratios are variables. The number 5, 12, 4, 6, 10 and 8 in the above expressions are **constants**.

- **A pronumeral or a variable is a letter, a box, or a blank which takes the place of a number.**
- **A constant is a number with a fixed value, that is, a value that does not change.**

Here are some more examples:

1) In the expression, □ + 34, the variable is □ ; the constant is 34.

2) In the expression $\frac{13}{x}$, the variable is x ; the constant is13.

3) In 3 + ____ + 4, the variable is _____; the constants are 3 and 4.

4) In the expression b + 14, the variable is b and the constant is 14.

5) In y + 50, the variable is y and the constant is 50.

Example 1

What possible value or values can replace the variable **N** in the statement below?

N is a whole number less than 10.

Solution: The whole numbers less than 10 are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Therefore, **N** = {0, 1, 2, 3, 4, 5, 6, 7, 8 and 9}.

Example 2

What possible value or values can replace the variable **x** in the statement below?

x is an integer between -1 and 1.

Solution: The integer between -1 and 1 is 0.

Therefore, **x** = {0}.

Example 3

What is the value of **m**, if m is a multiple of 2 greater than 10 but less than 20?

Solution: The multiples of 2 greater than 10 but less than 20 are 12, 14, 16, and 18.

Therefore, **m** = {12, 14, 16, 18}

Example 4

Find the whole number **w** which is between $\frac{1}{2}$ and $\frac{3}{2}$.

Solution: The whole number between $\frac{1}{2}$ and $\frac{3}{2}$ is 1.

Therefore, **w** = {1}

NOW DO PRACTICE EXERCISE 19

Practice Exercise 19

1. Give the possible value or values of the variables in the following.

   a. **x** is an even number less than 10.
   b. **y** is an odd number less than 15.
   c. **n** is a whole number greater than 5 but less than 12.
   d. **a** is a multiple of 3 greater than 9 but less than 24.
   e. **P** is the first prime number.
   f. **b** is a number which is twice 6.
   g. **z** is the least common multiple of 3,6 and 9.
   h. **w** is the greatest common factor of 7, 14 and 21.
   i. **c** is the number 5 less than 18.

2. Give the variable in each expression.

   a. 16 – x

   b. 36 – □

   c. ______ + y

   d. $\frac{32}{m}$

   e. s + 124

   f. 16 k

   g. 3(x)

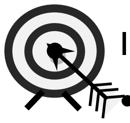
   h. 56 – ?

   i. 2 + 4s

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 6

Lesson 20: Algebraic Expressions

In Lesson 19, you learnt to identify pronumerals, variables and constants and to use letters to represent numbers.



In this lesson you will:

- translate words into algebraic expressions and vice versa.

In Sub-strand 2, we studied expressions like the following:

1. (13 x 13)
2. (8) x (-3) – (12 – 6)
3. (-30) x (-8) + (-32 + 4)

These expressions are called **numerical expressions.**

Other expressions have variables or pronumerals like the following:

1. n
2. 5x
3. 2a – b, and others

These are called **algebraic expressions or open phrases**.

Both numerical expressions and algebraic expressions are mathematical expressions.

- **A numerical expression may consist of a single number with or without operation symbols, or two or more numbers with operation and grouping symbols.**
- **An algebraic expression may consist of numbers and one or more variables joined by an operation sign and sometimes, using grouping symbols.**

In a numerical expression, multiplication may be indicated by using a cross (x), adjacent parentheses ( )( ) or by a dot midway between the two numbers. For example, "five times ten, may be written as 5 x 10, (5)(10) or 5 · 10.

In algebraic expressions, a cross (x) is not used to indicate multiplication. It is used as a variable to represent an unknown quantity or numbers. For example, „the product of x and y‟ may be written simply as (x)(y) or xy. The letters x and y are called factors of xy.

In cases where the expression contains a number and a variable, the number always precedes the variable. For example, „the product of 9 and n‟, may be written as 9n.

In writing algebraic expressions, you must always follow the order prescribed.

Examples

	English phrases	Algebraic expressions
1.	eleven subtracted from **a**	a - 11
2.	the sum of **n** and three	n + 3
3.	the product of 5 and **y**	5y
4.	the sum of **x** and **y** divided by two	$\frac{x + y}{2}$
5.	eight times the sum of **a** and **b**	8(a + b)

Sometimes we have to represent an unknown number not specified by a variable in a mathematical phrase before we can translate it into an algebraic expression.

Example 1 Let us translate the mathematical expression "nine less than a number" into an algebraic expression.

Step: If we let the variable **n** represent the number and use the numeral 9 and the minus sign or symbol to indicate **less than**, the phrase translates to n – 9.

Therefore, nine less than a number = n – 9

Example 2

Mathematical Phrase	Algebraic expressions
the sum of three numbers	a + b + c
the product of three numbers	xyz
12 subtracted from three times a number	3x – 12

We can also state in our own words the meaning of a given algebraic expression and often we can state the facts in several different ways.

Example 1 The algebraic expression **a + b** can be stated as follows:

the sum of a and b

a plus b

a increased by b

b more than a

Example 2 The algebraic expression **m – n** can be stated as follows:

m minus n

m less n

m decreased by n

m diminished by n

n subtracted from m

Example 3 The algebraic expression **mn** can be stated as follows:

m multiplied by n
the product of m and n
m times n

Example 4 The algebraic expression $\frac{x}{y}$ or x ÷ y can be stated as follows:

x divided by y
the quotient of x and y
x over y

The following are also examples of algebraic expressions:

1. $7x^3 – x^2 + 10$
2. a^2b^3
3. $5n^2$
4. d – e

The expression $7x^3 – x^2 + 10$ is said to have three terms. These three terms are $7x^3$, $- x^2$ and +10.

NOW DO PRACTICE EXERCISE 20

Practice Exercise 20

1. Write the following as algebraic expressions.

   a. Twice b.
   b. The product of w,y and z.
   c. Increase m by 5.
   d. Divide d by 4.
   e. Decrease k by 9.
   f. The sum of m and n.
   g. Three times x divided by twice y
   h. Fifteen minus the product of a and b
   i. The sum of the squares of m and n
   j. The product of the sum and difference of x and y

2. Translate the following into words.

   a. by
   b. $3x + 6$
   c. def
   d. $z – 3$
   e. $2a + 3b$
   f. $\frac{xy}{z}$
   g. $2mn$
   h. $2x – 8$
   i. $\frac{1}{5}y + 10$
   j. $x + y + z$

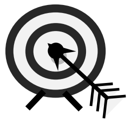
3. Match the expression on the left with the correct answer on the right.

a.	Add 2 to 6x	i.	$3m - 2$
b.	Multiply 2x by 5	ii.	$a + b + 3$
c.	decrease 3m by 2	iii.	bw
d.	the sum of a, b and 3.	iv.	$6x + 2$
e.	the product of b and w.	v.	$10x$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4

Lesson 21: Substitution

In Lesson 20, you learnt how to translate mathematical phrases into algebraic expressions.

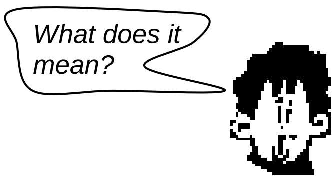


In this lesson you will:

- define substitution
- Find the value of an expression using substitution.

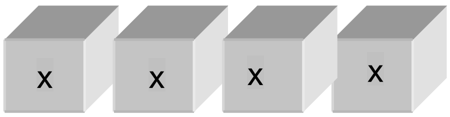
In Lesson 19, we learnt that a variable or a pronumeral can have different values. In this lesson, we will see that an algebraic expression can have a different value or values depending on the values we substitute for each pronumeral or variable. This process is called **substitution**.

NEW WORD:
You say " sabs – ti – tus- ion"



Look at the example to better understand the meaning of the word substitution.

Suppose we have 4 boxes of apples (x apples in each) we can work out the number of apples if we are told that x = 8.



$x + x + x + x = 4x$
$= 4(8)$
$= 32$

Therefore, there would be 32 apples if x = 8.

Now let us do the following examples.

Example 1
.
If x is replaced by the numeral 5, work out the value of each expression.

a. $5x$ b. $3x + 6$ c. $4x^2$ d. $x(2x – 8)$

Solution:

If we substitute 5 for x, each expression becomes:

a. $5x \longrightarrow 6(5)$
$= 30$

b. $3x + 6 \longrightarrow 3(5) + 6$
$= 15 + 6$
$= 21$

c. $4x^2 \longrightarrow 4(5^2)$
$= 4(25)$
$= 100$

d. $x(2x – 8) \longrightarrow 5[2(5) - 8]$
$= 5(10 – 8)$
$= 5(2)$
$= 10$

Note: The multiplication signs must be written in each expression once the values have been substituted for the pronumerals.

From the examples, we can derive the meaning of the word substitution.

Substitution is a method of replacing the variable in the expression by a given number.

Here are other examples.

Example 2

Work out the value of the following expressions if m = 2 and n =5.

a. mn   b. 3m + 7n   c. $\frac{10m}{n}$

d. (m – n)(m + n)

Solution:

a. mn = 2 x 5
= 10

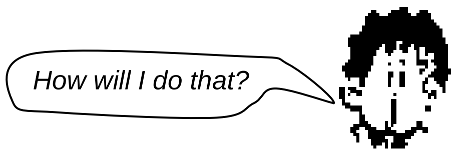
b. 3m + 7n = 3(2) + 7(5)
= 6 + 35
= 41

c. $\frac{10m}{n} = \frac{10(2)}{5}$
$= \frac{20}{5}$
= 4

d. (m – n)(m + n) = (2 – 5)(2 + 5)
= (-3)(7)
= -21

Example 3

Complete the table using the formula r = 3s.



s	1	2	3	4
r				

For a table of values, we simply substitute the top numbers one at a time into the formula to find the value of the bottom pronumeral.

Solution: If s = 1, then r = 3(1) = 3
If s = 2, then r = 3(2) = 6
If s = 3, then r = 3(3) = 9
If s = 4, then r = 3(4) = 12

s	1	2	3	4
r	3	6	9	12

NOW DO PRACTICE EXERCISE 21

Practice Exercise 21

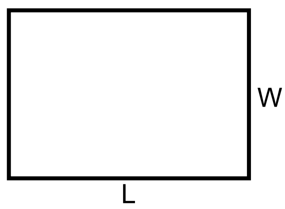
1. Work out the expression if x is replaced by 6.

(a) $3x$
(b) x^2
(c) $4x + 1$
(d) $6 – x$
(e) $\frac{x}{3}$
(f) $x^2 - x$
(g) $3(x + 1)$
(h) $\frac{1}{3}x + 2$
(i) $7x$
(j) $16 – 2x$

---

2. The perimeter of a rectangle is given by the formula $P = 2L + 2W$ where L is the length and W is the width of the rectangle.

Find the value of P if:



(a) $L = 5$, $W = 3$
(b) $L = 21$, $W = 15$
(c) $L = 7$, $W = 6$
(d) $L = 10$, $W = 6$

---

3. Complete the table using the given formula.

(a) $b = 4a$

a	0	1	2	3
b				

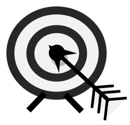
(b) $y = \frac{x}{2}$

x	2	4	6	8
y				

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 22: Evaluation of Numerical and Algebraic Expressions

In Lesson 21, we defined substitution and learnt to work out the value of expressions using substitution.



In this lesson you will:

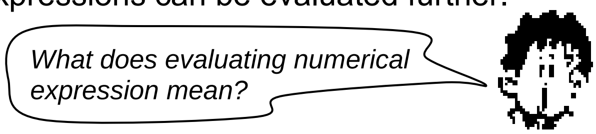
- find the value of a numerical expression in its simplest form.
- evaluate algebraic expressions

In Lesson 20, you learnt that a numerical expression may consist of a single number with or without operation symbols, or two or more numbers with operation and grouping symbols.

Here again are some examples of numerical expressions.

1. (3 + 7) -- 2
2. [(8)(2)] – 7
3. 24 ÷ [(3)(2)]
4. 8 + [(2)(6)] – (10 – 8)
5. 18 – [3 + (8 ÷2) – 1]

All these examples of numerical expressions can be evaluated further.



To evaluate numerical expressions is to find the value in the simplest form.

Look at the following examples.

To evaluate	Solution	Steps
1. (3 + 7) -2	= 10 – 2	add 3 and 7 to get 10
	= 4	subtract 2 from 10
2. [(8)(2)] – 7	= 16 – 7	multiply 8 by 2 to get 16
	= 9	subtract 7 from 16
3. 24 ÷ [(3)(2)]	= 24 ÷ 6	multiply 2 by 3 to get 6
	= 4	divide 24 by 6
4. 8 + [(2)(6)] – (10 – 8)	= 8 + 12 – 2	subtract 8 from 10 to get 2
	= 20 – 2	add 8 and 12 to get 20
	= 18	subtract 2 from 20
5. 18 – [3 + (8 ÷2) – 1]	= 18 - [3 + 4 – 1]	divide 8 by 2 to get 4
	= 18 - [7 – 1]	add 3 and 4 to get 7
	= 18 – 6	subtract 1 from 7 to get 6
	= 12	subtract 6 from 18 to get 12

REMEMBER:

In simplifying numerical expressions,

- **Addition and subtraction are performed from left to right.**
- **Multiplication and division are performed from left to right in the order in which they occur before addition and subtraction.**
- **Grouping symbols as [ ], ( ) and { } are removed by simplifying the expressions enclosed, starting with the innermost pair.**

Example 1 Evaluate x + 12, when x = 4.

Solution:	x + 4 = 4 + 12	4 replaces x
	= 16	add 4 and 12

Example 2 Evaluate x + 12, when x = (-3).

Solution:	x + 12 = (-3) + 12	- 3 replaces x
	= 9	add -3 and 12

Example 3 Evaluate 3x + 5, if x = 2

Solution:	3x + 5 = 3(2) + 5	2 replaces x
	= 6 + 5	3 x 2 = 6
	= 11	add 6 and 5

Evaluating algebraic expression means replacing the variable in the expression by a given number, then simplifying the resulting numerical expression.

Example 4 Evaluate 6(3x – 2), when x = 5

Solution:	6(3x -2) = 6[(3 · 5) – 2]	5 replaces x
	= 6 (15 – 2)	3 · 5 = 15
	= 6(13)	15 – 2 = 13
	= 78	multiply 6 by 13

Example 5 Evaluate x(y +8), when x = 4 and y = (-2).

Solution:	x(y + 8) = 4[(-2) + 8]	4 replaces x and (-2) replaces y
	= 4(6)	(-2) + 8 = +6
	= 24	multiply 4 by 6

In evaluating algebraic expressions with grouping symbols such as brackets [ ], parentheses ( ) and braces { }, perform the operation within the grouping symbol before performing other operations.

NOW DO PRACTICE EXERCISE 22

Practice Exercise 22

1. Simplify the following numerical expression.

(a) (18 – 15) + 2

(b) 3 + (22 – 8)

(c) 46 – (10 · 3)

(d) (8 + 4) · 5

(e) (6 · 2) ÷ 4

(f) [(9 – 8) ÷ 2] + (6 · 3)

(g) (15 + 18) ÷ 3 [(-5) · 20]

(h) 5 – (3 – 42) – 4

(i) [(32 + 13) – 25] – (13 – 15)

(j) (40 – 9) – (15 – 20)

---

2. Evaluate each expression for the given value of the variable.

A. Given: x = 3

(a) 5x + 1
(b) 8 – 2x
(c) 7x – 10
(d) 3x – x
(e) 8(x +2)

B. Given: n = 4

(a) n + 25
(b) (-n) + 7
(c) 19 + n – n
(d) 5(n +3)
(e) n^2 + 2n

C. Given: a = 2, b = 3 and c = (-1)

(a) 3(2a – c)

(b) 2a – 3b

(c) a + b + c

(d) a(b + c)

(e) 5a – c

(f) b + 2c

(g) 2a – 3b – 4c

(h) 9(2a – b) + 3c

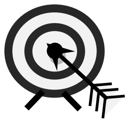
(i) 5(c – a – b)

(j) 7a – (4b · 3c)

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 23: Solving Equations with only One Variable.

You learnt to find the value of numerical and algebraic expressions in their simplest form.



In this lesson you will:

- solve equations that contains only one unknown

Mathematical expressions can also be related or joined by relation symbols like = (equal), ≠ (unequal) , $<$ (less than) and $>$ greater than. The symbols ≤ and ≥ can also be used to relate mathematical expressions.

If any of the symbols is used to relate two mathematical expressions, the result is called a **mathematical sentence.**

Look at the examples of mathematical sentences.

1. $3x = 2 + 4$
2. $m – 2 > 5$
3. $s -4 \neq (-s)$
4. $2x – 1 = 4x + 7$
5. $x < 3y$

A mathematical sentence which uses the symbol "=" is called an **equation.** Sentences which use the symbols ≤ , ≥, $<$ and $>$ are called **inequations** or **inequalities.**

In this lesson, we will concentrate on **equations with only one variable.** Inequalities will be studied and discussed further in your Upper Secondary Mathematics Course.

An equation with only one variable is an equation with only one letter, in which the highest power of that letter is the first power.

A mathematical sentence may be **true, false** or **open**.

Consider the equation $x + 7 = 15$. The variable or pronumeral x whose value is what we want to find represents the **unknown** or **missing number**.

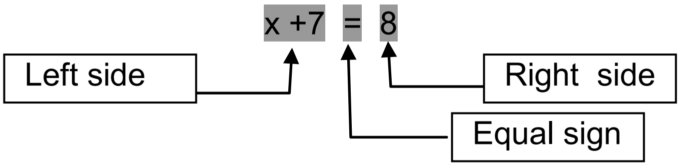
The equation $x + 7 = 15$ is an example of an open sentence since it contains a variable and cannot be said to be true or false until the replacement for the variable is made.

Example

Suppose you are told that x is a number in the replacement set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, which of the numbers would make the sentence true? Which of these would make the sentence false?

- **The solution or root of an equation is a number which when put in place of the letter or variable makes the two sides of the equation equal.**
- **The solution set of an equation that has only one variable is the set of all numbers that are solutions of that equation.**

An equation has three parts:



The equal sign in an equation means that the number represented by the left side is the same as the number represented by the right side.

The equation x + 7 = 15 will be true if and only if x = 8. If we evaluate the equation, we will have

$$x + 7 = 15$$
$$8 + 7 = 15$$
$$15 = 15$$

Notice that the left side of the equation is equal to the right side of the equation.

We say that the value of the variable **x** which is 8 satisfies the equation x +7 =15. We say, 8 is the **solution** or **root** of the equation. The **solution set** of the equation is {8}.

To check the solution of an equation, do the following steps:

1. Replace the unknown letter in the given equation by the number found in the solution.
2. Perform the indicated operations on both sides of the equal sign.
3. If the resulting number on each side of the = sign is the same, then the solution is correct.

In solving equations with only one variable the following principles of equality are used:

Addition Rule: The same number may be added to both sides.

Subtraction Rule: The same number may be subtracted from both sides.

Multiplication Rule: Both sides may be multiplied by a non-zero number.

Division Rule: Both sides may be divided by the same non-zero number.

Let us now illustrate how these principles are used to solve equations with one variable.

Example 1 Solve the equation $x – 3 = 12$.

Solution:

$$x - 3 = 12 \quad \text{given equation}$$
$$x - 3 + 3 = 12 + 3 \quad \text{add 3 to both sides}$$
$$x + 0 = 15 \quad \text{additive inverse}$$
$$x = 15$$

Check:

$$15 - 3 = 12$$
$$12 = 12$$

Example 2 Solve the equation $x + 3 = 12$.

Solution:

$$x + 3 = 12 \quad \text{given equation}$$
$$x + 3 - 3 = 12 - 3 \quad \text{subtract 3 to both sides}$$
$$x + 0 = 9 \quad \text{additive inverse}$$
$$x = 9$$

Check:

$$9 + 3 = 12$$
$$12 = 12$$

Example 3 Solve the equation $\frac{x}{3} = 6$.

Solution:

$$\frac{x}{3} = 6 \quad \text{given equation}$$
$$(3)\frac{x}{3} = 6(3) \quad \text{multiply both sides by 3}$$
$$x = 18$$

Check:

$$\frac{18}{3} = 6$$
$$6 = 6$$

Example 4 Solve the equation $3x = 15$.

Solution:

$$3x = 15 \quad \text{given equation}$$
$$3x \div 3 = 15 \div 3 \quad \text{divide both sides by 3}$$
$$x = 5$$

Check:

$$3(5) = 15$$
$$15 = 15$$

It is always advisable to look back and check to see if the root satisfies the given equation. Evaluate the equation by substituting the root for the unknown. If both sides of the equation are equal, then the root is correct.

Example 5 Solve the equation $5x + 8 = 33$.

Solution:	$5x + 8 = 33$	given equation
	$5x + 8 - 8 = 33 - 8$	subtract 8 from both sides of
	$5x = 25$	the equation
	$\frac{5x}{5} = \frac{25}{5}$	divide both sides of the
	$x = 5$	equation by 5

Check:

$$5x + 8 = 33$$
$$5(5) + 8 = 33$$
$$25 + 8 = 33$$
$$33 = 33$$

Example 6 Solve $4x - 8 = x - 2$.

Solution:	$4x - 8 = x - 2$	given equation
	$4x - 8 + 8 = x - 2 + 8$	add 8 to both sides of the equation and simplify
	$4x = x + 6$	
	$4x - x = x - x + 6$	subtract x from both sides of the equation and simplify
	$3x = 6$	
	$\frac{3x}{3} = \frac{6}{2}$	divide both sides of the
	$x = 2$	equation by 2 and simplify.

Check:

$$4x - 8 = x - 2$$
$$4(2) - 8 = 2 - 2$$
$$8 - 8 = 0$$
$$0 = 0$$

Try to perform some of the steps or processes mentally. We may not always show the principle being used. Just remember that any operation done on one side of the equation should also be done on the opposite side of the equation.

Example 7 Solve $x + 15 = 30 + 4x$.

Solution:

$$x + 15 = 30 + 4x$$
$$x + 15 - 15 - 4x = 30 + 4x - 15 - 4x$$
$$x - 4x = 30 - 15$$
$$-3x = 15$$
$$x = -5$$

Check:

$$x + 15 = 30 + 4x$$
$$-5 + 15 = 30 + 4(-5)$$
$$10 = 30 - 20$$
$$10 = 10$$

Example 8 Solve the equation $7x + 9 - 5x = 15 - 4x$.

Solution:

$$7x + 9 - 5x = 15 - 4x$$
$$2x + 9 = 15 - 4x$$
$$2x + 4x + 9 = 15$$
$$6x + 9 = 15$$
$$6x = 15 - 9$$
$$6x = 6$$
$$x = 1$$

Check:

$$7x + 9 - 5x = 15 - 4x$$
$$7(1) + 9 - 5(1) = 15 - 4(1)$$
$$7 + 9 - 5 = 15 - 4$$
$$16 - 5 = 11$$
$$11 = 11$$

NOW DO PRACTICE EXERCISE 23

Practice Exercise 23

1. Solve the following equations and check the solutions.

a. $x + 18 = 8$

b. $y + 28 = 50$

c. $x – 35 = 35$

d. $39 + x = 47$

e. $x + 12 = 29$

f. $7x = 70$

g. $4k = \frac{3}{8}$

h. $4n = 23$

i. $\frac{x}{2} = -4$

j. $\frac{1}{4}x = 9$

2. Solve for the root of each equation.

a. $3a + 7 = 16$

b. $6m + 10 = 52$

c. $7x – 7 = 14$

d. $2x + 5 = - 3$

e. $\frac{x}{8} – 5 = 12$

f. $3m – 5 = 2m + 12$

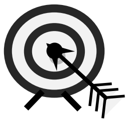
g. $6 – x = x + 22$

h. $12 c – 5 = 6c + 1$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 24: Solving Expressions with Two Variables

In Lesson 23, you learnt to solve expressions with only one variable by applying the principles of equality.



In this lesson you will:

- solve expressions that contain two variables or pronumerals

You learnt in Lesson 20 that a mathematical expression can have variables as part of the expression. Likewise, in Lesson 21, you learnt to find the value of an expression by substitution which means replacing the variables in the expression by a given number.

For example, if x =3, and y =5, the expression 7x + y – 4 becomes 7(3) + 5 – 4 which is equal to 21 + 5 – 4 or 22.

To evaluate an expression with two or more variables, replace each letter by its number value and carry out all the operations in the correct order.

In this lesson, you will find the value of a given expression by assigning any values for one unknown to find the other. This can be better understood by completing the table using a given **rule or formula**.

A formula or a rule is an equation in which a particular variable is expressed in terms of other variable.

Look at the examples below.

Example 1 Suppose we have the following table:

a	b
2	15
5	45
11	105
8	75
10	95
20	195

The formula for this is b = 10a – 5.

If the value of a = 7, what would be the value of b?

To answer this, we replace the value of **a** in the formula with 7 and then work out the answer. Remember 10a means 10 x a.

Solution: b = (10 x 7) – 5
b = 70 – 5
b = 65
So **b = 65**

Example 2

Suppose the formula is y = 8(x + 1). If you wish to find the value for **y** when **x** is 10, you would substitute 10 for **x** in the formula.

This means y = 8(10 + 1)
= 8(11)
= 88
So **y = 88**

If the **x** value is 0, what would be the **y** value?

Solution: y = 8(0 + 1)
= 8(1)
= 8
So **y = 8**

Example 3

Use the formula y = 5(x + 7) to complete the table below.

x	1	2	3	7	13
y					

Solution: Substitute the given values into the formula to find the value of **y**.

If x = 1, then y = 5(x + 7)
= 5(1 + 7)
= 5(8)
= 40

If x = 2, then y = 5(x + 7)
= 5(2 + 7)
= 5(9)
= 45

If x = 3, then y = 5(x + 7)
= 5(3 + 7)
= 5(10)
= 50

If x = 7, then y = 5(x+ 7)
= 5(2 + 7)
= 5(14)
= 70

If x = 13, then y = 5(x + 7)
= 5(13 + 7)
= 5(20)
= 100

Answer;

x	1	2	3	7	13
y	**40**	**45**	**50**	**70**	**100**

Example 4

Use the rule given to work out the missing **y** values and complete the table below.

Rule: Multiply each x value by itself.

x	10	7	6	1	12
y					

Solution:
a. 10 x 10 = 100
b. 7 x 7 = 49
c. 6 x 6 = 36
d. 1 x 1 = 1
e. 12 x 12 = 144

Answer:

x	10	7	6	1	12
y	**100**	**49**	**36**	**1**	**144**

NOW DO PRACTICE EXERCISE 24

Practice Exercise 24

1. Answer True or False for each of these statements.

   a. If we substitute a = 2 into b = 16 + a we get b = 32 __________
   b If we substitute a = 4 into b = 5a we get b = 20 __________
   c. If we substitute a = 9 into b = a + 11 we get b = 20 __________
   d. If we substitute x = 6 into y = 4(x – 5) we get y =19 __________
   e. If we substitute x = 7 into y = 7 – x we get x = 1 __________

2. Look at the tables and work out which rule has been used.

   a.

x	2	1	7	10	100
y	4	1	49	100	10 000

A. Multiply each x value by two.
B. Add two to each x value.
C. Add one to each x value, then subtract one
D. Multiply each x value by itself.

   b.

x	4	22	5	3	39
y	10	28	11	9	45

E. Multiply each x value by two, then add two.
F. Add sixteen to each x value, then divide by two.
G. Add six to each x value.
H. Multiply each x value by itself, subtract six.

3. Use the following rules to complete the tables.

   a. y = x – 7

x	7	9	12	8	15
y					

   b. y = 4x

x	5	4	3	2	1
y					

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

SUB-STRAND 4: SUMMARY

This summarises some of the important ideas and concepts to remember.

- A **pronumeral** or a **variable** is a letter, a box, or a blank which takes the place of a number.
- A **constant** is a number with a fixed value, that is, a value that does not change.
- **Substitution** is a method of replacing the variable in the expression by a given number.
- In simplifying numerical expressions, remember that:
  - a. Addition and subtraction are performed from left to right.
  - b. Multiplication and division are performed from left to right in the order in which they occur before addition and subtraction.
  - c. Grouping symbols as [ ], ( ) and { } are removed by simplifying the expressions enclosed, starting with the innermost pair.
- **Evaluating algebraic expression** means replacing the variable in the expression by a given number, then simplifying the resulting numerical expression.
- In evaluating algebraic expressions with grouping symbols such as brackets [ ], parentheses ( ) and braces { }, perform the operation within the grouping symbol before performing other operations.
- A mathematical sentence which uses the symbol "=" is called an **equation.**
- An equation has three parts: the equal sign, the expression to the right of the equal sign (=) or the right hand side of the equation and the expression to the left of the equal sign or the left hand side of the equation.
- In solving equations with only one variable the following principles of equality are used:

  Addition Rule: The same number must be added to both sides.

  Subtraction Rule: The same number must be subtracted from both sides.

  Multiplication Rule: Both sides must be multiplied by the same non-zero number.

  Division Rule: Both sides must be divided by the same non-zero number.
- A **formula** or a rule is an equation in which a particular variable is expressed in terms of other variables.

REVISE LESSON 1 - 6 THEN DO SUB-STRAND TEST 4 IN ASSIGNMENT 6

ANSWERS TO PRACTICE EXERCISES 19 - 24

Practice Exercise 19

1. a. x = {2, 4, 6, 8}
   b. y = {1, 3, 5, 7, 9, 11, 13}
   c. n = {6, 7, 8, 9 ,10, 11}
   d. a = {12, 15, 18, 24}
   e. P = {2}
   f. b = {12}
   g. z = {18}
   h. w = {7}.
   i. c = {13}

2.

a.	x	f.	k
b.	□	g.	x
c.	______ and y	h.	?
d.	m	i.	s
e.	s		

Practice Exercise 20

1.

a.	2b	f.	m + n
b.	wyz	g.	3x ÷ 2y or $\frac{3x}{2y}$
c.	m + 5	h.	15 – ab
d.	$\frac{d}{4}$	i.	$m^2 + n^2$
e.	k – 9	j.	(x + y)(x – y)

2. a. product of b and y
   b. three times a number x plus 6
   c. product of the numbers d, e and f
   d. three subtracted from a number z
   e. twice a number a plus three times a number b
   f. product of x and y divided by z
   g. twice the product of m and n
   h. twice a number x minus 8
   i. one-fifth of a number y plus 10
   j. the sum of the numbers x, y and z

3. a. iv
   b. v
   c. i
   d. ii
   e. iii

.

Practice Exercise 21

1. a. 18 f. 30
   b. 36 g. 21
   c. 25 h. 4
   d. 0 i. 42
   e. 2 j. 4

2. a. 16
   b. 72
   c. 28
   d. 32

3. a

a	0	1	2	3
b	0	4	8	12

b.

x	2	4	6	8
y	1	2	3	4

Practice Exercise 22A

1. (a) 5 (f) $18\frac{1}{2}$
   (b) 7 (g) $-\frac{11}{100}$
   (c) 16 (h) 40
   (d) 60 (i) 22
   (e) 3 (j) 36

Practice Exercise 22B

1. (a) 6 (b) 2 (c) 11 (d) 6 (e) 40

2. (a) 29 (b) 3 (c) 19 (d) 35 (e) 24

3. (a) 13 (f) 1
   (b) -5 (g) 2
   (c) 4 (h) 6
   (d) -6 (i) -30
   (e) 11 (j) 50

Practice Exercise 23

1. (a) -10
   (b) 22
   (c) 70
   (d) 8
   (e) 17
   (f) 10
   (g) $\frac{3}{32}$
   (h) $\frac{23}{4}$
   (i) -8
   (j) 36

2. (a) 3
   (b) 7
   (c) 3
   (d) -4
   (e) 136
   (f) 17
   (g) -8
   (h) 1

Practice Exercise 24

1. (a) False
   (b) True
   (c) True
   (d) False
   (e) False

2. (a) (D) Multiply each x value by itself.

   (b) (G) Add six to each x value.

3. (a) y = x - 7

x	7	9	12	8	15
y	**0**	**2**	**5**	**1**	**8**

   (b) y = 4x

x	5	4	3	2	1
y	**20**	**16**	**12**	**8**	**4**

END OF STRAND 6

REFERENCES

- NDOE (1995) Secondary School Mathematics 7A, Department of Education, PNG
- NDOE (1995) Secondary School Mathematics 7B, Department of Education, PNG
- Sue Gunningham and Pat Lilburn, **Mathematics 7A and 7B,** Outcome Edition, Oxford University Press, Australia and New Zealand
- Mcseveny, R. Conway and S. Wilkens, **New Signpost Mathematics 7,** Pearson Education Australia Pty Ltd., www. longman.com.au
- Thompson, and E. Wrightson, revised by S. Tisdell, **Developmental Mathematics Book 1 and 2,** McGraw Hill Fourth Edition
- Antonio Coronel, P. Manalastas and Jose Marasigan, **Mathematics 1 An Integrated Approach,** Bookmark Inc. Makati, Manila, Philippines
- **Mathematics 1 for First Year High School, Textbook and Teachers Manual**, SEDP Series DECS Republic of the Philippines
- Estrellita I. Misa and Bernardino Q. Li, **Moving Ahead with Mathematics,** Mathematics Textbook for First Year High School, Public School Edition, FNB Educational Inc. Quezon City, Philippines
- K. Swan, R. Adamson and G.Asp, **Nelson Maths 7**
- Paul Barns and Chris Lynagh, **Maths for Qld 1,** Longman Outcomes, Pearson Education Australia